A Certain Circle Can Be Represented By The Following Equation X^2+y^2+18x+14y+105=0 What Is The Center

A Certain Circle Can Be Represented By The Following Equation X^2 + y^2 + 18x + 14y + 105 = 0. What Is The Center?

Understanding the properties of a circle and how to find its center from a given equation is a fundamental aspect of coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or a math enthusiast exploring geometric concepts, deciphering the equation of a circle is an essential skill. In this article, we will delve into the process of identifying the center of a circle based on its algebraic equation, specifically focusing on the example provided: X^2 + y^2 + 18x + 14y + 105 = 0.

By the end of this comprehensive guide, you'll understand how to convert a general circle equation into its standard form, extract the center coordinates, and appreciate the geometric significance of these algebraic steps.

Understanding the Equation of a Circle

Before diving into the specifics of the given equation, it's important to review the standard forms of a circle's equation and how they relate to its geometric properties.

Standard Form of a Circle Equation

The most common form to represent a circle in coordinate geometry is the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:

- (h, k) is the center of the circle,
- r is the radius.

This form makes it straightforward to identify the center and radius directly.

General Form of a Circle Equation

Sometimes, the equation of a circle appears in the general quadratic form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

where D, E, F are constants. To determine the circle's center and radius, we need to convert this general form into the standard form by completing the square.

Analyzing the Given Equation

Let's revisit the specific circle equation:

$$
X^2 + y^2 + 18x + 14y + 105 = 0
$$

Note: The use of uppercase X and lowercase y is typical; here, we'll treat X as the variable x (assuming a typo or formatting inconsistency). The standard approach is to work with:

$$
x^2 + y^2 + 18x + 14y + 105 = 0
$$

Our goal is to rewrite this in standard form to identify the center.

Converting to Standard Form

The process involves completing the square for both x and y terms.

Step 1: Group the x and y terms

$$
x^2 + 18x + y^2 + 14y = -105
$$

Step 2: Complete the square for x and y

For $x^2 + 18x$:

- Take half of 18: $\frac{18}{2} = 9$
- Square it: $9^2 = 81$

For $y^2 + 14y$:

- Take half of 14: $\frac{14}{2} = 7$
- Square it: $7^2 = 49$

Add these squares to both sides of the equation to maintain equality:

$$
x^2 + 18x + 81 + y^2 + 14y + 49 = -105 + 81 + 49
$$

Simplify the right side:

$$
-105 + 81 + 49 = 25
$$

Step 3: Write the equation in completed square form

$$
(x + 9)^2 + (y + 7)^2 = 25
$$

Now, the equation is in standard form:

$$
(x + 9)^2 + (y + 7)^2 = 5^2
$$

Identifying the Center and Radius

From the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

we can directly read off:

- Center: $(-9, -7)$
- Radius: $r = 5$

Therefore, the center of the circle is at $(-9, -7)$.

Significance of the Center in Geometry

The center of a circle is a fundamental geometric property that defines the circle's position within the coordinate plane. It serves as the fixed point from which all points on the circle are equidistant, with the distance being the radius.

Geometric Importance of the Center

- The center acts as the pivot point for symmetry.
- It is crucial for calculating the circle's area and circumference.
- Knowing the center allows for graphing the circle easily.
- It is essential in transformations, such as translations and rotations.

Methods to Find the Center from the Equation

While completing the square is the most straightforward algebraic method, other approaches include:

- **Graphical Method:** Plotting the circle based on the equation.
- **Using Derivatives:** For more advanced contexts, calculus can be used to find the maximum or minimum points, which correspond to the center.
- **Coordinate Geometry Techniques:** Analyzing the coefficients in the general form.

However, algebraically, completing the square remains the most accessible and reliable method for equations in general form.

Additional Examples and Practice

To reinforce understanding, consider practicing with similar equations:

1. Convert \(x^2 + y^2 - 6x + 8y + 9 = 0\) into standard form and find its center and radius.

2. Given the equation \(x^2 + y^2 + 4x - 10y + 12 = 0\), determine the circle's center and radius.

Performing these exercises will deepen your grasp of the process.

Summary and Key Takeaways

- The given equation \(x^2 + y^2 + 18x + 14y + 105 = 0\) can be converted into standard form by completing the square.
- Completing the square involves grouping \(x\) and \(y\) terms, adding the necessary squares, and balancing the equation.
- The standard form reveals the circle's center at \((-9, -7)\) and radius 5.
- The center's coordinates are vital for understanding the circle's position, symmetry, and for graphing purposes.
- Mastery of converting general quadratic equations into standard form is essential for solving geometric problems involving circles.

Conclusion

In summary, the process of identifying the center of a circle from its algebraic equation involves rewriting the general quadratic form into the standard circle form through completing the square. For the specific equation \(x^2 + y^2 + 18x + 14y + 105 = 0\), the center is \(\boxed{(-9, -7)}\), and the radius is 5. This method not only helps in solving geometric problems but also enhances your understanding of the intrinsic relationship between algebraic expressions and geometric figures.

Whether you're analyzing mathematical problems or applying these principles in real-world contexts, mastering the technique of converting and interpreting circle equations is a valuable skill in coordinate geometry.

Frequently Asked Questions

What is the standard form of the circle's equation given as X^2 + y^2 + 18x + 14y + 105 = 0?

To convert to standard form, complete the square for x and y terms: (X + 9)^2 - 81 + (y + 7)^2 - 49 + 105 = 0, which simplifies to (X + 9)^2 + (y + 7)^2 = 25.

How do you find the center of the circle from its equation?

In the standard form (X - h)^2 + (y - k)^2 = r^2, the center is at (h, k). After completing the square, the center for the given equation is at (-9, -7).

