

7. The Sides Of A Rectangle Are In The Ratio 5:2. If The Perimeter Of The Rectangle Is 42 Cm, Find The

7. The Sides Of A Rectangle Are In The Ratio 5:2. If The Perimeter Of The Rectangle Is 42 Cm, Find The

Introduction

Understanding geometric concepts such as the properties of rectangles is essential in mathematics, particularly in solving real-world problems involving measurements and ratios. In this article, we'll explore a specific problem involving rectangles where the sides are in a ratio of 5:2, and the perimeter is given as 42 centimeters. The goal is to find the lengths of the sides based on these conditions. By breaking down the problem step-by-step, applying algebraic methods, and understanding the concepts involved, readers can develop a clear approach to solving similar problems in geometry.

Understanding the Problem

Before diving into the calculations, it's important to clearly understand what the problem states:

- The rectangle's sides are in a ratio of 5:2.
- The perimeter of the rectangle is 42 centimeters.
- The task is to find the lengths of the sides.

Key Concepts Involved

- Ratio of sides: The ratio between the length and width of the rectangle.
- Perimeter of a rectangle: The total distance around the rectangle, which is calculated as 2 times the sum of length and width.
- Algebraic representation: Using variables to represent the sides and ratios for setting up equations.

Step-by-Step Approach to Solving the Problem

To find the sides of the rectangle, follow these steps:

Step 1: Define Variables

Let's denote:

- L as the length of the rectangle.
- W as the width of the rectangle.

Since the sides are in the ratio 5:2, we can express:

- $L = 5x$
- $W = 2x$

where x is the common multiplying factor.

Step 2: Write the Perimeter Equation

The perimeter P of a rectangle is given by:

$$P = 2(L + W)$$

Given that the perimeter is 42 cm, substitute the expressions for L and W :

$$42 = 2(5x + 2x)$$

Simplify inside the parentheses:

$$42 = 2(7x)$$

Which simplifies to:

$$42 = 14x$$

Step 3: Solve for the Common Factor x

Divide both sides of the equation by 14:

$$x = \frac{42}{14} = 3$$

Step 4: Calculate the Actual Length and Width

Now, substitute $x = 3$ into the expressions for L and W :

- $L = 5x = 5 \times 3 = 15 \text{ cm}$
- $W = 2x = 2 \times 3 = 6 \text{ cm}$

Final Answer:

- Length = 15 cm
- Width = 6 cm

Additional Insights and Related Concepts

Understanding Ratios in Geometry

Ratios are fundamental in comparing quantities, especially when dealing with similar figures or proportional relationships. In this problem, the sides' ratio helps set up an algebraic framework to solve for actual lengths.

Significance of Perimeter in Geometry

The perimeter provides a measure of the boundary length of a shape, which is crucial in various applications such as fencing, framing, or material estimation.

Applications of the Problem

- Design and Architecture: Calculating dimensions based on ratios and total boundary length.
- Manufacturing: Determining material dimensions for rectangular components.
- Education: Teaching students to translate word problems into algebraic equations.

Common Mistakes to Avoid

When solving similar problems, keep in mind:

- Incorrectly setting up the ratio: Ensure the ratio is expressed correctly in terms of a common variable.
- Miscalculating the perimeter: Remember that the perimeter formula involves doubling the sum of the sides.
- Forgetting to simplify: Always simplify equations step-by-step to avoid errors.
- Ignoring units: Maintain consistent units throughout calculations.

Practice Problems

To reinforce understanding, try solving these problems:

1. If the sides of a rectangle are in the ratio 3:4 and the perimeter is 56 cm, find the lengths of the sides.
2. A rectangle has sides in the ratio 7:3, and its perimeter is 50 cm. Determine the side lengths.
3. The length of a rectangle is twice its width, and its perimeter is 48 cm. Find the length and width.

Tips for Solving Ratio and Perimeter Problems

- Always start by defining variables for the sides.
- Express the sides in terms of a common variable based on the given ratio.
- Write the perimeter equation carefully and substitute the expressed sides.
- Simplify step-by-step to isolate the variable.
- Verify your answer by plugging back into the perimeter formula.

