A Convex Spherical Mirror Has A Radius Of Curvature Of Magnitude 44.0 Cm.(a) Determine The Position Of

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Introduction to Convex Spherical Mirrors

Convex spherical mirrors are widely used in various applications such as vehicle side mirrors, security mirrors, and optical devices. Understanding their properties, especially the relationship between their radius of curvature, focal length, and the position of images, is fundamental in optics. This article explores how to determine the position of the image formed by a convex mirror with a known radius of curvature, focusing on the case where the radius of curvature is 44.0 cm.

Basics of Spherical Mirrors

Types of Spherical Mirrors

Spherical mirrors are classified based on the nature of the reflecting surface:

- **Concave Mirrors:** Curved inward, converging light rays to a focal point, used in reflectors and telescopes.
- **Convex Mirrors:** Curved outward, diverging light rays, commonly used in vehicle side mirrors for a wider field of view.

Sign Convention

In optics, sign conventions are essential for consistent calculations:

- Distances measured along the principal axis are positive if measured toward the mirror and negative if measured away from the mirror.

- The focal length (f) is positive for converging mirrors (concave) and negative for diverging mirrors (convex).

- The radius of curvature (R) relates to the focal length as: $f = R / 2$.

- Image distances (v) are positive if the image is real and on the same side as the object; negative if virtual and on the opposite side.

Understanding the Geometry of a Convex Mirror

Radius of Curvature and Focal Length

Given the radius of curvature $R = 44.0\, \text{cm}$, the focal length f of the convex mirror is:

$$
f = - \frac{R}{2} = - \frac{44.0\, \text{cm}}{2} = -22.0\, \text{cm}
$$

The negative sign indicates that the mirror is convex, diverging light rays.

Object and Image Positions

Suppose an object is placed in front of the convex mirror at a distance u from the mirror. The goal is to find the position of the image v.

The mirror equation relates object distance u, image distance v, and focal length f:

$$
\frac{1}{f} = \frac{1}{v} + \frac{1}{u}
$$

Rearranged to find v:

$$
v = \left(\frac{1}{\frac{1}{f} - \frac{1}{u}} \right)
$$

Knowing f and u, one can determine v.

Determining the Image Position: Step-by-Step Approach

Step 1: Identify the Object Distance u

In practical scenarios, the object could be a real object placed at a certain distance from the mirror. For this analysis, assume the object is at a distance u from the mirror. For example, let's consider $u = 100\, \text{cm}$.

Step 2: Apply the Mirror Equation

Using the known values:

$$
f = -22.0\, \text{cm}
$$
$$
u = 100\, \text{cm}
$$

Calculate v:

$$
\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-22.0} - \frac{1}{100} \approx -0.04545 - 0.01 = -0.05545
$$

$$
v = \frac{1}{-0.05545} \approx -18.03\, \text{cm}
$$

Interpretation: The negative value indicates that the image is virtual and located behind the mirror at approximately 18.03 cm.

Step 3: Generalize for Different Object Distances

The same method applies for any object distance u:

- As u increases (object moves farther away), the image moves closer to the focal point.
- As u decreases (object approaches the mirror), the image moves further behind the mirror.

Significance of the Image Position

Virtual and Upright Images

For convex mirrors:

- The images are virtual, meaning they appear to be behind the mirror.
- The images are upright and reduced in size compared to the actual object.

Implications in Practical Applications

- In vehicle mirrors, the virtual image allows the driver to see a wide area.
- The image's position affects how the driver perceives the distance and size of objects.

Additional Factors Affecting Image Position

Object Distance Variations

Changing the object distance u influences the image position v. Closer objects produce images farther behind the mirror, while distant objects produce images nearer to the mirror.

Mirror Manufacturing Tolerances

In real-world applications, slight variations in the radius of curvature can affect the focal length and, consequently, the image position.

Practical Example: Calculating Image Position for a Specific Object Distance

Suppose an object is placed 50 cm in front of the convex mirror:

- $u = 50\, \text{cm}$
- $f = -22.0\, \text{cm}$

Calculate v:

$$
\frac{1}{v} = \frac{1}{-22.0} - \frac{1}{50} \approx -0.04545 - 0.02 = -0.06545
$$

$$
v \approx \frac{1}{-0.06545} \approx -15.28\, \text{cm}
$$

The image is virtual, upright, and approximately 15.28 cm behind the mirror.

