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When it comes to probability and combinatorics, understanding how to analyze scenarios involving drawing items from a collection is fundamental. In this article, we explore a classic problem involving a bag of marbles with different colors and analyze the various outcomes and probabilities associated with drawing three marbles at random. This problem not only highlights key concepts in probability theory but also offers practical insights into real-world situations where random sampling is involved.

Understanding the Problem: The Composition of the Marble Bag

Before diving into calculations, it’s essential to understand the initial setup of the problem:

- The bag contains a total of 15 marbles.
- The marbles are divided into three color groups:
- Red marbles: 5
- Blue marbles: 4
- Green marbles: 6

The question posed is: If three marbles are drawn out of the bag at random, what are the possible outcomes, and what are their probabilities?

Basic Concepts in Probability and Combinatorics

To analyze this problem, let’s review some fundamental concepts:

Total Number of Ways to Draw 3 Marbles

When drawing 3 marbles from 15 without replacement, the total number of possible combinations can be calculated using the combination formula:

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}

\]

Where:
- \(n \) is the total number of items (here 15),
- \(k \) is the number of items to choose (here 3).

Thus, total possible outcomes:

\[
\binom{15}{3} = \frac{15!}{3!(15-3)!} = 455
\]

This means there are 455 different ways to draw 3 marbles from the bag.

Possible Outcomes Based on Color Combinations

The drawn marbles can be any combination of red, blue, and green. The possible color combinations for 3 marbles are:

1. 3 Red
2. 2 Red + 1 Blue
3. 2 Red + 1 Green
4. 1 Red + 2 Blue
5. 1 Red + 1 Blue + 1 Green
6. 3 Blue
7. 2 Blue + 1 Green
8. 1 Blue + 2 Green
9. 3 Green

Each of these combinations has a different probability, which we'll calculate next.

Calculating Probabilities of Different Outcomes

To compute the probability of each outcome, we need to determine the number of favorable ways to draw that specific combination and then divide by the total number of combinations (455).

Outcome 1: Drawing 3 Red Marbles

Number of ways:
\[
\binom{5}{3} = 10
\]
Probability:
\[
P(\text{3 Red}) = \frac{10}{455} \approx 0.022

\]

Outcome 2: Drawing 2 Red and 1 Blue

Number of ways:
\[
\binom{5}{2} \times \binom{4}{1} = 10 \times 4 = 40
\]
Probability:
\[
P(\text{2 Red, 1 Blue}) = \frac{40}{455} \approx 0.088
\]

Outcome 3: Drawing 2 Red and 1 Green

Number of ways:
\[
\binom{5}{2} \times \binom{6}{1} = 10 \times 6 = 60
\]
Probability:
\[
P(\text{2 Red, 1 Green}) = \frac{60}{455} \approx 0.132
\]

Outcome 4: Drawing 1 Red and 2 Blue

Number of ways:
\[
\binom{5}{1} \times \binom{4}{2} = 5 \times 6 = 30
\]
Probability:
\[
P(\text{1 Red, 2 Blue}) = \frac{30}{455} \approx 0.066
\]

Outcome 5: Drawing 1 Red, 1 Blue, and 1 Green

Number of ways:
\[
\binom{5}{1} \times \binom{4}{1} \times \binom{6}{1} = 5 \times 4 \times 6 = 120
\]
Probability:
\[
P(\text{1 Red, 1 Blue, 1 Green}) = \frac{120}{455} \approx 0.264
\]

Outcome 6: Drawing 3 Blue Marbles

Number of ways:

$$
\binom{4}{3} = 4
$$

Probability:

$$
P(\text{3 Blue}) = \frac{4}{455} \approx 0.009
$$

Outcome 7: Drawing 2 Blue and 1 Green

Number of ways:

$$
\binom{4}{2} \times \binom{6}{1} = 6 \times 6 = 36
$$

Probability:

$$
P(\text{2 Blue, 1 Green}) = \frac{36}{455} \approx 0.079
$$

Outcome 8: Drawing 1 Blue and 2 Green

Number of ways:

$$
\binom{4}{1} \times \binom{6}{2} = 4 \times 15 = 60
$$

Probability:

$$
P(\text{1 Blue, 2 Green}) = \frac{60}{455} \approx 0.132
$$

Outcome 9: Drawing 3 Green Marbles

Number of ways:

$$
\binom{6}{3} = 20
$$

Probability:

$$
P(\text{3 Green}) = \frac{20}{455} \approx 0.044
$$

Summary of Probabilities for All Outcomes

Outcome	Number of Ways	Probability	Approximate Percentage
3 Red	10	10/455	2.2%
2 Red + 1 Blue	40	8.8%
2 Red + 1 Green	60	13.2%
1 Red + 2 Blue	30	6.6%
1 Red + 1 Blue + 1 Green	120	26.4%
3 Blue	4	0.9%
2 Blue + 1 Green	36	7.9%
1 Blue + 2 Green	60	13.2%
3 Green	20	4.4%

Applications and Real-World Implications

Understanding the probabilities of various outcomes in drawing marbles can be translated into many real-world scenarios, such as:

- Quality Control: Randomly selecting items from a batch to determine quality levels.
- Game Design: Creating fair randomization processes in board games and lotteries.
- Risk Assessment: Evaluating probable outcomes in systems involving multiple variables.
- Education: Teaching fundamental probability concepts through tangible examples.

