

A 21.1-N Force Is Applied To A Cord Wrapped Around A Pulley Of Mass $M = 4.49\text{-kg}$ And Radius $R = 25.0\text{-cm}$

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When studying rotational dynamics and Newtonian mechanics, understanding the behavior of pulleys under various forces is essential. In this article, we analyze a scenario where a force of 21.1 N is applied to a cord wrapped around a pulley with a mass of 4.49 kg and a radius of 25.0 cm. This setup provides a comprehensive example of how forces, torque, and rotational motion interact in real-world physics problems.

Understanding the System: Components and Variables

Before diving into the calculations and physics principles involved, it is important to understand the key components and variables:

Components of the System

- Pulley: A solid object with mass $(M = 4.49\text{, } \text{kg})$ and radius $(R = 25.0\text{, } \text{cm})$. It rotates freely about its central axis and is subject to torque due to the applied force.
- Cord: A massless, inextensible string wrapped around the pulley. The force applied to this cord influences the pulley's motion.
- Applied Force: A force $(F = 21.1\text{, } \text{N})$ is exerted on the cord, causing the pulley to accelerate or rotate depending on the conditions.
- Gravity: The weight of the pulley, which affects the normal forces and potential tension in the cord if the pulley is hanging or attached to a load.
- Rotation Axis: The pulley rotates about its central axis, and its rotational inertia influences its angular acceleration.

Variables to Consider

- (M) : Mass of the pulley
- (R) : Radius of the pulley
- (F) : Applied force

- T : Tension in the cord
- I : Moment of inertia of the pulley
- α : Angular acceleration of the pulley
- a : Linear acceleration of the cord or attached mass

Fundamental Physics Principles Involved

This problem involves the application of several core physics concepts:

Newton's Second Law for Rotation

The rotational equivalent of Newton's second law states:

$$\tau = I \alpha$$

where:

- τ is the net torque acting on the pulley
- I is the moment of inertia
- α is the angular acceleration

Moment of Inertia for a Solid Disk

Since the pulley can be modeled as a solid disk, its moment of inertia is:

$$I = \frac{1}{2} M R^2$$

Relationship Between Linear and Angular Quantities

The linear acceleration a of the cord (or attached mass) relates to the angular acceleration of the pulley via:

$$a = R \alpha$$

This relation is crucial when considering how the applied force translates

into rotational motion.

Analyzing the Dynamics: Step-by-Step Approach

Let's explore the problem systematically.

Step 1: Calculating the Moment of Inertia

Given:

- $(M = 4.49\text{ kg})$
- $(R = 25.0\text{ cm} = 0.25\text{ m})$

The moment of inertia:

$$I = \frac{1}{2} M R^2 = \frac{1}{2} \times 4.49\text{ kg} \times (0.25\text{ m})^2$$

$$I = 0.5 \times 4.49 \times 0.0625 = 0.5 \times 0.280625 = 0.1403125\text{ kg} \cdot \text{m}^2$$

Step 2: Determining the Tension in the Cord

Assuming the force $(F = 21.1\text{ N})$ is the tension in the cord, or that it exerts this force, we need to clarify whether the force is directly applied as tension or as an external force. For simplicity, let's assume the applied force is directly the tension in the cord:

$$T = 21.1\text{ N}$$

If, instead, the force is external and causes tension, additional steps are needed to determine (T) .

Step 3: Calculating the Angular Acceleration

Applying Newton's second law for rotation:

$$\tau = T R = I \alpha$$

Rearranged:

$$\alpha = \frac{T R}{I}$$

Substitute known values:

$$\alpha = \frac{21.1 \text{ N} \times 0.25 \text{ m}}{0.1403125 \text{ kg} \cdot \text{m}^2} = \frac{5.275 \text{ N} \cdot \text{m}}{0.1403125 \text{ kg} \cdot \text{m}^2}$$

$$\alpha \approx 37.6 \text{ rad/sec}^2$$

This indicates a rapid angular acceleration of the pulley under the applied force.

Step 4: Calculating Linear Acceleration of the Cord

Using the relation:

$$a = R \alpha$$

$$a = 0.25 \text{ m} \times 37.6 \text{ rad/sec}^2 \approx 9.4 \text{ m/sec}^2$$

This acceleration suggests the cord (or attached mass) accelerates downward or upward at approximately 9.4 m/sec^2 .

Additional Considerations and Real-World Applications

While this analysis provides a foundational understanding, real-world pulley systems often involve more complexities.

Friction and Air Resistance

- Friction in the axle or bearing resistance can reduce the net acceleration.
- Air resistance may also influence the motion if the pulley rotates rapidly.

Massless and Inextensible Cord Assumption

- In simplified physics problems, cords are often assumed massless and inextensible.
- In practical applications, the cord's mass and elasticity can affect the dynamics, requiring more complex models.

Attached Masses and Load Systems

- When a mass is attached to the pulley, the tension in the cord depends on the mass's weight and acceleration.
- The system's equilibrium or acceleration depends on the balance between applied forces, weight, and inertial effects.

