9) A 6 Ft Tall Man Walks At The Rate Of 5 Ft/sec Toward A Street Light That Is 16 Ft Above The Ground.

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Introduction

Understanding the behavior of shadows and light in relation to moving objects is a classic problem in physics and calculus. When a man walks towards a street light, his shadow length changes dynamically as he approaches the light source. This problem is a perfect illustration of related rates, a concept in differential calculus that involves understanding how different quantities change with respect to time. In this article, we will analyze a scenario where a 6-foot tall man walks toward a street light that stands 16 feet above the ground, moving at a constant speed of 5 feet per second. We will explore how to determine the length of his shadow at any given moment, how the shadow length varies as he approaches the streetlight, and what the rates of change are under different circumstances.

Problem Breakdown and Assumptions

Given Data

- Height of the man, $h_m = 6$ ft
- Height of the street light, $H = 16$ ft
- Man's walking speed toward the street light, $v_m = 5$ ft/sec
- Initial distance of the man from the street light, x_0 (assumed or specified)

Assumptions

1. The street light is stationary and emits light uniformly downward and outward.
2. The man walks directly towards the street light along a straight line.
3. The shadow is formed on the ground behind the man, extending away from the street light.
4. Air resistance and other external forces are negligible.
5. All measurements are in feet, and time is in seconds.

Setting Up the Geometry

Coordinate System and Variables

To analyze the problem, we establish a coordinate system:

- Place the street light at the origin, $(0, 0)$.
- The ground is the x-axis, with the street light at $(0, 0)$.
- The man walks along the positive x-axis toward the light; thus, his position at time t is $x(t)$.

Since the man walks toward the street light at 5 ft/sec, his position at time t is:

$$x(t) = x_0 - v_m t$$

where x_0 is his initial distance from the street light.

The height of the street light is fixed at $H = 16$ ft, and the man’s height is $h_m = 6$ ft.

Deriving the Shadow Length

Understanding the Shadow Formation

The shadow of the man is formed on the ground behind him, extending away from the street light. To find the length of the shadow, consider the similar triangles formed by:

- The street light, its tip, and the ground.
- The man, his shadow tip, and the ground.

Key Variables

- $s(t)$: the length of the man's shadow at time t.
- $x(t)$: the man's position relative to the street light.
- $S(t)$: the position of the tip of the shadow on the ground.

Since the man is at $x(t)$, the tip of his shadow on the ground is at:

$$S(t) = x(t) + s(t)$$

Applying Similar Triangles

Formulating the Relationship

The two similar triangles are:

1. The triangle formed by the street light, its top, and the ground point directly below its top.
2. The triangle formed by the man, his head, and the tip of his shadow.

The ratios of corresponding sides give:

$$
\frac{\text{height of street light}}{\text{distance from street light to shadow tip}} = \frac{\text{height of man}}{\text{distance from man to shadow tip}}
$$

Expressed mathematically:

$$
\frac{16}{S(t)} = \frac{6}{S(t) - x(t)}
$$

Note that:

- $S(t)$ is the distance from the street light to the shadow tip.
- $x(t)$ is the position of the man relative to the street light.

Rearranged, the shadow length $s(t)$ is:

$$
s(t) = S(t) - x(t)
$$

From the proportion:

$$
16 (S(t) - x(t)) = 6 S(t)
$$

Simplify:

$$
16 S(t) - 16 x(t) = 6 S(t)
$$

$$
(16 S(t) - 6 S(t)) = 16 x(t)
$$

$$
10 S(t) = 16 x(t)
$$

$$
S(t) = \frac{16}{10} x(t) = \frac{8}{5} x(t)
$$

Now, the shadow length $s(t)$:

$$
s(t) = S(t) - x(t) = \frac{8}{5} x(t) - x(t) = \left(\frac{8}{5} - 1 \right) x(t) = \frac{3}{5} x(t)
$$

Expressing Shadow Length Over Time

Since $x(t) = x_0 - v_m t$, the shadow length:

$$
s(t) = \frac{3}{5} \left(x_0 - v_m t \right)
$$

This formula allows us to compute the shadow length at any time t.

Calculating Rates of Change

Rate of Shadow Length Change

Differentiate $s(t)$ with respect to time:

$$
\frac{d s}{dt} = \frac{3}{5} \frac{d}{dt} (x_0 - v_m t) = - \frac{3}{5} v_m
$$

Given $v_m = 5$ ft/sec:

$$
\frac{d s}{dt} = - \frac{3}{5} \times 5 = -3 \text{ ft/sec}
$$

Interpretation: The shadow length decreases at a rate of 3 ft/sec as the man approaches the street light.

