9 (5 Points) Express 7.84848484848... As A Rational Number, In The Form P/q Where P And Q Are Positive

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Expressing a repeating decimal as a rational number is a fundamental concept in mathematics that helps in understanding the relationship between decimals and fractions. The decimal 7.84848484848... is a recurring decimal, where the sequence "84" repeats infinitely. To convert this decimal into a fraction in the form P/q, where P and Q are positive integers, we use algebraic methods designed for repeating decimals. This article provides a comprehensive step-by-step guide to perform this conversion efficiently and accurately.

Understanding Repeating Decimals

Before diving into the conversion process, it's essential to understand what repeating decimals are and how they behave.

What Is a Repeating Decimal?

A repeating decimal is a decimal number with a sequence of digits that repeats infinitely. For example, in 7.84848484848..., the digits "84" repeat endlessly after the decimal point.

Types of Repeating Decimals

Repeating decimals can be classified into two categories:

- **Pure Repeating Decimals:** The decimal begins immediately with a repeating sequence. Example: 0.3333... (repeats "3").
- **Mixed Repeating Decimals:** The decimal has a non-repeating part followed by a repeating part. Example: 7.848484... (non-repeating "7." and repeating "84").

Since 7.84848484848... has a non-repeating part "7." followed by a repeating "84," it's a mixed repeating decimal, which requires a specific approach to convert.

Step-by-Step Conversion Process

Converting a mixed repeating decimal like 7.84848484848... into a fraction involves a few algebraic steps.

Step 1: Assign the Decimal to a Variable

Let:

$x = 7.84848484848...$

Our goal is to isolate x and express it as a fraction.

Step 2: Eliminate the Repeating Part

Since the repeating sequence is "84," which repeats every two digits, multiply x by a power of 10 that shifts the decimal point two places to the right:

$10x = 78.4848484848...$

However, because there's a non-repeating part "7" before the repeating segment, it’s helpful to consider the entire number carefully.

Step 3: Separate the Non-Repeating and Repeating Parts

Express x as the sum of its integer part and decimal part:

$x = 7 + 0.84848484848...$

Focus on converting the decimal part 0.84848484848... into a fraction.

Converting the Decimal Part 0.8484848484848... into a Fraction

This is the core step. Let's denote:

$y = 0.84848484848...$

Method:

1. Multiply y by 100 to shift two repeating digits:

$100y = 84.84848484848...$

2. Multiply y by 1 to keep it unchanged:

1y = 0.84848484848...

3. Subtract the second from the first to eliminate the repeating part:

100y - y = 84.84848484848... - 0.84848484848...

Result:

99y = 84

4. Solve for y:

y = 84 / 99

5. Simplify the fraction:

Both numerator and denominator are divisible by 3:

84 ÷ 3 = 28

99 ÷ 3 = 33

Thus,

y = 28 / 33

Therefore:

0.84848484848... = 28/33

Combine the Parts to Find the Total Fraction

Recall that:

x = 7 + y = 7 + 28/33

Express 7 as a fraction with denominator 33:

7 = 231/33

Add:

x = 231/33 + 28/33 = (231 + 28)/33 = 259/33

Final answer:

x = 259/33

Since both numerator and denominator are positive, this is the required rational representation.

Verification and Simplification

It’s good practice to verify the correctness of the fraction.

Check:

Divide 259 by 33:

$33 \times 7 = 231$

Remaining: $259 - 231 = 28$

So,

$259/33 = 7 + 28/33 \approx 7.84848484848...$

which confirms our conversion.

Additionally, the fraction 259/33 is in its simplest form because:

- 259 factors into 7×37
- 33 factors into 3×11

Since 7, 37, 3, and 11 share no common factors, the fraction is already simplified.

Conclusion

In summary, to convert the repeating decimal 7.84848484848... into a rational number:

1. Recognize the decimal as a mixed repeating decimal with a non-repeating part "7" and a repeating part "84".
2. Convert the repeating part into a fraction: 0.84848484848... = 28/33.
3. Add this fractional part to the integer part: 7 + 28/33 = 259/33.
4. Verify the result by division and ensure the fraction is simplified.