Further Exploration: Conditional Probability and Expected Values

Once familiar with basic outcomes, one can explore more advanced topics:

Conditional Probability

Calculating the probability of drawing a specific color given certain prior outcomes. For example:
- If the first marble drawn is red, what is the probability that the second is blue?

Expected Value of Red Marbles in 3 Draws

Calculating the average number of red marbles expected in multiple draws, which involves multiplying possible red counts by their probabilities.

Conclusion

Analyzing the scenario of drawing three marbles from a bag containing different colored marbles provides valuable insights into combinatorics and probability. By understanding how to calculate the total number of outcomes and the likelihood of each specific event, we develop skills applicable across various fields, from statistics and mathematics to everyday decision-making. Whether for academic purposes, game development, or risk management, mastering these concepts enhances our ability to interpret and predict random events accurately.

Remember, the key to mastering probability problems is to break down the problem into manageable parts, understand the total possible outcomes, and systematically compute the likelihood of each scenario. With practice, these calculations become intuitive, empowering you to analyze complex random processes confidently.

Frequently Asked Questions

What is the total number of marbles in the bag?

The total number of marbles is 5 red + 4 blue + 6 green = 15 marbles.

What is the probability of drawing three red marbles in a row without replacement?

The probability is (5/15) (4/14) (3/13) = 60/2730 ≈ 0.022, or about 2.2%.

What is the probability of drawing exactly two blue marbles and one green marble in three draws?

Number of ways: C(4,2) C(6,1) = 6 6 =36. Total combinations: C(15,3)= 455. Probability = 36/455 ≈ 0.079, or 7.9%.

What is the probability of drawing at least one green marble in three draws?

First, find the probability of drawing no green marbles: C(9,3)/C(15,3) = 84/455. So, the probability of at least one green is 1 - 84/455 = 371/455 ≈ 81.54%.

If three marbles are drawn, what is the probability that all three are of different colors?

Number of ways: (Red-Blue-Green) = 546=120. Total combinations: C(15,3)=455. Probability = 120/455 ≈ 26.37%.

What is the probability that all three drawn marbles are green?

Probability = C(6,3)/C(15,3) = 20/455 ≈ 4.4%.

What is the expected number of green marbles in three draws?

Expected value = 3 (6/15) = 3 0.4 = 1.2 green marbles.

If three marbles are drawn, what is the probability that exactly one of them is blue?

Number of ways: C(4,1) C(11,2) = 4 55 = 220. Total combinations: 455. Probability = 220/455 ≈ 48.35%.

What is the probability of drawing three marbles of the same color?

Sum of probabilities: all red (C(5,3)/C(15,3)=10/455), all blue (C(4,3)/455=4/455), all green (C(6,3)/455=20/455). Total = (10+4+20)/455=34/455 ≈ 7.47%.

Additional Resources

A Bag Contain 5 Red Marbles, 4 Blue Marbles And 6 Green Marbles. If Three Marbles Are Drawn Out Of The Bag Without Replacement, What Is the Probability That...?

Understanding the probability of drawing certain combinations of marbles from a bag is a classic problem in combinatorics and probability theory. This scenario involves a bag containing a total of 15 marbles—5 red, 4 blue, and 6 green—and exploring the likelihood of various outcomes when three marbles are drawn without replacement. Whether you're a student preparing for exams, a teacher designing probability exercises, or a math enthusiast interested in real-world applications, this guide will provide a comprehensive breakdown of how to analyze such problems effectively.

Overview of the Problem

Imagine a bag with the following composition:

- Red marbles: 5
- Blue marbles: 4
- Green marbles: 6

Total marbles: 5 + 4 + 6 = 15

The task involves drawing three marbles from this bag without replacement. "Without replacement" means once a marble is drawn, it is not put back into the bag before the next draw. This aspect influences the probabilities because the total number of marbles and their distribution change after each draw.

Key Questions Usually Asked

- What is the probability that all three drawn marbles are of the same color?
- What is the probability that exactly two marbles share the same color, and the third is different?
- What is the probability that the drawn marbles are all different colors?
- What is the probability of drawing a specific color combination?

Understanding how to approach these questions requires a solid grasp of combinatorial calculations, particularly combinations, and how probabilities are derived from counting favorable outcomes over total possible outcomes.

Fundamental Concepts Needed

Total Number of Ways to Draw 3 Marbles

The total number of ways to select any 3 marbles from 15 is:

$$
\text{Total outcomes} = \binom{15}{3}
$$

where $\binom{n}{k}$ denotes the binomial coefficient, representing the number of combinations of n items taken k at a time.