Rate of Change of Shadow as the Man Approaches

At a specific moment when the man is at position $x(t)$, the rate at which his shadow length changes is constant at -3 ft/sec, regardless of his position, because the rate of change depends solely on his speed and the proportionality constant.

Practical Examples and Applications

Example 1: Initial Shadow Length

Suppose the man starts 50 ft away from the street light ($x_0 = 50$ ft). The initial shadow length:

\[

$$
s(0) = \frac{3}{5} \times 50 = 30 \text{ ft}
$$

He walks toward the street light at 5 ft/sec. After 4 seconds:

$$
x(4) = 50 - 5 \times 4 = 30 \text{ ft}
$$

Corresponding shadow length:

$$
s(4) = \frac{3}{5} \times 30 = 18 \text{ ft}
$$

Example 2: Shadow Length Near the Street Light

When the man is very close to the street light, say, $x(t) = 5$ ft:

$$
s(t) = \frac{3}{5} \times 5 = 3 \text{ ft}
$$

The shadow is relatively short, illustrating how shadows diminish as the man approaches the lamp.

Extending the Model: Variations and Additional Considerations

Different Heights or Speeds

- If the streetlight height changes, the ratios and formulas would adapt accordingly.
- If the man walks at a different speed, the derivative and rate of shadow change would scale proportionally.

Dynamic Scenarios

- In real-world applications, factors such as the angle of the sun, multiple light sources, or

obstacles could influence shadow behavior.
- For more complex models, consider the sun’s position or multiple objects casting shadows.

Applications in Real Life

- Urban planning and street lighting design.
- Safety measures based on visibility and shadow coverage.
- Animation and computer graphics involving dynamic shadows.

Conclusion

Analyzing the scenario where a man walks toward a streetlight involves understanding the principles of similar triangles, related rates, and differential calculus. By establishing the geometric relationships and applying calculus, we determined that the shadow length decreases at a constant rate of 3 ft/sec as the man approaches the streetlight at 5 ft/sec. The key takeaways include:

- The shadow

Frequently Asked Questions

How long does it take for the 6 ft tall man to reach the street light if he is 20 ft away initially?

Since he walks at 5 ft/sec and is initially 20 ft away, time taken = distance / speed = 20 ft / 5 ft/sec = 4 seconds.

What is the length of the shadow cast by the man when he is a certain distance from the streetlight?

The shadow length depends on similar triangles formed by the streetlight, the man, and their shadows. Using proportionality, shadow length = (height of streetlight / height of man) × distance from the streetlight.

How does the shadow length change as the man approaches the street light?

As the man gets closer to the street light, his shadow length decreases because the ratio of the streetlight's height to the distance between the man and the streetlight decreases.

At what distance from the streetlight is the man's shadow exactly twice his height?

Let x be the distance from the man to the streetlight when his shadow equals twice his height (which is 12 ft). Using similar triangles, 12 / (x) = 16 / (x + shadow length). Solving for x gives the specific distance where shadow length is 12 ft.

Can we determine the rate of change of the shadow length as the man walks toward the streetlight?

Yes, by differentiating the shadow length with respect to time using related rates, considering the man's speed and the geometry of the situation, we can find how quickly the shadow length changes at any given moment.

Additional Resources

A 6 Ft Tall Man Walks At The Rate Of 5 Ft/sec Toward A Street Light That Is 16 Ft Above The Ground

Understanding the dynamics of a tall individual walking toward a street light involves a fascinating blend of physics, perception, and real-world application. This scenario, while seemingly straightforward, offers rich insights into concepts such as shadow length, angles of elevation, and motion analysis. In this comprehensive review, we will explore the scenario of a 6-foot tall man walking at a steady pace of 5 feet per second toward a street light that is elevated 16 feet above the ground. We will analyze the problem from multiple perspectives, including physics, mathematics, and practical implications, providing a detailed breakdown suitable for students, educators, engineers, or enthusiasts interested in motion and light behavior.

Understanding the Scenario: Basic Description and Context

The core of this scenario involves a man walking toward a fixed light source, which is a street lamp positioned 16 feet above ground level. The man’s height is 6 feet, and he proceeds at a consistent velocity of 5 feet per second.

Key elements include:
- The man’s height: 6 ft
- Walking speed: 5 ft/sec
- Street light height: 16 ft
- Direction: Toward the streetlight
- Objective: To analyze how the shadow length changes as he approaches the streetlight.

This setup is commonplace in physics problems involving similar triangles, shadow projection, and angular measurements. It combines the concepts of linear motion with geometric optics and provides a platform to understand how shadows evolve as the object (the man) moves closer to the light source.

Mathematical Foundations and Derivations

Establishing the Geometry

The key to analyzing this scenario lies in understanding the geometric relationships among the streetlight, the man, and his shadow.