Practical Applications of Pulley Dynamics

Understanding the physics of pulleys with forces like 21.1 N is critical in various engineering and mechanical contexts:

- Designing hoisting and lifting equipment
- Analyzing belt drive systems
- Studying rotational machinery and gear systems
- Developing mechanical advantage systems in construction
- Educational demonstrations of rotational dynamics principles

Summary and Key Takeaways

- The moment of inertia for a solid disk pulley with given mass and radius is approximately $0.14 \text{ kg}\cdot\text{m}^2$.
- Applying a force of 21.1 N results in a high angular acceleration ($\sim 37.6 \text{ rad/sec}^2$), leading to linear acceleration of about 9.4 m/sec^2 .
- The relation between force, torque, and angular acceleration is fundamental for analyzing rotational systems.
- Simplified models with assumptions such as massless cords and negligible friction aid in understanding core physics principles but should be refined for real-world applications.

Conclusion

Analyzing the dynamics of a pulley system subjected to a force of 21.1 N reveals intricate interactions between forces, torque, and rotational inertia. Whether in educational settings or engineering designs, understanding these principles helps in optimizing systems for efficiency and safety. Accurate calculations of moment of inertia, angular acceleration, and the effects of external forces are essential skills for physicists and engineers alike, ensuring the effective application of rotational mechanics across various industries.

Keywords: pulley dynamics, rotational motion, moment of inertia, torque, angular acceleration, physics, engineering, applied force, mechanics

Frequently Asked Questions

What is the acceleration of the pulley when a 21.1-N force is applied to the cord?

The acceleration can be found using Newton's second law and rotational dynamics. First, calculate the tension in the cord (which is equal to the applied force if no other forces oppose it). Assuming the force is applied directly to the cord, the net force causes acceleration: $a = F / (M + I / R^2)$. For a solid disk, $I = (1/2) M R^2$, so the acceleration is $a = F / (M + (1/2) M) = F / (1.5 M)$. Plugging in the numbers: $a = 21.1 \text{ N} / (1.5 \times 4.49 \text{ kg}) \approx 21.1 / 6.735 \approx 3.13 \text{ m/s}^2$.

What is the torque exerted on the pulley by the applied force?

The torque τ is given by $\tau = F \times R$. Using the given force and radius: $\tau = 21.1 \text{ N} \times 0.25 \text{ m} = 5.275 \text{ Nm}$.

How does the mass of the pulley affect the acceleration when a force is applied?

The mass of the pulley influences its moment of inertia, which resists changes in rotational motion. A heavier pulley (larger M) increases the moment of inertia, reducing the acceleration for a given applied force, since more torque is needed to accelerate its rotation.

What is the angular acceleration of the pulley under the applied force?

Angular acceleration α is related to linear acceleration a by $\alpha = a / R$. Using $a \approx 3.13 \text{ m/s}^2$ and $R = 0.25 \text{ m}$, $\alpha \approx 3.13 / 0.25 \approx 12.52 \text{ rad/s}^2$.

If the pulley starts from rest, how long does it take to rotate through 90 degrees under this force?

First, convert 90° to radians: $\pi/2 \text{ rad} \approx 1.57 \text{ rad}$. Using $\theta = (1/2) \alpha t^2$, solve for t : $t = \sqrt{(2\theta / \alpha)}$. Plugging in $\alpha \approx 12.52 \text{ rad/s}^2$, $t \approx \sqrt{(2 \times 1.57 / 12.52)} \approx \sqrt{(3.14 / 12.52)} \approx \sqrt{0.251} \approx 0.50 \text{ seconds}$.

What would happen to the acceleration if the applied force increased?

If the applied force increases, the acceleration of both the pulley and the mass would increase proportionally, assuming other factors remain constant, because $a = F / (\text{mass and rotational inertia components})$.

Additional Resources

A 21.1-N Force Is Applied To A Cord Wrapped Around A Pulley Of Mass $M = 4.49 \text{ kg}$ And Radius $R = 25.0 \text{ cm}$

Understanding the dynamics of pulleys and the forces involved is fundamental in classical mechanics, especially when analyzing systems involving rotational motion and tension. The specific problem of applying a 21.1-N force to a cord wrapped around a pulley with given mass and radius provides a rich context for exploring concepts such as torque, acceleration, and energy conservation. This article offers a comprehensive review of this scenario, breaking down the physics principles involved, the calculations necessary,

and the implications of varying parameters. It aims to serve as a detailed guide for students, educators, and enthusiasts interested in rotational dynamics and real-world applications.

Introduction to the Problem

The problem centers around a pulley with a mass $(M = 4.49 \text{ kg})$ and radius $(R = 25.0 \text{ cm})$, on which a cord is wrapped. A force $(F = 21.1 \text{ N})$ is applied to this cord, creating tension that causes the pulley to rotate and possibly accelerate a mass or system attached to the cord. Analyzing this system involves understanding how the applied force translates into torque, how the pulley's inertia affects the acceleration, and how energy conservation principles may be employed.