Conclusion

This problem exemplifies how ratios and basic algebra can be combined to solve geometric problems involving rectangles. By understanding the relationships between sides and perimeter, and translating the problem into algebraic equations, solutions become more straightforward. Remember to carefully define variables, set up your equations correctly, and solve systematically. With practice, these types of problems will become more intuitive, enhancing both your algebraic and geometric problem-solving skills.

Summary

Step	Description	Example/Formula
1	Define variables for sides based on ratio	$L = 5x, W = 2x$
2	Write perimeter equation	$P = 2(L + W)$
3	Substitute expressions and solve for x	$42 = 14x$
		$\Rightarrow x = 3$
4	Calculate actual side lengths	$L = 15 \text{ cm}, W = 6 \text{ cm}$

Final Remarks

Mastering problems involving ratios and perimeters equips students with critical problem-solving skills applicable in many areas of mathematics and real-world applications. By following structured approaches and practicing different scenarios, learners can develop confidence and competence in geometry.

Disclaimer: This article is for educational purposes and aims to provide a comprehensive understanding of solving ratio-based rectangle perimeter problems.

Frequently Asked Questions

What are the lengths of the sides of a rectangle with a ratio of 5:2 if its perimeter is 42 cm?

Let the sides be $5x$ and $2x$. Perimeter $P = 2(5x + 2x) = 14x$. Given $P = 42$ cm, so $14x = 42$, which gives $x = 3$. Therefore, sides are $5 \times 3 = 15$ cm and $2 \times 3 = 6$ cm.

How do you find the individual side lengths of a rectangle when the sides are in a 5:2 ratio and the perimeter is known?

Set the sides as $5x$ and $2x$. Use the perimeter formula $P = 2(5x + 2x)$. Substitute the perimeter value, solve for x , then multiply to find each side length.

If the perimeter of a rectangle is 42 cm and the sides are in the ratio 5:2, what are the dimensions of the rectangle?

The sides are 15 cm and 6 cm. This is found by setting sides as $5x$ and $2x$, then solving $14x = 42$, which yields $x = 3$, leading to sides 15 cm and 6 cm.

Can the sides of a rectangle in ratio 5:2 with a perimeter of 42 cm be fractional? Why or why not?

Yes, they can be fractional if the ratio or perimeter is fractional. However, in this case, since the ratio and perimeter are whole numbers, the sides are whole numbers (15 cm and 6 cm).

What is the importance of understanding ratios and perimeter in solving rectangle problems like this?

Understanding ratios helps determine the relative lengths of sides, and knowing the perimeter allows for calculating actual side lengths, making it easier to solve geometric problems efficiently.

Additional Resources

The Sides Of A Rectangle Are In The Ratio 5:2. If The Perimeter Of The Rectangle Is 42 Cm, Find The

Understanding how to analyze geometric figures such as rectangles, especially when given specific ratios and measurements, is fundamental in mathematics. This problem involves working with ratios, perimeters, and algebraic expressions to find unknown side lengths of a rectangle. In this comprehensive guide, we will explore the step-by-step process to solve such problems, focusing on the key concepts involved and illustrating the methodology with clarity. Whether you're a student preparing for exams or someone interested in sharpening your problem-solving skills, this article will serve as an invaluable resource.

Introduction: The Importance of Ratios and Perimeter in Geometry

Rectangles are one of the most basic and widely studied shapes in geometry. They have four sides with opposite sides equal and four right angles. Understanding the relationships between their sides and properties like perimeter and area is essential for solving real-world problems involving design, construction, and various engineering applications.

This particular problem introduces two core concepts:

- Side ratios: The relationship between the lengths of sides in a given ratio.
- Perimeter calculation: The total length around the rectangle, given as 42 cm in this case.

The key to solving such problems lies in translating the given information into algebraic expressions, forming equations, and then solving those equations systematically.

Step 1: Understanding the Given Data and What Is Required

The problem states:

- The sides of a rectangle are in the ratio 5:2.
- The perimeter of the rectangle is 42 cm.
- Find the lengths of the sides.

From this, it is clear that:

- The sides are proportional, with one side being 5 parts and the other being 2 parts.
- The total perimeter is known, which is the sum of all sides.