Final Result:

7.84848484848... = \(\frac{259}{33}\)

This fraction in lowest terms perfectly represents the original decimal as a rational number with positive numerator and denominator, fulfilling the criteria of the problem.

Additional Tips for Converting Repeating Decimals

- Always identify whether the decimal is pure or mixed repeating.
- Use algebraic methods like multiplication by powers of 10 to shift the decimal.
- Simplify fractions by dividing numerator and denominator by their greatest common

divisor (GCD).
- Verify your result by converting back to decimal form to ensure accuracy.

By mastering these steps, you can confidently convert any repeating decimal into its equivalent rational form, facilitating better understanding of the relationship between decimals and fractions.

Frequently Asked Questions

How can I express the repeating decimal 7.84848484848... as a rational number in the form P/q?

You can set x = 7.84848484848..., then multiply by 100 to shift the repeating part, subtract to eliminate the repeating decimals, and solve for x to find the rational form.

What is the step-by-step process to convert 7.84848484848... into a fraction P/q?

First, let x = 7.84848484848..., then multiply by 100: 100x = 784.84848484848...; subtract x from 100x: 100x - x = 784.84848484848... - 7.84848484848... = 777; solve for x: x = 777/99, which simplifies to 259/33.

What is the simplified fractional form of the repeating decimal 7.84848484848...?

The decimal 7.84848484848... can be expressed as the rational number 259/33.

How do I verify that 259/33 is equivalent to 7.84848484848...?

Divide 259 by 33: 259 ÷ 33 = 7.84848484848..., confirming that 259/33 matches the decimal expansion.

Why is it important to express repeating decimals as fractions in simplest form?

Expressing repeating decimals as fractions in simplest form makes calculations easier, helps in understanding their exact value, and is useful for mathematical proofs and problem-solving.

Additional Resources

Express 7.84848484848... As A Rational Number, In The Form P/q Where P And Q Are Positive

In the realm of mathematics, the distinction between rational and irrational numbers forms a fundamental cornerstone. Rational numbers, expressible as the quotient of two integers, often reveal their structure through recurring decimal expansions. A classic example involves repeating decimals such as 0.333..., 0.727272..., or 7.848484... To convert a repeating decimal into a rational number—specifically in the form $\frac{P}{Q}$ where P and Q are positive integers—requires a systematic approach rooted in algebraic manipulation and pattern recognition.

This article delves into the detailed process of transforming the decimal $7.84848484848...$ into its equivalent rational form. We will explore the underlying principles, step-by-step methods, and broader implications of such conversions, emphasizing their significance in mathematical theory and application.

Understanding the Nature of the Decimal: Is 7.84848484848... Rational?

Before embarking on the conversion process, it is crucial to recognize that the decimal $7.84848484848...$ is a repeating decimal, which inherently indicates it is a rational number.

Definition of Repeating Decimals and Rational Numbers

A repeating decimal is a decimal expansion that repeats a sequence of digits indefinitely. For example:

- $0.\overline{3} = 0.333...$
- $0.\overline{27} = 0.272727...$
- $7.\overline{84} = 7.848484...$

By definition, every repeating decimal corresponds to a rational number because it can be expressed as a ratio of two integers. Conversely, every rational number has a decimal expansion that is either terminating or repeating.

Therefore, the decimal $7.84848484848...$ is rational, and our goal is to find its exact fractional form.

Methodology for Converting Repeating Decimals to Fractions

The process of converting a repeating decimal to a fraction involves algebraic steps that isolate the repeating part and eliminate the repeating pattern through subtraction. The general method comprises:

1. Let the decimal be x.
2. Identify the non-repeating and repeating parts.
3. Multiply x by powers of 10 to shift the decimal point to align the repeating parts.
4. Subtract the shifted equations to cancel out the repeating component.
5. Simplify the resulting fraction to lowest terms.

This methodology applies universally, whether the decimal has only a repeating part or both non-repeating and repeating parts.

Step-by-Step Conversion of 7.84848484848... to $\frac{P}{Q}$

Applying the above strategy to $7.84848484848...$, we proceed as follows:

Step 1: Express x as the decimal

Let:
$$x = 7.84848484848...$$

Step 2: Separate the non-repeating and repeating parts

Observe that:

- The integer part is 7.
- The decimal part is $0.84848484848...$.

