A 1.2kg Stone Is Tied To String And Swung In A Vertical Circle With A Radius Of 0.85 M. The String Can

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Introduction

Understanding the physics behind swinging a stone in a vertical circle involves concepts from circular motion, energy conservation, and tension analysis. When a stone is attached to a string and swung in a vertical path, the forces acting on it vary depending on its position in the circle, and the energy transformations between kinetic and potential energy play a crucial role. This article explores the detailed dynamics of this scenario, providing insights into the forces involved, energy considerations, and the implications for the tension in the string at different points in the motion.

Fundamental Concepts in Circular Motion

Uniform Circular Motion

- When an object moves along a circular path at a constant speed, it exhibits uniform circular motion.
- The key features are a centripetal acceleration directed toward the center of the circle and a tension or force providing the necessary centripetal force.

Centripetal Force and Acceleration

- Centripetal Force (F_c): The inward force required to keep an object moving in a circle.
- Formula: $F_c = \frac{m v^2}{r}$

Where:
- m = mass of the object
- v = tangential velocity
- r = radius of the circle

Energy Considerations in Vertical Circular Motion

Potential and Kinetic Energy

- As the stone moves in the vertical circle, it exchanges potential energy (PE) and kinetic energy (KE).
- The highest point of the circle has maximum potential energy and minimum kinetic energy.
- The lowest point has maximum kinetic energy and minimum potential energy.

Conservation of Mechanical Energy

- Total mechanical energy remains constant if we neglect air resistance and other dissipative forces.
- Equation: $KE + PE = \text{constant}$

Calculating Energy at Different Points

- At the bottom (lowest point):
- PE = 0 (taking this as reference)
- KE = $\frac{1}{2} m v^2$

- At the top (highest point):
- PE = $m g h$, where $h = 2 r$ (height difference)
- KE = $\frac{1}{2} m v_{top}^2$

Analyzing the Tension in the String

Tension Components at Different Positions

- The tension in the string must provide the centripetal force and support the weight of the stone.
- The magnitude of tension varies depending on the position:

1. At the Bottom of the Circle:
- Tension must support the weight and provide the centripetal force.

2. At the Top of the Circle:
- Tension supports the weight and contributes to centripetal acceleration.

Tension at the Bottom

- $T_{bottom} = \frac{m v_{bottom}^2}{r} + m g$

Tension at the Top

- $T_{top} = \frac{m v_{top}^2}{r} - m g$

(Note the sign difference; tension at the top must support the weight, which acts downward, and the centripetal force must be directed inward.)

Determining the Required Velocity for Complete Circular Motion

Minimum Speed at the Top

- To just complete the circle without the string going slack, the tension at the top must be greater than or equal to zero.
- The minimum velocity at the top:

$$
v_{top, min} = \sqrt{g r}
$$

Calculating the Velocity at the Bottom

- Using energy conservation, the velocity at the bottom can be found:

$$
v_{bottom} = \sqrt{v_{top}^2 + 2 g h}
$$

- Since $h = 2 r$, the velocity at the bottom:

$$
v_{bottom} = \sqrt{v_{top}^2 + 4 g r}
$$

Applying Values

Given:
- Mass, $m = 1.2\,kg$
- Radius, $r = 0.85\,m$
- Gravitational acceleration, $g = 9.8\,m/s^2$

Calculate $v_{top, min}$:

$$
v_{top, min} = \sqrt{9.8 \times 0.85} \approx \sqrt{8.33} \approx 2.89\,m/s
$$

Calculate v_{bottom}:

$$
v_{bottom} = \sqrt{(2.89)^2 + 4 \times 9.8 \times 0.85} \approx \sqrt{8.33 + 33.32} \approx \sqrt{41.65} \approx 6.46\,m/s
$$

Tension in the String at Different Positions

Tension at the Bottom

- Using $T_{bottom} = \frac{m v_{bottom}^2}{r} + m g$:

$$
T_{bottom} = \frac{1.2 \times (6.46)^2}{0.85} + 1.2 \times 9.8
$$

\[

$$
T_{bottom} = \frac{1.2 \times 41.76}{0.85} + 11.76 \approx \frac{50.11}{0.85} + 11.76 \approx 58.96 + 11.76 \approx 70.72\,N
$$

Tension at the Top

- Using $T_{top} = \frac{m v_{top}^2}{r} - m g$:

$$
T_{top} = \frac{1.2 \times (2.89)^2}{0.85} - 1.2 \times 9.8
$$

$$
T_{top} = \frac{1.2 \times 8.33}{0.85} - 11.76 \approx \frac{9.996}{0.85} - 11.76 \approx 11.76 - 11.76 = 0\,N
$$

This indicates that at the minimum speed, the tension at the top can be zero, meaning the string is just taut, and the stone is on the verge of losing contact.

Practical Implications and Safety Considerations

Maximum Tension and Material Strength

- The maximum tension occurs at the bottom of the circle, which in our calculations is approximately 70.72 N.
- The string must have a tensile strength exceeding this value to prevent breaking.
- Material selection and safety factors are critical in real-world applications.

