9. Solve Each System Of Equations Using Elimination. You Must Show Your Work For Credit. (From Unit 2,

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Solving systems of equations is a fundamental skill in algebra that allows us to find the point(s) where two or more equations intersect. Among the various methods to solve systems, the elimination method is particularly efficient for systems where adding or subtracting equations can eliminate one variable, making it easier to solve for the remaining variables. In this guide, we will explore the step-by-step process of solving systems of equations using elimination, emphasizing clarity and thoroughness to ensure you understand each part of the process. Whether you're preparing for an exam or seeking to strengthen your algebraic skills, mastering elimination is essential. Remember, showing your work is crucial for credit and understanding, so every step will be explained in detail.

Understanding the Elimination Method

The elimination method involves manipulating the given system of equations to eliminate one variable, allowing you to solve for the other. This method is especially useful when the coefficients of one variable are opposites or can be easily made opposites through multiplication.

Key Steps in the Elimination Process

1. Arrange the equations in standard form: \(ax + by = c\).
2. Adjust the coefficients of one variable to be opposites, often by multiplying equations by suitable numbers.
3. Add or subtract the equations to eliminate one variable.
4. Solve the resulting single-variable equation.
5. Substitute the found value back into one of the original equations to find the other variable.
6. Check your solution by substituting the variables into both original equations.

Step-by-Step Example: Solving a System Using

Elimination

Let's consider an example to demonstrate the elimination method thoroughly.

Example System:

$$
\begin{cases}
3x + 4y = 10 \\
5x - 4y = 14
\end{cases}
$$

Step 1: Write equations in standard form.

Both equations are already in standard form:

- Equation 1: $3x + 4y = 10$
- Equation 2: $5x - 4y = 14$

Step 2: Identify coefficients for elimination.

Notice that the coefficients of y are $+4$ and -4. These are opposites, which means we can directly add the equations to eliminate y.

Step 3: Add equations to eliminate y.

Adding both equations:

$$
(3x + 4y) + (5x - 4y) = 10 + 14
$$

Simplify:

$$
3x + 5x + 4y - 4y = 24
$$

$$
8x = 24
$$

Step 4: Solve for x.

Divide both sides by 8:

$$
x = \frac{24}{8} = 3
$$

Step 5: Substitute $x = 3$ into one of the original equations to find y.

Using the first equation:

$$
3(3) + 4y = 10

\]

Simplify:

\[
9 + 4y = 10
\]

Subtract 9 from both sides:

\[
4y = 1
\]

Divide both sides by 4:

\[
y = \frac{1}{4}
\]

Step 6: Write the solution as an ordered pair.

\[
\boxed{(x, y) = \left(3, \frac{1}{4}\right)}
\]

Step 7: Verify the solution.

Substitute into the second original equation:

\[
5(3) - 4\left(\frac{1}{4}\right) = 14
\]

Calculate:

\[
15 - 1 = 14
\]

Since both equations are satisfied, the solution is correct.

Additional Examples of Solving Systems Using Elimination

Let's explore a few more examples, including cases where coefficients are not initially opposites and require multiplication.

Example 1: Coefficients Not Opposite

\[
\begin{cases}
2x + 3y = 7 \\
4x + 5y = 11

\end{cases}
\]

Step 1: Make the coefficients of \(x\) or \(y\) opposites. Let's eliminate \(x\). The coefficients are 2 and 4, so multiply the first equation by 2:

\[
(2x + 3y) \times 2 \Rightarrow 4x + 6y = 14
\]

Now, the system becomes:

\[
\begin{cases}
4x + 6y = 14 \\
4x + 5y = 11
\end{cases}
\]

Step 2: Subtract the second equation from the first:

\[
(4x + 6y) - (4x + 5y) = 14 - 11
\]

Simplify:

\[
4x - 4x + 6y - 5y = 3
\]

\[
0x + y = 3
\]

\[
y = 3
\]

Step 3: Substitute \(y = 3\) into one original equation:

Using \(2x + 3y = 7\):

\[
2x + 3(3) = 7
\]

\[
2x + 9 = 7
\]

Subtract 9:

\[
2x = -2
\]

Divide by 2:

\[
x = -1
\]

Solution:

\[
\boxed{(x, y) = (-1, 3)}
\]

Verification:

Plug into the second original equation:

\[
4(-1) + 5(3) = -4 + 15 = 11
\]

Correct!

Example 2: Both variables need to be eliminated

Suppose the system is:

\[
\begin{cases}
x + y = 5 \\
2x + 2y = 10
\end{cases}
\]

Notice the second equation is just twice the first, indicating infinitely many solutions or dependent equations.

Elimination approach:

Multiply the first equation by 2:

\[
2x + 2y = 10
\]

Compare with the second:

\[
2x + 2y = 10
\]

They're identical, which means the system has infinitely many solutions—every point on the line \(x + y = 5\).

Conclusion:

- The system is dependent, and the solution set is all points satisfying \(x + y = 5\).