Summary and Key Takeaways

- The radius of curvature R of a convex mirror directly determines its focal length $f = - R/2$.
- The position of the virtual image v depends on the object distance u and the mirror's focal length.
- The mirror equation $1/f = 1/v + 1/u$ enables calculation of the image position.
- For convex mirrors with $R = 44.0\, \text{cm}$, the focal length is $-22.0\, \text{cm}$, and images are always virtual, upright, and diminished.
- Adjusting the object distance alters the image's position, impacting practical applications like vehicle mirrors.

Conclusion

Understanding the relationship between the radius of curvature, focal length, and image position in convex spherical mirrors is vital for both theoretical physics and practical applications. By applying the mirror equation and sign conventions, one can accurately determine where the virtual image appears relative to the mirror for any given object distance. In the case of a mirror with a radius of curvature of 44.0 cm, the focal length of $-22.0\, \text{cm}$ serves as a critical parameter in these calculations, ensuring effective design and utilization of such optical devices.

Frequently Asked Questions

What is the significance of the radius of curvature in a convex spherical mirror?

The radius of curvature determines the curvature of the mirror surface, affecting how it reflects light and forms images; a larger radius results in a flatter mirror, while a smaller radius produces a more curved surface.

How do you determine the focal length of a convex mirror with a radius of curvature of 44.0 cm?

The focal length (f) of a convex mirror is given by $f = R / 2$, so for $R = 44.0$ cm, $f = 22.0$ cm.

What is the formula used to find the position of the image formed by a convex mirror?

The mirror formula is $1/f = 1/do + 1/di$, where do is the object distance, di is the image distance, and f is the focal length.

If an object is placed 100 cm in front of the convex mirror with a radius of curvature 44.0 cm, where will the virtual image be formed?

Using the mirror formula: $1/22 = 1/100 + 1/di$, so $1/di = 1/22 - 1/100 = (50/1100 - 11/1100) = 39/1100$, thus $di \approx 28.2$ cm behind the mirror.

Is the image formed by a convex mirror real or virtual?

The image formed by a convex mirror is always virtual, upright, and diminished.

How does the size of the image change as the object moves farther from the convex mirror?

As the object moves farther away, the image becomes smaller and closer to the focal point behind the mirror.

What is the magnification produced by a convex mirror with a radius of curvature of 44.0 cm for an object placed at 100 cm?

Magnification (m) = -di / do. Using di ≈ 28.2 cm, m = -28.2 / 100 ≈ -0.28, indicating a virtual, upright, reduced image.

Can a convex mirror form a real image? Why or why not?

No, a convex mirror cannot form a real image because the reflected rays diverge, and the image appears to be located behind the mirror.

What practical applications utilize convex spherical mirrors with a radius of curvature around 44.0 cm?

Convex mirrors of this size are commonly used in vehicle side mirrors, security mirrors, and decorative applications due to their wide field of view and safety features.

How do you calculate the position of the virtual image in a convex mirror given the object distance and known radius of curvature?

First, find the focal length (f = R/2), then use the mirror formula 1/f = 1/do + 1/di to solve for di, which will be negative, indicating a virtual image behind the mirror.

Additional Resources

A Convex Spherical Mirror Has A Radius Of Curvature Of Magnitude 44.0 Cm. (a) Determine The Position Of

Understanding how convex spherical mirrors work is fundamental in optics, especially in everyday devices like vehicle side mirrors, security mirrors, and optical instruments. In this article, we delve into the detailed process of determining the position of an object relative to a convex mirror that has a specified radius of curvature. By examining the principles of mirror optics, the relationships between object distance, image distance, and mirror curvature, and applying relevant equations, we aim to clarify how to find the position of an object when given the mirror's radius of curvature.

Fundamentals of Convex Spherical Mirrors

Before tackling the problem directly, it’s crucial to understand the basic properties and principles governing convex spherical mirrors.

What Is a Convex Spherical Mirror?

A convex spherical mirror is a mirror with a reflective surface that bulges outward, forming part of a sphere’s surface. Its outward curvature causes incident light rays to diverge after reflection, producing virtual, erect, and diminished images. These mirrors are widely used in rear-view mirrors of vehicles, security setups, and optical devices because they provide a wider field of view.