Effect of Air Resistance and Energy Losses

- In reality, air resistance and friction reduce the energy, requiring a higher initial speed to complete the circle.
- Continuous swinging would need energy input (e.g., a push) to compensate for these losses.

Ensuring Safe Operation

- Use a durable string with a tensile strength well above the maximum tension.
- Avoid swinging the stone with excessive force that could break the string or cause injury.
- Always consider the radius and mass when designing such experiments or activities.

Additional Factors and Advanced Considerations

Non-Uniform Circular Motion

- If the stone is swung with varying speed, the tension varies accordingly.
- Understanding the relationship between angular velocity and linear velocity is essential.

Effects of String Mass

- If the string has significant mass, its weight and distribution affect the tension calculations.
- Usually, for lightweight strings, this effect is negligible.

Angular Momentum Conservation

- In the absence of external torque, angular momentum about the center of the circle remains constant.
- This principle helps analyze the motion and predict velocities at different points.

Conclusion

Swinging a stone in a vertical circle involves complex interplay between forces, energy, and material strength. By applying principles of physics—such as conservation of energy, circular motion dynamics, and tension analysis—one can accurately determine the forces at play and ensure safe operation. The calculations demonstrate that the tension in the string varies significantly depending on the position in the circle, reaching a maximum at the bottom and approaching zero at the top for the minimum velocity case. Proper understanding and careful planning are essential for both educational demonstrations and practical applications involving vertical circular motion.

References

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- Tipler, P. A., & Mosca, G. (2008). Physics for Scientists and Engineers. W. H. Freeman.

Frequently Asked Questions

What is the maximum tension in the string when the stone is at the bottom of the vertical circle?

The maximum tension occurs at the bottom of the circle and can be calculated using $T = m(g + v^2/r)$, where v is the velocity at the bottom, determined by energy conservation. This tension accounts for both the weight of the stone and the centripetal force required for circular motion.

How does the tension in the string vary as the stone moves through different points in the vertical circle?

The tension is greatest at the bottom of the circle due to the combined effect of gravity and the centripetal force, and it is least at the top where gravity partially offsets the required centripetal force, resulting in lower tension.

What is the minimum speed the stone must have at the top of the circle to complete the vertical loop without the string slackening?

The minimum speed at the top is found by setting the tension to zero and applying energy conservation, resulting in v = sqrt(g r), which ensures the stone just maintains contact with the string without slackening.

How would increasing the radius of the circle affect the tension in the string during the stone's motion?

Increasing the radius would increase the required centripetal force at a given speed, leading to higher tension in the string at points where the velocity remains constant, especially at the bottom of the circle.

What factors limit the maximum speed the stone can achieve without breaking the string?

The maximum speed is limited by the tensile strength of the string. When the tension exceeds the string's maximum tensile strength, the string will break. Therefore, the material properties and the mass of the stone determine this maximum safe speed.

Additional Resources

Physics in Action: Exploring the Dynamics of a 1.2kg Stone Swung in a Vertical Circle

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Introduction: The Intriguing World of Circular Motion

When we think of simple experiments involving everyday objects, few are as captivating as swinging a stone tied to a string in a vertical circle. This seemingly straightforward activity, however, encapsulates complex principles of physics—ranging from tension and gravity to centripetal force and energy conservation. In this article, we delve deeply into the mechanics of a 1.2kg stone attached to a string with a radius of 0.85 meters, exploring how it behaves when swung in a vertical circle, and what factors influence its motion and safety considerations.

Understanding the Basic Setup

Imagine a sturdy string, capable of withstanding significant tension, securely tied to a stone weighing 1.2 kilograms. The string's length is precisely 0.85 meters, defining the radius of the circular path.

When swung, the stone traces a vertical circle, completing a full rotation while experiencing various forces at different points in the path.

This setup is not merely a recreational activity; it embodies core physical concepts that are crucial in fields such as engineering, amusement park ride design, and even celestial mechanics. To appreciate the intricacies, we must first understand the fundamental forces at play.

Fundamental Forces and Concepts

Centripetal Force

Any object moving in a circle requires a centripetal force directed toward the center of the circle. In this case, the tension in the string provides this force, counteracting the outward pull of inertia.

Mathematically, the centripetal force F_c is given by:

$$
F_c = \frac{m v^2}{r}
$$

where:
- m = mass of the stone (1.2 kg)
- v = tangential velocity at a given point
- r = radius of the circle (0.85 meters)

Understanding how v varies at different points in the circle is key to analyzing the tension and the forces involved.

Gravity and Its Role

Gravity acts downward throughout the motion, with a constant force:

$$
F_g = m g
$$

where g is acceleration due to gravity ($\approx 9.81\, \text{m/s}^2$).

The interplay between tension and gravity varies as the stone moves from the bottom to the top of the circle.