What is the radius of the circle described by the equation X^2 + y^2 + 18x + 14y + 105 = 0?

From the standard form, the radius is √25 = 5.

Can you verify that the point (-9, -7) is the center of the circle?

Yes, after completing the square, the equation is centered at (-9, -7), confirming that this is the circle's center.

What steps are involved in converting the given general form to standard form to identify the circle's center?

First, group x and y terms, then complete the square for each group, add the necessary constants to both sides, and rewrite the equation in the form (X - h)^2 + (y - k)^2 = r^2 to find the center at (h, k).

Is the circle's center at (-9, -7) based on the original equation X^2 + y^2 + 18x + 14y + 105 = 0?

Yes, after completing the square, the center is at (-9, -7).

Additional Resources

A Certain Circle Can Be Represented By The Following Equation X^2 + y^2 + 18x + 14y + 105 = 0. What Is The Center?

Introduction: Decoding the Equation of a Circle

In the realm of coordinate geometry, the equations of geometric figures serve as the foundational language that describes their properties and positions in the plane. Among these figures, the circle holds a prominent place due to its symmetry and simplicity. The general equation of a circle in the Cartesian plane is expressed as:

$$ x^2 + y^2 + 2gx + 2fy + c = 0 $$

where (h, k) denotes the center of the circle and r its radius. Understanding how to interpret and manipulate this equation is crucial for mathematicians, students, and professionals alike.

In this article, we explore a specific circle given by the equation:

$$ x^2 + y^2 + 18x + 14y + 105 = 0 $$

Our goal is to determine the center of this circle. We will walk through the process step-by-step, highlighting the mathematical techniques involved and elucidating the underlying concepts in a clear, reader-friendly manner.

Recognizing the Standard Form of a Circle Equation

Before diving into calculations, it's essential to recognize the form of the given equation. The general form for a circle's equation in the Cartesian plane is:

$$ x^2 + y^2 + 2gx + 2fy + c = 0 $$

Here, the coefficients $2g$ and $2f$ relate directly to the circle’s center (h, k), where:

$$ h = -g \quad \text{and} \quad k = -f $$

The radius r can be found using the relation:

$$ r = \sqrt{g^2 + f^2 - c} $$

However, the given equation isn't immediately in this form, so our first step is to rewrite it into the standard form by completing the square for both x and y terms.

Step 1: Rewrite the Equation

Given:

$$ x^2 + y^2 + 18x + 14y + 105 = 0 $$

To facilitate completing the square, group the x and y terms:

$$ (x^2 + 18x) + (y^2 + 14y) = -105 $$

Our goal now is to complete the square for each group.

Step 2: Completing the Square

Completing the square is a technique used to convert quadratic expressions into perfect squares, which makes it easier to identify the center and radius of the circle.

For the x-terms:

- Take the coefficient of x, which is 18.
- Half it: $\frac{18}{2} = 9$.
- Square it: $9^2 = 81$.

Add and subtract 81 within the equation:

$$x^2 + 18x + 81 - 81$$

which simplifies as:

$$(x + 9)^2 - 81$$

For the y-terms:

- Coefficient of y: 14.
- Half: $\frac{14}{2} = 7$.
- Square: $7^2 = 49$.

Add and subtract 49:

$$y^2 + 14y + 49 - 49$$

which simplifies as:

$$(y + 7)^2 - 49$$

Step 3: Rewrite the Entire Equation

Inserting these completed squares back into the original expression:

$$(x + 9)^2 - 81 + (y + 7)^2 - 49 = -105$$

Combine the constants:

$$(x + 9)^2 + (y + 7)^2 - 81 - 49 = -105$$

$$(x + 9)^2 + (y + 7)^2 - 130 = -105$$

Add 130 to both sides:

$$ (x + 9)^2 + (y + 7)^2 = 25 $$

Step 4: Identify the Center and Radius

Now, the equation is in the standard form of a circle:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

Comparing:

$$ (x + 9)^2 + (y + 7)^2 = 25 $$

to the standard form:

$$ (x - (-9))^2 + (y - (-7))^2 = r^2 $$

we find:

- Center (h, k): $(-9, -7)$
- Radius r: $\sqrt{25} = 5$

Conclusion: The Center of the Circle

The key insight is that by completing the square, we transformed the original quadratic equation into the standard circle form, revealing the circle's center directly.

Therefore, the center of the circle described by the equation:

$$ x^2 + y^2 + 18x + 14y + 105 = 0 $$

is at the point $(-9, -7)$.

Why Is Knowing the Center Important?

Understanding the center of a circle is fundamental for numerous applications:

- Geometric Constructions: Locating points of symmetry and designing figures.
- Navigation and Mapping: Determining central points in spatial layouts.
- Physics and Engineering: Analyzing forces, motion, or fields centered around a point.
- Mathematical Analysis: Studying properties like tangents, chords, and

angles.

The process demonstrated here — completing the square — is a powerful technique used not only in geometry but across algebra and calculus, facilitating the interpretation of equations and their geometric counterparts.

Additional Insights: Radius and Diameter

While our focus was on the center, it's worth noting the radius:

$$ r = 5 $$

and the diameter:

$$ d = 2r = 10 $$

which can help in plotting or further analyzing the circle.

Final Remarks

The ability to interpret and manipulate equations of circles is a vital skill in mathematics. By systematically completing the square, we can extract meaningful geometric information from algebraic expressions. In this specific case, the circle's center at $(-9, -7)$ offers insights into its position relative to the coordinate plane, laying the groundwork for more advanced analyses or applications.

Whether you're a student tackling a homework problem or a professional working with geometric models, mastering these techniques enhances your understanding of the intrinsic connection between algebra and geometry.

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