- When the man casts a shadow, a right triangle is formed with:
- The streetlight at the apex (top of the vertical pole).
- The tip of the shadow on the ground.
- The point where the man is standing.

- The height of the streetlight (16 ft) and the height of the man (6 ft) influence the angles and lengths involved.

Suppose:
- $x(t)$ is the distance from the man to the streetlight at time t.
- $s(t)$ is the length of the man’s shadow at time t.

When the man is at a distance $x(t)$, the total length from the streetlight to the tip of the shadow is $x(t) + s(t)$.

Using similar triangles:

$$
\frac{\text{height of streetlight}}{\text{total shadow length}} = \frac{\text{man's height}}{\text{shadow length}}
$$

Expressed as:

$$
\frac{16}{x(t) + s(t)} = \frac{6}{s(t)}
$$

From this, solving for $s(t)$:

$$
16 s(t) = 6 (x(t) + s(t))
$$

$$
16 s(t) = 6 x(t) + 6 s(t)
$$
$$
(16 - 6) s(t) = 6 x(t)
$$
$$
10 s(t) = 6 x(t)
$$
$$
s(t) = \frac{3}{5} x(t)
$$

This relation shows that the shadow length at any instant is proportional to his distance from the streetlight.

Relating Motion and Shadow Length

Since the man walks toward the streetlight at a constant speed:

$$
\frac{d x}{dt} = -5 \text{ ft/sec}
$$

(Note: The negative sign indicates that $x(t)$ decreases as he approaches the streetlight.)

Using the relation $s(t) = \frac{3}{5} x(t)$, differentiate both sides with respect to time:

$$
\frac{d s}{dt} = \frac{3}{5} \frac{d x}{dt}
$$

Substituting the value of $\frac{d x}{dt}$:

$$
\frac{d s}{dt} = \frac{3}{5} \times (-5) = -3 \text{ ft/sec}
$$

This indicates that the shadow length decreases at 3 ft/sec as the man approaches the streetlight.

Analysis of Shadow Dynamics

The derived relationships allow us to analyze how the shadow behaves over time, which has practical implications in visibility, safety, and even in understanding how light interacts with moving objects.

Key observations:
- The shadow shortens as the man gets closer.
- The rate of change of the shadow length (-3 ft/sec) is proportional to his approach velocity.
- When he is far from the light, his shadow is longer; as he gets closer, the shadow diminishes.

Practical Applications and Implications

Understanding the behavior of shadows in such scenarios is relevant in various fields:

Urban Planning and Street Lighting Design

Designers need to consider how tall objects cast shadows, especially during different times of the day or night. Accurate modeling ensures that shadows do not obstruct visibility or cause safety issues.

Pros:
- Helps optimize streetlight placement.
- Ensures pedestrians and drivers are not affected by excessive shadows.

Cons:
- Simplistic models assume perfect point sources; real lights have distribution patterns.
- Environmental factors like wind or obstacles are not considered.

Safety and Visibility

Knowing how shadows evolve can influence safety in pedestrian zones, especially for individuals walking at night or in poorly lit areas.

Features:
- Shadows can obscure hazards or pathway markings.
- Awareness of shadow behavior helps in designing better lighting schemes.

Physics Education and Demonstrations

This scenario is commonly used as a teaching example in physics classrooms to illustrate similar triangles, rate of change, and the application of derivatives.

Extensions and Advanced Topics

The basic analysis can be extended to include more complex considerations:

- Multiple light sources: How shadows interact when multiple lights are involved.
- Variable walking speeds: Analyzing scenarios where the man accelerates or decelerates.
- Different heights: Considering objects or persons of different heights.
- Realistic light sources: Incorporating non-point light sources with specific beam patterns.

Limitations and Assumptions

While the scenario provides valuable insights, it relies on several simplifying assumptions:

- The streetlight is modeled as a point source located at the top.
- The ground is perfectly flat and level.
- The man walks in a straight line directly toward the light.
- No external factors such as wind, obstacles, or non-uniform lighting.

These assumptions are necessary for mathematical tractability but may not fully represent real-world conditions.

Conclusion

The scenario of a 6-foot tall man walking at 5 ft/sec toward a street light 16 ft above ground is a classic example in physics illustrating the relationship between motion and light projection. Through geometric analysis, we find that the shadow length decreases proportionally as he approaches, with the shadow shrinking at a rate of 3 ft/sec. This problem exemplifies how similar triangles and derivatives can be combined to understand real-world phenomena. Its applications extend from urban planning to educational demonstrations, showcasing the importance of fundamental physics principles in everyday contexts. While simplified, this analysis offers a robust foundation for more complex studies and emphasizes the value of mathematical modeling in understanding the interplay of light, motion, and geometry.

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