A Committee That Consists Of Five Members Are To Be Chosen From 6 Boys And 5 Girls. Find The Number Of

A Committee That Consists Of Five Members Are To Be Chosen From 6 Boys And 5 Girls. Find The Number Of

Choosing a committee from a group of individuals involves understanding combinatorial principles and applying basic mathematical formulas. In this article, we explore the problem of selecting a five-member committee from a pool of six boys and five girls. We will analyze how to calculate the total number of possible committees, considering different scenarios such as gender-based restrictions or preferences. This comprehensive guide will help you grasp the concepts of combinations, permutation, and their applications in real-world decision-making processes.

Understanding the Problem Statement

Before diving into calculations, it is essential to understand the problem's core elements:

- Total boys available: 6
- Total girls available: 5
- Total committee members to be chosen: 5

The primary goal is to determine the number of different committees that can be formed under various conditions. These conditions may include:

- Selecting any 5 members regardless of gender.
- Selecting committees with specific gender distributions (e.g., at least one girl, only boys, etc.).
- Considering constraints such as minimum or maximum number of boys or girls.

By analyzing these scenarios, we can develop a comprehensive understanding of the combinatorial calculations involved.

Fundamental Concepts in Combinatorics

To solve the problem, we need to familiarize ourselves with key combinatorial concepts:

Combination (nCr)

- Represents the number of ways to choose r elements from a set of n elements without regard to order.
- Formula:

$$
\binom{n}{r} = \frac{n!}{r!(n-r)!}
$$

- Example: Selecting 3 boys from 6 boys:

$$
\binom{6}{3} = \frac{6!}{3! \times 3!} = 20
$$

Permutation (nPr)

- Represents the number of ways to choose r elements from n elements with regard to order.
- Not commonly used for committee selection without regard to order, but important in other contexts.

Applying Combinations to Committee Selection

- When order doesn't matter, and we're only concerned with which members are selected, combinations are used.
- Total arrangements are calculated by multiplying combinations for different groups when needed.

Calculating Total Number of Committee Combinations

The problem involves selecting any five members from a pool of 11 individuals (6 boys + 5 girls). We will explore different scenarios based on gender composition.

Scenario 1: No Restrictions

- Question: How many ways are there to select 5 members from 11 candidates?
- Calculation:

$$
\binom{11}{5} = \frac{11!}{5! \times 6!} = 462
$$

- Result: There are 462 possible committees when selecting any 5 members without restrictions.

Scenario 2: Committees with a Specific Number of Boys and

Girls

Suppose the problem specifies particular gender counts, such as:

- At least 2 boys and 3 girls
- Exactly 3 boys and 2 girls
- Other combinations

Let's analyze these in detail.

Case-by-Case Analysis of Gender Combinations

Given the total of 6 boys and 5 girls, and the need to select 5 members, the possible gender distributions are:

Number of Boys	Number of Girls	Total Members
0	5	5
1	4	5
2	3	5
3	2	5
4	1	5
5	0	5

Note: Since there are only 6 boys and 5 girls, any distribution exceeding these counts is invalid.

Calculating Each Valid Combination

For each distribution, the total number of committees is the product of the combinations of selecting the corresponding boys and girls:

$$
\text{Number of committees} = \binom{\text{number of boys}}{k} \times \binom{\text{number of girls}}{5 - k}
$$

Where k is the number of boys selected in the committee.

Let's compute these:

1. 0 Boys, 5 Girls:
\[

$$
\binom{6}{0} \times \binom{5}{5} = 1 \times 1 = 1
$$

2. 1 Boy, 4 Girls:
$$
\binom{6}{1} \times \binom{5}{4} = 6 \times 5 = 30
$$

3. 2 Boys, 3 Girls:
$$
\binom{6}{2} \times \binom{5}{3} = 15 \times 10 = 150
$$

4. 3 Boys, 2 Girls:
$$
\binom{6}{3} \times \binom{5}{2} = 20 \times 10 = 200
$$

5. 4 Boys, 1 Girl:
$$
\binom{6}{4} \times \binom{5}{1} = 15 \times 5 = 75
$$

6. 5 Boys, 0 Girls:
$$
\binom{6}{5} \times \binom{5}{0} = 6 \times 1 = 6
$$

Total number of committees with any gender distribution:

$$
1 + 30 + 150 + 200 + 75 + 6 = 462
$$

This confirms our previous total, as expected.

Additional Constraints and Customized Calculations

The problem can be extended to include various restrictions, which are common in real-world scenarios.

1. Committees with At Least One Girl

- Total committees excluding those with no girls:

$$
\text{Total committees} - \text{committees with 5 boys}
$$

- Committees with 5 boys:

$$
\binom{6}{5} \times \binom{5}{0} = 6 \times 1 = 6
$$

- Therefore,

$$
462 - 6 = 456
$$

Result: There are 456 committees with at least one girl.

2. Committees with At Least One Boy

- Committees with no boys (all girls):

$$
\binom{6}{0} \times \binom{5}{5} = 1 \times 1 = 1
$$

- Committees with at least one boy:

$$
462 - 1 = 461
$$

Result: There are 461 committees with at least one boy.