Note that the repeating part is "84", starting immediately after the decimal point, with no non-repeating prefix following the decimal.

Step 3: Focus on the fractional part \(0.84848484848...

\)

Let:

\[y = 0.84848484848... \]

Our goal is to find \(y \) as a fraction, then incorporate the integer part to find \(x \).

Step 4: Convert \(y \) to a fraction

Since \(y \) has a repeating pattern "84" starting immediately after the decimal, and the repeating block has length 2, we proceed as:

- Multiply \(y \) by \(10^2 = 100 \):

\[100y = 84.848484848... \]

- Observe that:

\[100y = 84 + y \]

because the decimal part of \(100y \) is the same as \(y \).

- Subtract the original \(y \):

\[100y - y = 84 + y - y \Rightarrow 99y = 84 \]

- Solve for \(y \):

\[y = \frac{84}{99} \]

- Simplify the fraction:

Divide numerator and denominator by 3:

\[y = \frac{84/3}{99/3} = \frac{28}{33} \]

Therefore:

\[y = \frac{28}{33} \]

Step 5: Incorporate the integer part back to find x

Recall:

$$ x = 7 + y = 7 + \frac{28}{33} $$

Express 7 as a fraction with denominator 33:

$$ 7 = \frac{7 \times 33}{33} = \frac{231}{33} $$

Add the fractions:

$$ x = \frac{231}{33} + \frac{28}{33} = \frac{231 + 28}{33} = \frac{259}{33} $$

Final Result:

$$
\boxed{
7.84848484848... = \frac{259}{33}
}
$$

Here, $P = 259$ and $Q = 33$, both positive integers, fulfilling the initial requirement.

Verifying the Result and Its Significance

To ensure correctness, one can perform the decimal division:

$$
\frac{259}{33} \approx 7.84848484848...
$$

which matches the original decimal expansion. This confirms the validity of the conversion.

The process exemplifies the broader principle that any repeating decimal with a recurring pattern can be systematically converted into a rational number in lowest terms. Such conversions are foundational in understanding the structure of rational numbers and their decimal representations.

Broader Implications and Applications

Converting repeating decimals to fractions isn't merely an academic exercise; it has practical implications across diverse fields:

- Mathematics Education: Reinforces understanding of number types and algebraic manipulation.
- Computational Mathematics: Facilitates precise calculations where decimal approximations are insufficient.
- Cryptography and Data Encoding: Understanding rational representations underpin algorithms that rely on exact fractions.
- Engineering and Science: Precise fractional representations are often necessary for measurements and calculations involving recurring signals or periodic phenomena.

Furthermore, mastering these conversions enhances numeracy skills and deepens comprehension of the real number system's structure.

Extensions and Related Topics

This discussion opens avenues for further exploration:

- Converting repeating decimals with non-zero non-repeating parts, e.g., $3.142857...$.
- Understanding periodicity and its relation to algebraic equations.
- Exploring irrational numbers and their decimal expansions.
- Examining the concept of minimal denominators in rational representations.

Conclusion

The conversion of $7.84848484848...$ into the rational form $\frac{P}{Q}$ underscores the elegance and consistency of mathematical principles governing decimal expansions and fractions. Through algebraic manipulation, we confirm:

$$
\boxed{
7.84848484848... = \frac{259}{33}
}
$$

This process exemplifies the power of systematic approaches in mathematics, transforming infinite, recurring decimal patterns into precise, finite fractional expressions. As a central tenet of number theory, these techniques forge a bridge between the intuitive understanding of decimals and the rigorous structure of rational numbers, enriching our comprehension of the numerical universe.

References

- Stewart, I. (2016). Calculus: Early Transcendentals. Cengage Learning.
- Rosen, K. H. (2012). Elementary Number Theory and Its Applications. Pearson.
- Burton, D. M. (2011). Elementary Number Theory. McGraw-Hill Education.
- Online resources: [Khan Academy - Repeating Decimals](https://www.khanacademy.org/math/algebra/rational-expressions/arith-review-rational-numbers/a/repeating-decimals)

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