Step 2: Assigning Variables to the Sides

To translate the ratio into algebraic expressions, we define:

- Let x be the common multiplier for the ratio.
- Therefore, the length of the longer side is $5x$.
- The length of the shorter side is $2x$.

This approach simplifies calculations by representing sides as multiples of a common factor.

Step 3: Expressing the Perimeter in Terms of Variables

The perimeter (P) of a rectangle is calculated as:

$$P = 2 \times (\text{length} + \text{breadth})$$

Substituting the algebraic expressions:

$$42 = 2 \times (5x + 2x)$$

Simplify inside the brackets:

$$42 = 2 \times (7x)$$

Further simplifying:

$$42 = 14x$$

This equation relates the total perimeter to the common multiplier x .

Step 4: Solving for the Common Multiplier

To find x , divide both sides of the equation by 14:

$$x = \frac{42}{14}$$

$$x = 3$$

This provides the value of x , which allows us to find the actual lengths of the sides.

Step 5: Calculating the Actual Side Lengths

Now, substitute $x = 3$ into the earlier expressions:

- Longer side: $(5x = 5 \times 3 = 15)$ cm

- Shorter side: $(2x = 2 \times 3 = 6)$ cm

Therefore, the sides of the rectangle are 15 cm and 6 cm.

Step 6: Verifying the Solution

Always verify your solution by checking whether the perimeter matches the given value:

$$[P = 2 \times (15 + 6) = 2 \times 21 = 42 \text{ cm}]$$

Since this matches the given perimeter, the solution is correct.

Additional Insights and Variations

While this problem is straightforward, similar problems may involve different ratios, perimeters, or even areas. Here are some variations and tips:

1. When Given Area Instead of Perimeter

- Use the ratio to express sides as algebraic expressions.
- Set up equations for area: $(\text{Area} = \text{length} \times \text{breadth})$.
- Solve the resulting quadratic equation if necessary.

2. When the Ratio Involves Different Values

- Always assign variables based on the ratio.
- Remember that ratios are proportional, so use the common multiplier to express actual lengths.

3. Dealing with Complex Ratios

- For ratios involving more than two sides or more complex relationships, break down the ratio into multiple variables.
- Use simultaneous equations to solve for each variable.

Practical Applications of Such Problems

Problems involving ratios and perimeters are not just academic exercises—they have real-world applications such as:

- Designing rectangular plots of land with specific proportional dimensions.
- Manufacturing products where components must fit within certain ratio specifications.

- Architectural planning where proportional relationships affect structural design.

Summary and Key Takeaways

- Assign variables based on the ratio to express side lengths algebraically.
- Use the perimeter formula to form an equation relating the sides.
- Solve for the common multiplier to find actual side lengths.
- Verify your results against the given data to ensure accuracy.
- Practice with different ratios and given measurements to strengthen problem-solving skills.

Final Thoughts

Understanding the relationship between ratios and perimeter is crucial for solving many geometric problems efficiently. By methodically translating ratios into algebraic expressions and applying basic formulas, you can tackle a wide range of questions confidently. Remember, practice makes perfect—so work through various problems to develop your mathematical intuition and precision.

In conclusion, the sides of a rectangle in the ratio 5:2 with a perimeter of 42 cm are 15 cm and 6 cm, respectively. This problem exemplifies how algebraic techniques can simplify complex ratio-based questions, turning them into straightforward calculations. Keep practicing these methods to confidently handle similar geometric challenges in your studies and real-life applications.

7 The Sides Of A Rectangle Are In The Ratio 5 2 If The Perimeter Of The Rectangle Is 42 Cm Find The

7 The Sides Of A Rectangle Are In The Ratio 5 2 If The Perimeter Of The Rectangle Is 42 Cm Find The

Related Articles

- [A Coin Is Tossed Three Times. An Outcome Is Represented By A String Of The Sort HTT \(meaning A Head On](#)
- [3. What Is The Distance Around One Of The Semicircles At Either End Of The Soccer Field? HINT: Youneed](#)
- [A Circle Has A Circumference Of 18. It Has An Arc Of Length 6.What Is The Central Angle Of](#)

[The Arc, In](#)

[Back to Home](#)