3. Committees with Equal Number of Boys and Girls

- Since total is 5, equal division is not possible (as 2.5 each is not valid), but near-equal distributions are:

Boys	Girls	Count
2	3	150
3	2	200

- Total committees with near-equal gender distribution:

$$

$$150 + 200 = 350$$

Real-World Applications of Combination Calculations in Committee Selection

Understanding how to calculate combinations for selecting committees has practical implications in various fields:

- Educational institutions: Forming student committees, project teams, or academic panels.
- Corporate environments: Selecting teams for projects or task forces.
- Event organizations: Creating diverse panels or planning committees.
- Government and policymaking: Forming advisory groups with specific gender or skill distributions.

By mastering the principles of combinatorics, decision-makers can evaluate the number of possible arrangements efficiently and optimize their selection processes.

Strategies for Efficient Calculation of Combinations

When dealing with larger groups or more complex restrictions, the following strategies can streamline calculations:

- Break down the problem into manageable parts: Use case-by-case analysis.
- Apply symmetry and complementary counting: Count the total options and subtract unwanted scenarios.
- Use factorial simplification: Recognize common factors in binomial coefficients.
- Leverage computational tools: Use calculators or software for large factorial computations.

Summary and Key Takeaways

- The total number of committees can be calculated using the combination formula $\binom{n}{r}$.
- For the problem of choosing 5 members from 6 boys and 5 girls, the total possible committees are 462.
- Considering gender distributions provides insight into the number of committees fitting specific criteria.
- Constraints like "at least one girl" or "at least one boy" can be easily incorporated through subtraction.

Frequently Asked Questions

How many ways can a committee of five members be chosen from 6 boys and 5 girls?

The total number of ways is calculated by choosing any 5 members from the 11 individuals, which is C(11, 5) = 462.

What is the number of committees consisting of exactly 3 boys and 2 girls?

Number of such committees is C(6, 3) C(5, 2) = 20 10 = 200.

How many committees can be formed with at least 2 girls?

Total committees with at least 2 girls are the sum of committees with 2, 3, 4, or 5 girls: C(6, 3)C(5, 2) + C(6, 4)C(5, 1) + C(6, 5)C(5, 0) = 200 + 90 + 6 = 296.

How many committees are formed with no girls (all boys)?

Number of committees with all boys is C(6, 5) = 6.

How many committees include at least one girl?

Total committees minus those with no girls: 462 - 6 = 456.

What is the number of committees formed with exactly 4 boys and 1 girl?

Number of such committees is C(6, 4) C(5, 1) = 15 5 = 75.

Are there more committees with at least 3 boys or at least 3 girls?

Number of committees with at least 3 boys can be calculated similarly; overall, there are more committees with at least 3 boys (or girls) depending on the combinations, but exact counts require specific calculations.

How many committees can be formed with more girls than boys?

Committees with 3 girls and 2 boys or 4 girls and 1 boy or 5 girls and 0 boys: C(5, 3)C(6, 2) + C(5, 4)C(6, 1) + C(5, 5)C(6, 0) = 1015 + 56 + 11 = 150 + 30 + 1 = 181.

What is the total number of committees with at least one girl and at least one boy?

Total committees minus those with only boys or only girls: 462 - (committees with only boys + committees with only girls) = 462 - (6 + 0) = 456.

Additional Resources

A Committee That Consists Of Five Members Are To Be Chosen From 6 Boys And 5 Girls

---

Introduction

In the realm of combinatorial mathematics and decision-making processes, the problem of selecting a committee from a larger group presents a fascinating case study. Specifically, the problem statement: "A committee that consists of five members is to be chosen from 6 boys and 5 girls" invites exploration into the principles of counting, probability, and strategic selection. Such problems are not only theoretical exercises but also have practical applications in organizational planning, event management, and social sciences.

This article aims to explore the problem in depth, analyzing various aspects such as the total number of possible committees, the constraints involved, and the implications of different selection criteria. We will examine the problem from multiple perspectives, including general combinatorial calculations, possible restrictions (such as gender-based quotas), and extensions to more complex scenarios.

Understanding the Problem: Basic Principles

At the core of the problem is a straightforward combinatorial question: How many different committees of five members can be formed from a group of 11 individuals (6 boys and 5 girls)?

- Total group size: 11 (6 boys + 5 girls)
- Committee size: 5 members

The fundamental approach involves the concept of combinations, denoted as $\binom{n}{k}$, which represents the number of ways to choose k elements from a set of n elements without regard to order.

Basic Calculation: Total Number of Committees

Without any restrictions, the total number of possible committees is calculated as:

\[
\text{Total committees} = \binom{11}{5}

\]

Calculating this:

\[
\binom{11}{5} = \frac{11!}{5! \times (11 - 5)!} = \frac{11!}{5! \times 6!}
\]

Breaking down the factorials:

\[
= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} = \frac{55440}{120} = 462
\]

Result: There are 462 possible committees when selecting any 5 members from the group of 11 individuals.