Energy Conservation and Velocity Calculation

One of the most elegant principles in physics—conservation of mechanical energy—states that in the absence of non-conservative forces like air resistance, the total mechanical energy remains constant.

Suppose the stone is swung such that it starts from the lowest point with an initial tangential velocity v_0. As it moves upward, kinetic energy converts into potential energy, and vice versa.

Total energy at any point:

$$
E = KE + PE = \frac{1}{2} m v^2 + m g h
$$

At the lowest point (initial):

$$
E_{bottom} = \frac{1}{2} m v_0^2
$$

At the highest point:

$$
E_{top} = \frac{1}{2} m v_{top}^2 + m g (2r)
$$

because the height difference from bottom to top is $2r$.

Calculating the velocity at the top:

Assuming the initial velocity v_0 at the bottom is sufficient to complete the circle, the velocity at the top is:

$$
\frac{1}{2} m v_0^2 = \frac{1}{2} m v_{top}^2 + m g (2r)
$$

Rearranged:

$$
v_{top}^2 = v_0^2 - 2 g (2r)
$$

This equation allows us to determine the minimum initial velocity v_0 required to ensure the stone reaches the top with a positive velocity, preventing it from losing contact with the string.

Determining the Minimum Velocity for Complete Vertical Circle

A critical consideration is the minimum velocity at the top of the circle needed to maintain tension in the string. If the tension drops to zero, the string becomes slack, and the motion ceases to be a perfect circle.

Condition at the top:

- Tension $T_{top} \geq 0$
- The net inward force must provide the necessary centripetal force:

$$
T_{top} + mg = \frac{m v_{top}^2}{r}
$$

At the minimum condition (when tension just equals zero):

$$
0 + mg = \frac{m v_{top}^2}{r}
$$

$$
v_{top}^2 = g r
$$

$$
v_{top} = \sqrt{g r}
$$

Plugging in the known values:

$$
v_{top} = \sqrt{9.81 \times 0.85} \approx \sqrt{8.3385} \approx 2.89\, \text{m/s}
$$

To ensure the stone maintains contact throughout, the initial velocity must be such that at the top:

$$
v_{top} \geq 2.89\, \text{m/s}
$$

Calculating the initial velocity v_0:

Using the energy conservation equation:

$$
v_0^2 = v_{top}^2 + 4 g r
$$

$$
v_0^2 = (2.89)^2 + 4 \times 9.81 \times 0.85
$$

$$
v_0^2 = 8.34 + 33.34 = 41.68
$$

$$
v_0 \approx \sqrt{41.68} \approx 6.46\, \text{m/s}
$$

Interpretation:
The stone must be given an initial tangential velocity of at least approximately 6.46 m/s at the bottom to complete the vertical circle without losing contact at the top.

Calculating Tension at Different Points in the Motion

Understanding the tension in the string at various points provides insight into the stresses involved and safety margins.

At the bottom:

$$
T_{bottom} = \frac{m v_{bottom}^2}{r} - m g
$$

where v_{bottom} is the velocity at the bottom, which can be calculated from energy conservation:

$$
v_{bottom}^2 = v_{top}^2 + 4 g r
$$

Using $v_{top} = 2.89\, \text{m/s}$:

$$
v_{bottom}^2 = (2.89)^2 + 4 \times 9.81 \times 0.85 \approx 8.34 + 33.34 = 41.68
$$

$$
v_{bottom} \approx 6.46\, \text{m/s}
$$

Thus:

$$
T_{bottom} = \frac{1.2 \times (6.46)^2}{0.85} - 1.2 \times 9.81
$$

$$
T_{bottom} = \frac{1.2 \times 41.68}{0.85} - 11.77
$$

$$
T_{bottom} = \frac{50.02}{0.85} - 11.77 \approx 58.84 - 11.77 \approx 47.07\, \text{N}
$$

This tension is significantly higher than the weight of the stone (~11.77 N), indicating the string must withstand a force roughly four times the weight at the bottom.

At the top:

$$
T_{top} = \frac{m v_{top}^2}{r} - m g
$$

$$
T_{top} = \frac{1.2 \times (2.89)^2}{0.85} - 1.2 \times 9.81

\]
\[
T_{top} = \frac{1.2 \times 8.34}{0.85} - 11.77 \approx \frac{10.00}{0.85} - 11.77 \approx 11.76 - 11.77 \approx -0.01\, \text{N}
\]

Since tension cannot be negative, this confirms that at the minimum velocity, tension at the top is effectively zero, meaning the string is just taut.

Practical Implications and Safety Considerations

While the physics paints a fascinating picture, practical application demands attention to safety and material limits.

Material Strength:

- The string must withstand the maximum tension, approximately 47 N at the bottom.
- Selecting a string with a significant safety margin—preferably rated for at least 10 times the expected tension—is advisable.

Initial Velocity:

- Achieving initial velocities near 6.5 m/s (about 23.4 km/h or 14.5 mph) requires careful effort.
- Launching the stone with controlled force ensures safety and consistency.

Environmental Factors:

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