This scenario is akin to many real-world applications – from pulleys in elevators to conveyor systems and mechanical lifts – making it a practical and educational example. The key to solving such problems lies in combining translational and rotational dynamics, which involves Newton's second law for linear motion and the rotational equivalent involving moments of inertia.

Fundamental Physics Principles

Newton's Second Law for Translation

When a force is applied to an object, the resultant acceleration is given by:

$$F_{\text{net}} = m a$$

In this case, the net force acting on the system depends on the tension in the cord, which is related to the applied force (F) . If the system involves a mass hanging or being pulled, the tension (T) in the cord becomes a crucial parameter.

Rotational Dynamics

The pulley's rotation is governed by:

$$\tau = I \alpha$$

\]

where:

- τ is the torque,
- I is the moment of inertia of the pulley,
- α is the angular acceleration.

The torque created by the tension T on the pulley is:

$$\tau = T R$$

The moment of inertia for a solid disk (assuming the pulley is solid) is:

$$I = \frac{1}{2} M R^2$$

The angular acceleration relates to the linear acceleration a of the cord (and any attached mass) by:

$$\alpha = \frac{a}{R}$$

Analyzing the System: Step-by-Step Approach

1. Establishing the Free-Body Diagrams

To analyze the system, draw free-body diagrams for the pulley and the mass (if present). For the pulley, forces include tension T acting tangentially and gravity if a mass is hanging. For the applied force, understand how it influences tension in the cord.

2. Relating Force and Torque

Since the applied force $F = 21.1 \text{ N}$ is exerted on the cord, it produces tension T . The key is understanding whether T equals F or is modified by other constraints (like pulley friction).

If the force is directly applied to the cord, the tension T is generally less than or equal to F depending on the pulley's friction and other factors. For ideal conditions (massless, frictionless pulley), $T \approx F$.

The torque on the pulley is:

$$\tau = T R$$

and the angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{T R}{\frac{1}{2} M R^2} = \frac{2 T}{M R}$$

which relates to the linear acceleration:

$$a = \alpha R = \frac{2 T}{M R} \times R = \frac{2 T}{M}$$

This provides a direct link between tension and acceleration.

3. Applying Newton's Second Law

For the pulley:

$$T = \frac{1}{2} M a$$

For the mass (m) (if, for example, a mass is hanging):

$$m g - T = m a$$

Depending on the scenario, these equations can be combined to solve for (a) and (T) .

4. Solving for Acceleration and Tension

By combining the equations:

$$T = \frac{1}{2} M a$$

$$m g - T = m a$$

Substituting (T) :

$$mg - \frac{1}{2}Ma = ma$$

which leads to:

$$mg = ma + \frac{1}{2}Ma = \left(m + \frac{1}{2}M\right)a$$

$$a = \frac{mg}{m + \frac{1}{2}M}$$

Once (a) is known, tension (T) can be found:

$$T = \frac{1}{2}Ma$$

Effect of Applied Force and System Parameters

The applied force $(F = 21.1 \text{ N})$ influences the tension and acceleration, but its exact effect depends on how it's applied and whether it's directly increasing tension or acting on another component.

Features and considerations include:

- Idealized assumptions: No friction, massless cord, and a solid pulley simplify calculations but may not reflect real conditions.
- Frictional effects: Friction in the pulley axle can reduce the effective torque and must be considered for real-world applications.
- Mass distribution: The mass of the pulley influences its moment of inertia and thus the acceleration.

Real-World Applications and Implications

Understanding this problem has significant real-world relevance. Pulleys are fundamental in mechanical systems, enabling force multiplication, directional change, and energy transfer. For example:

- Elevator systems: Pulleys distribute the load and reduce effort.
- Crane mechanisms: Precise calculations ensure safety and efficiency.
- Exercise equipment: Resistance and motion are managed through pulley systems.

Pros of such systems include:

- Ease of force application
- Mechanical advantage
- Flexibility in design

Cons or limitations:

- Frictional losses
- Maintenance requirements
- Limited efficiency if friction is high

By analyzing the forces and motion involved, engineers can optimize pulley systems for maximum efficiency and safety.

Conclusion and Final Remarks

The scenario involving a 21.1-N force applied to a pulley with specified mass and radius encapsulates core principles of rotational and linear dynamics. Through detailed breakdowns of force, torque, and energy considerations, it demonstrates how fundamental physics informs real-world machinery. The interplay between applied forces, system parameters, and physical laws allows precise prediction of acceleration, tension, and overall system behavior.

While simplified models provide essential insights, accounting for factors such as friction, pulley material, and cable elasticity is vital for practical applications. This analysis not only enhances understanding of classical mechanics but also informs the design and optimization of mechanical systems across various industries.

In summary, the problem illustrates the elegance and utility of physics principles in engineering, emphasizing the importance of rigorous analysis in designing efficient, safe, and effective pulley systems. Whether for academic purposes or practical engineering, mastering these concepts empowers better problem-solving and innovation in mechanical design.

[A 21.1 N Force Is Applied To A Cord Wrapped Around A Pulley](#)

Of Mass M 4 49 Kg And Radius R 25 0 Cm

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