Key Properties of Convex Mirrors

- Focal Length (f): The distance from the mirror's surface to its focal point, where incident rays appear to diverge from after reflection.
- Radius of Curvature (R): The radius of the sphere of which the mirror is a part. It relates directly to the mirror's focal length.
- Image and Object Distances: The positions of the object and its reflected image relative to the mirror are governed by the mirror equation.
- Virtual Images: Convex mirrors always produce virtual images that are upright and reduced in size, located behind the mirror.

Relation Between Radius of Curvature and Focal Length

The foundation of analyzing mirror images lies in the relationship between the radius of curvature (R) and the focal length (f).

Mirror Equation and Sign Conventions

The standard mirror equation is:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Where:
- d_o = object distance (positive if object is in front of the mirror)
- d_i = image distance (positive if image is real and in front of the mirror; negative if virtual and behind)
- f = focal length of the mirror

For convex mirrors:
- The focal length f is positive.
- The radius of curvature R relates to f as:

$$R = 2f$$

This relation holds because the mirror's focal point is located at half the radius of curvature, on the principal axis.

Calculating the Focal Length from the Radius of Curvature

Given that the radius of curvature R is 44.0 cm:

$$f = \frac{R}{2} = \frac{44.0\, \text{cm}}{2} = 22.0\, \text{cm}$$

Since the mirror is convex, the focal length is taken as positive:

$$f = +22.0\, \text{cm}$$

Determining the Position of the Object

The core of the problem involves finding the object position relative to the mirror, given the mirror's properties. Typically, in such problems, the object distance d_o is specified or to be determined based on a known image position. If the problem provides a specific image position or other data, the process unfolds accordingly.

Suppose, for illustrative purposes, the goal is to determine the object position when a certain image is formed at a known location. Let's explore the general approach.

Step 1: Establish Known Quantities

- Radius of curvature: $R = 44.0\, \text{cm}$
- Focal length: $f = +22.0\, \text{cm}$

Additional data needed (for example):

- Image distance (d_i): the position where the virtual image appears
- Object distance (d_o): to be determined

Step 2: Applying the Mirror Equation

Rearranged for object distance:

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}$$

Or, equivalently, if the image distance is known:

$$d_o = \left(\frac{1}{f} - \frac{1}{d_i} \right)^{-1}$$

Given the nature of convex mirrors, the image distance d_i is negative, indicating a virtual image behind the mirror.

Step 3: Example Calculation

Suppose the virtual image is located at $d_i = -30\, \text{cm}$:

$$d_i = -30\, \text{cm} \quad (\text{virtual, behind the mirror})$$

Calculating the object distance:

$$\frac{1}{d_o} = \frac{1}{22} - \frac{1}{-30} = \frac{1}{22} + \frac{1}{30}$$

Calculating:

$$\frac{1}{22} \approx 0.04545$$
$$\frac{1}{30} \approx 0.03333$$

Sum:

$$0.04545 + 0.03333 = 0.07878$$

Thus:

$$d_o = \frac{1}{0.07878} \approx 12.7\, \text{cm}$$

This indicates the object is approximately 12.7 cm in front of the mirror.

Implications and Practical Applications

Understanding the positions of objects relative to convex mirrors has real-world implications:

- Safety and Visibility: Vehicle side mirrors are convex precisely because they allow drivers to see a wider field of view, albeit with smaller, virtual images.
- Design Parameters: Engineers select specific radii of curvature and placement to optimize the field of view and image clarity.
- Optical Instrumentation: Precise calculations enable the design of optical tools such as periscopes and surveillance systems.

Conclusion

Determining the position of an object relative to a convex spherical mirror involves a clear understanding of the mirror's properties, the relationships between radius of curvature, focal length, and image/object distances, and the application of the mirror equation. With a radius of curvature of 44.0 cm, the focal length is 22.0 cm, guiding calculations that reveal how objects are perceived through these reflective surfaces.

Whether for safety, scientific, or technological purposes, mastering these principles allows for accurate predictions of image locations, enhancing the effective use of convex mirrors in various domains. As demonstrated, even a straightforward property like the radius of curvature can unlock detailed insights into the behavior of light and images, underpinning many modern optical applications.

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