Considering Gender-Based Constraints

While the total number of committees provides a baseline, real-world scenarios often impose additional constraints, such as ensuring gender representation. Suppose the problem specifies certain conditions, such as:

- The committee must include at least one girl
- The committee must include exactly two boys and three girls
- The committee must include a certain number of boys and girls

Each of these constraints alters the calculation, leading to more nuanced combinatorial considerations.

Case 1: Committees with at least one girl

Complementary counting offers an efficient method here. Instead of directly counting committees with at least one girl, we calculate the total committees and subtract the number of committees with no girls (i.e., all boys).

- Number of committees with no girls: select all 5 members from the 6 boys

\[
\binom{6}{5} = 6
\]

- Therefore, number of committees with at least one girl:

\[
462 - 6 = 456

\]

Result: There are 456 committees with at least one girl.

Case 2: Committees with exactly two boys and three girls

Here, the selection involves choosing:

- 2 boys from 6
- 3 girls from 5

The total number of such committees is:

\[
\binom{6}{2} \times \binom{5}{3}
\]

Calculations:

\[
\binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 15
\]

\[
\binom{5}{3} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10
\]

Multiplying:

\[
15 \times 10 = 150
\]

Result: There are 150 committees with exactly two boys and three girls.

Case 3: Committees with minimum number of girls

Suppose the requirement is that the committee must have at least three girls. Then, possible compositions are:

- 2 boys and 3 girls
- 1 boy and 4 girls
- 0 boys and 5 girls

Calculations for each:

1. 2 boys and 3 girls:

$$
\binom{6}{2} \times \binom{5}{3} = 15 \times 10 = 150
$$

2. 1 boy and 4 girls:

$$
\binom{6}{1} \times \binom{5}{4} = 6 \times 5 = 30
$$

3. 0 boys and 5 girls:

$$
\binom{6}{0} \times \binom{5}{5} = 1 \times 1 = 1
$$

Total committees with at least 3 girls:

$$
150 + 30 + 1 = 181
$$

Variations and Extensions

The above calculations demonstrate the versatility of basic combinatorial methods. However, real-world constraints may complicate matters further.

1. Gender Quotas and Proportional Representation

Organizations sometimes require minimum or maximum numbers of certain genders. For instance, a committee might need:

- At least 2 boys
- At least 2 girls

This creates multiple overlapping cases, which can be addressed using inclusion-exclusion principles.

2. Sequential Selection and Hierarchical Structures

In some contexts, the selection process might be hierarchical, involving first choosing a subgroup, then selecting the remaining members based on specific criteria.

3. Weighted or Probability-Based Selection

Instead of counting combinations, probabilistic models might be employed, especially when selection is random with known probabilities for each individual.

Practical Implications and Applications

Understanding the combinatorial possibilities in such selection problems is crucial in various domains:

- Educational Committees: Determining fair representation among students
- Corporate Boards: Ensuring diversity and compliance with regulations
- Event Planning: Creating diverse panels or committees
- Research Studies: Sampling strategies for balanced representation

Moreover, the principles discussed extend to larger and more complex groups, emphasizing the importance of combinatorial literacy in decision-making processes.

Limitations and Assumptions

While the calculations presented are mathematically sound, they rely on certain assumptions:

- All individuals are equally eligible and distinguishable
- The order of selection does not matter
- No additional constraints (e.g., skill, experience) are considered

In real-world scenarios, these factors may influence the actual number of feasible committees, requiring more sophisticated models.

Conclusion

The problem of selecting a five-member committee from a group of six boys and five girls exemplifies fundamental concepts in combinatorics, with applications spanning various fields. The total number of possible committees, without restrictions, is 462. When constraints such as gender balance or minimum representation are introduced, the calculations become more nuanced, often reducing the total number of feasible committees.

Understanding these combinatorial principles equips decision-makers, educators, and researchers with the tools necessary to analyze and optimize selection strategies in diverse contexts. As groups grow larger and constraints become more complex, the importance of systematic counting methods and probabilistic reasoning only increases.

In summary, "A Committee That Consists Of Five Members Are To Be Chosen From 6 Boys And 5 Girls" is not just a straightforward counting problem but a gateway into the rich, practical world of combinatorial analysis and decision science.

References:

- Graham, R. L., Knuth, D. E., & Patashnik, O. (1994). Concrete Mathematics. Addison-Wesley.
- Stanley, R. P. (2011). Enumerative Combinatorics. Cambridge University Press.

- Rosen, K. H. (2012). Discrete Mathematics and Its Applications. McGraw-Hill Education.

A Committee That Consists Of Five Members Are To Be Chosen From 6 Boys And 5 Girls Find The Number Of

A Committee That Consists Of Five Members Are To Be Chosen From 6 Boys And 5 Girls Find The Number Of

Related Articles

- A Shipping Crate Is A Perfect Cube With Volume Of 2,197 Cubic Feet. The Ceiling Of The Warehouse Is 14
- After Reading The Syllabus And The Rhetorical Modes Handout, Please Write A Paragraph Or Two Comparing
- A Runner Has Two Pairs Of Running Shoes. To Find Out Which Pair Will Allow Her To Run Faster, She Puts

Back to Home