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A Fair Coin Is Tossed Three Times. Let Be The Number Of Heads On The First Toss And Be The Total Number Of Heads In All Three Tosses

Consider a scenario where a fair coin is tossed three times in succession. Each toss is independent, with a probability of 0.5 for landing heads (H) and 0.5 for tails (T). In this context, let B represent the number of heads obtained on the very first toss, which can only be 0 (if tails) or 1 (if heads). Let A denote the total number of heads obtained after all three tosses. This setup allows us to analyze the joint and marginal probabilities, explore various conditional probabilities, and understand the underlying distributions governing the outcomes. The goal is to develop a comprehensive understanding of the probabilistic relationships involving B and A, and to illustrate how these variables interact within the framework of probability theory.

Understanding the Basic Setup and Variables

Defining the Random Variables

- **B**: The number of heads on the first toss.
- **A**: The total number of heads in all three tosses.

Since the coin is fair, each individual toss has two equally likely outcomes. The variable B is straightforward: it can be either 0 or 1, with probabilities:

- P(B=1) = 0.5
- P(B=0) = 0.5

The variable A, representing the total number of heads in three tosses, can take values 0,

1, 2, or 3. The possible values of A depend on the sequence of outcomes, which are all equally likely, given the independence and fairness of the coin.

Sample Space and Possible Outcomes

Enumerating All Outcomes

Each toss can be Heads (H) or Tails (T). The sample space Ω for three tosses contains 2^3 = 8 outcomes:

1. HHH