Tips for Effectively Using the Elimination Method

To ensure accuracy and efficiency, keep these tips in mind:

- Always write the equations in standard form before starting.
- Look for coefficients that are already opposites to minimize work.
- If coefficients are not opposites, consider multiplying to align coefficients.
- Perform operations carefully to avoid sign errors.
- Check your solutions by substituting back into both original equations.
- Be aware of special cases: dependent systems (infinite solutions) or inconsistent systems (no solution).

Common Mistakes to Avoid

- Incorrect multiplication: Failing to multiply all terms correctly when adjusting coefficients.
- Sign errors: Paying attention to signs during addition or subtraction.
- Skipping verification: Always verify solutions in the original equations.
- Misidentifying the method: Remember that elimination is best when coefficients align for easy elimination.

Conclusion

The elimination method is a powerful and systematic approach to solving systems of equations. By carefully manipulating the equations to eliminate one variable, you can reduce the problem to a single-variable equation, making it straightforward to find solutions. Remember to show all your work clearly, justify each step, and verify your solutions to ensure correctness. Mastery of elimination enhances your algebraic skills and prepares you for more advanced topics involving systems of equations, such as matrices and linear programming. Practice with different systems to become confident in identifying the best approach and executing the elimination method efficiently.

Frequently Asked Questions

How do you set up the elimination method when solving

a system of equations?

First, align the equations and multiply one or both equations by a constant if needed to make the coefficients of one variable opposites. Then, add the equations to eliminate that variable, allowing you to solve for the remaining variable.

Solve the system: 3x + 2y = 7 and 5x − 2y = 3 using elimination. Show your work.

Add the two equations: $(3x + 2y) + (5x - 2y) = 7 + 3$
$(3x + 5x) + (2y - 2y) = 10$
$8x + 0 = 10$
$x = 10/8 = 5/4$
Now substitute $x = 5/4$ into the first equation: $3(5/4) + 2y = 7$
$(15/4) + 2y = 7$
$2y = 7 - 15/4 = (28/4) - (15/4) = 13/4$
$y = (13/4) \div 2 = (13/4)\ (1/2) = 13/8$
Solution: $x = 5/4$, $y = 13/8$.

What should you do if the coefficients of a variable are the same in both equations during elimination?

If the coefficients are the same (and the constants are different), adding the equations will eliminate the variable. If the coefficients are the same and constants are also the same, the equations are dependent and have infinitely many solutions; if constants are different, the system has no solution.

Solve the system: 2x + 3y = 8 and 4x − y = 10 using elimination. Show your work.

Multiply the second equation by 3 to align y coefficients: $3(4x - y) = 3(10)$
$\rightarrow 12x - 3y = 30$
Now, add this to the first equation: $(2x + 3y) + (12x - 3y) = 8 + 30$
$(2x + 12x) + (3y - 3y) = 38$
$14x = 38$
$x = 38/14 = 19/7$
Substitute into the second original equation: $4(19/7) - y = 10$
$(76/7) - y = 10$
$-y = 10 - 76/7 = (70/7) - (76/7) = -6/7$
$y = 6/7$
Solution: $x = 19/7$, $y = 6/7$.

When solving a system via elimination, what should you do if the variables do not cancel out after addition?

You should multiply one or both equations by a constant to make the coefficients of one variable opposites, so that adding the equations will eliminate that variable.

Solve the system: x + 2y = 5 and 3x + 4y = 11 using elimination. Show your work.

Multiply the first equation by −3 to match the x coefficient: $-3(x + 2y) = -3(5) \rightarrow -3x - 6y = -15$
Add this to the second equation: $3x + 4y + (-3x - 6y) = 11 + (-15)$
$(3x - 3x) + (4y - 6y) = -4$
$0x - 2y = -4$
$-y = -4/2 = -2$
$y = 2$
Substitute y = 2 into the first original equation: $x + 2(2) = 5$
$x + 4 = 5$
$x = 1$
Solution: $x = 1$, $y = 2$.

Explain why the elimination method is useful for solving systems of equations.

The elimination method simplifies solving systems by systematically eliminating one variable through addition or subtraction, making it easier to solve for the remaining variable, especially with linear equations.

Solve the system: 4x − y = 9 and 2x + y = 1 using elimination. Show your work.

Add the two equations: $(4x - y) + (2x + y) = 9 + 1$
$(4x + 2x) + (-y + y) = 10$
$6x + 0 = 10$
$x = 10/6 = 5/3$
Substitute x = 5/3 into the second equation: $2(5/3) + y = 1$
$(10/3) + y = 1$
$y = 1 - 10/3 = (3/3) - (10/3) = -7/3$
Solution: $x = 5/3$, $y = -7/3$.

What are common mistakes to avoid when using the elimination method?

Common mistakes include not multiplying equations properly to align coefficients, forgetting to change signs when adding, and algebraic errors during substitution. Always double-check coefficients and calculations.

Additional Resources

Solve Each System Of Equations Using Elimination is a fundamental skill in algebra that empowers students and professionals alike to find solutions to two-variable systems efficiently. Mastering this method not only enhances problem-solving capabilities but also deepens understanding of how equations interact within a system. Whether tackling math homework, preparing for exams, or solving real-world problems involving multiple variables, the elimination method is a versatile tool that, when properly understood and applied, simplifies complex calculations and provides clear solutions.