Calculating:

$$
\binom{15}{3} = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455
$$

This is the denominator for all probability calculations involving drawing three marbles.

Combinatorial Notation and Principles

- Combination ($\binom{n}{k}$): Number of ways to choose k items from n without regard to order.
- Probability: Number of favorable outcomes divided by total outcomes.

Calculating Specific Probabilities

1. Probability that all three marbles are of the same color

Possible cases:

- All three are red
- All three are blue
- All three are green

Calculations:

- All red:

$$
\binom{5}{3} = 10
$$

- All blue:

$$
\binom{4}{3} = 4
$$

- All green:

$$
\binom{6}{3} = 20
$$

Total favorable outcomes:

$$
10 + 4 + 20 = 34
$$

Probability:

$$
P(\text{all same color}) = \frac{34}{455} \approx 0.0747
$$

---

2. Probability that exactly two marbles are of the same color, and the third is of a different color

This case involves multiple sub-cases:

- Two red marbles + one non-red marble
- Two blue marbles + one non-blue marble
- Two green marbles + one non-green marble

Let's analyze each separately.

Case A: Two red + one other (not red)

- Number of ways to choose 2 red marbles:

$$
\binom{5}{2} = 10
$$

- Number of remaining marbles (non-red):

$$
4 + 6 = 10
$$

- Number of ways to pick 1 non-red marble:

$$
\binom{10}{1} = 10
$$

Total for this case:

$$
10 \times 10 = 100
$$

Similarly, for blue:

Case B: Two blue + one other (not blue)

- $\binom{4}{2} = 6$
- Remaining non-blue marbles:

$$
5 + 6 = 11
$$

- Ways to pick 1 non-blue marble:

$$
\binom{11}{1} = 11
$$

- Total:

$$
6 \times 11 = 66
$$

Case C: Two green + one other (not green)

- $\binom{6}{2} = 15$
- Remaining non-green marbles:

$$
5 + 4 = 9
$$

- Ways to pick 1 non-green marble:

$$
\binom{9}{1} = 9
$$
- Total:

$$
15 \times 9 = 135
$$

Total favorable outcomes:

$$
100 + 66 + 135 = 301
$$

Probability:

$$
P(\text{exactly two same, one different}) = \frac{301}{455} \approx 0.6615
$$

---

3. Probability that all three marbles are of different colors

In this scenario, we select 1 red, 1 blue, and 1 green marble.

Number of ways:

$$
\binom{5}{1} \times \binom{4}{1} \times \binom{6}{1} = 5 \times 4 \times 6 = 120
$$

Total outcomes: 455 (as previously calculated).

Probability:

$$
P(\text{all different colors}) = \frac{120}{455} \approx 0.2637
$$

Additional Scenarios and Variations

4. Drawing at Least Two Marbles of the Same Color

This includes the cases where all three are the same color or exactly two are the same.

$$
P(\text{at least two same}) = P(\text{all same}) + P(\text{exactly two same, one different})
$$

$$
= \frac{34}{455} + \frac{301}{455} = \frac{335}{455} \approx 0.7363
$$

5. Probability of Drawing a Specific Color Combination

Suppose we are asked, for example, what is the probability of drawing 1 red, 1 blue, and 1 green in any order:

- Already calculated as 120 favorable outcomes.

- Probability:

$$
\boxed{\frac{120}{455} \approx 0.2637}
$$

Practical Applications and Insights

Understanding these probability calculations isn't just an academic exercise—these principles underpin many real-world decision-making processes, from quality control in manufacturing to game design, and even in statistical sampling.

Key Takeaways:

- Sample Space Size: Always start by calculating the total number of possible outcomes.
- Favorable Outcomes: Identify and count the specific configurations that satisfy your condition.
- Use of Combinations: When order doesn't matter (as in selecting marbles), combinations are the appropriate counting method.
- Conditional Probabilities: When drawing without replacement, probabilities are dependent on previous draws, but in this case, since we are calculating the total probability of certain outcomes, combinatorial counts suffice.

Tips for Solving Similar Problems:

- Break down complex scenarios into manageable cases.
- Use symmetry to simplify calculations.
- Verify that your counts do not overlap when summing probabilities.
- Remember that probabilities are always between 0 and 1, and the sum of probabilities of mutually exclusive events equals 1.

Final Thoughts

Analyzing the probability of drawing specific combinations from a finite set, like marbles in a bag, reinforces fundamental concepts in combinatorics and probability. By carefully counting outcomes and understanding the structure of the problem, you can derive meaningful insights and precise probabilities. Whether for academic purposes or practical applications, mastering these techniques enhances your analytical skills and deepens your understanding of randomness and chance.

In summary, when a bag contains 5 red, 4 blue, and 6 green marbles, and three marbles are drawn without replacement, the probability of various outcomes can be systematically calculated using combination formulas and basic probability principles. This approach provides a solid foundation for tackling a wide array of similar probabilistic challenges.

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