Understanding the Elimination Method

Before diving into the step-by-step process, it's essential to grasp what the elimination method entails. This technique involves manipulating a system of equations to eliminate one variable, making it easier to solve for the remaining variable. Once that variable is found, substituting back into one of the original equations yields the other variable.

When to Use the Elimination Method

- When the coefficients of one variable are the same or opposites in both equations.
- When the system is linear (both equations are first degree).
- When you want a systematic approach that can be applied to any pair of equations.

Step-by-Step Guide to Solving Systems Using Elimination

Let's explore the method through a structured approach, complete with examples and tips for ensuring accuracy.

Step 1: Write the System of Equations Clearly

Ensure both equations are in standard form:

$$ ax + by = c $$

where a, b, c are constants, and x, y are variables.

Example:

$$
\begin{cases}
3x + 4y = 10 \quad \text{(Equation 1)} \\
2x - 4y = -2 \quad \text{(Equation 2)}
\end{cases}
$$

Step 2: Align Equations and Prepare for Elimination

Identify which variable to eliminate. Usually, choose the one with coefficients that are the same or opposites.

In the example:

- Coefficient of y in Equation 1: 4
- Coefficient of y in Equation 2: -4

Since these are opposites, adding the equations will eliminate y.

Step 3: Make Coefficients Opposite (if they aren't already)

If coefficients are not opposites, multiply one or both equations by suitable constants to achieve this.

Suppose:

\[
\begin{cases}
2x + 3y = 7 \\
4x + 5y = 11
\end{cases}
\]

To eliminate \(x\):

- Multiply Equation 1 by 2:

\[(2x + 3y) \times 2 \Rightarrow 4x + 6y = 14 \]

- Now, subtract Equation 2 from this new equation:

\[(4x + 6y) - (4x + 5y) = 14 - 11 \]

\[4x - 4x + 6y - 5y = 3 \]

\[0x + y = 3 \Rightarrow y = 3 \]

Step 4: Add or Subtract Equations to Eliminate a Variable

Using the earlier example:

\[
\begin{cases}
3x + 4y = 10 \\
2x - 4y = -2
\end{cases}
\]

Add the two equations:

\[
(3x + 4y) + (2x - 4y) = 10 + (-2)
\]

\[
3x + 2x + 4y - 4y = 8
\]

\[
5x = 8
\]

Solve for \(x\):

\[
x = \frac{8}{5}
\]

Step 5: Substitute the Found Variable into One of the Original Equations

Plug \(x = \frac{8}{5}\) into either Equation 1 or Equation 2 to find \(y\).

Using Equation 1:

\[

$$
3x + 4y = 10
$$

Substitute x:

$$
3 \times \frac{8}{5} + 4y = 10
$$

$$
\frac{24}{5} + 4y = 10
$$

Subtract $\frac{24}{5}$ from both sides:

$$
4y = 10 - \frac{24}{5}
$$

Express 10 as $\frac{50}{5}$:

$$
4y = \frac{50}{5} - \frac{24}{5} = \frac{26}{5}
$$

Divide both sides by 4:

$$
y = \frac{\frac{26}{5}}{4} = \frac{26}{5} \times \frac{1}{4} = \frac{26}{20} = \frac{13}{10}
$$

Solution:

$$
x = \frac{8}{5} \quad \text{and} \quad y = \frac{13}{10}
$$

Tips for Success with Elimination

- Always check if coefficients are already opposites or equal; this saves time.
- Multiply equations by constants to create matching or opposite coefficients.
- Perform operations carefully to avoid sign errors.
- Always verify your solution by plugging values into the original equations.
- Practice with different systems to become comfortable with various coefficient combinations.

Common Challenges and How to Overcome Them

Challenge 1: Coefficients Not Suitable for Immediate Elimination

Solution: Multiply one or both equations by suitable numbers to align coefficients.

Challenge 2: Sign Errors During Addition or Subtraction

Solution: Double-check each step and write intermediate steps clearly. Use parentheses to keep track of signs.

Challenge 3: Fractions in Solutions

Solution: Simplify fractions where possible, and consider clearing denominators early on by multiplying through by the least common denominator.

Practice Problems

1. Solve the system:

$$
\begin{cases}
5x + 2y = 12 \\
3x - 2y = 4
\end{cases}
$$

2. Solve the system:

$$
\begin{cases}
4x + 3y = 7 \\
8x + 6y = 14
\end{cases}
$$

3. Solve the system:

$$
\begin{cases}
x - 2y = 3 \\
2x + y = 7
\end{cases}
$$

Conclusion

Solve each system of equations using elimination by systematically manipulating the equations to cancel out one variable, simplifying the path to a solution. The key steps involve aligning coefficients, performing strategic multiplications, and carefully combining equations. With practice, this method becomes a powerful tool for tackling a wide array of linear systems, offering clarity and efficiency. Always verify your solutions to ensure accuracy, and over time, you'll develop an intuitive sense of how to approach different systems with confidence.

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