A Fair Coin Is Tossed Twice. Calculate The Probabilities. (Enter Your Probabilities As Fractions.) (a)

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When analyzing the outcomes of tossing a fair coin twice, understanding the underlying probability principles is essential. Each toss of a fair coin has two equally likely outcomes: Heads (H) or Tails (T). Since the coin is fair, the probability of getting Heads or Tails in a single toss is exactly 1/2. When the coin is tossed twice, the total number of possible outcomes increases, and each outcome's probability can be calculated based on the independence of the individual tosses. This article explores the full set of possible outcomes, their probabilities, and how to compute the probabilities of various combined events, all expressed as fractions.

Understanding the Sample Space

Defining the Sample Space

- The sample space is the set of all possible outcomes of an experiment.
- For tossing a fair coin twice, each outcome is a sequence of two results, each being either H or T.
- Since each toss is independent and has 2 outcomes, the total number of outcomes is $2 \times 2 = 4$.

Listing the Outcomes

1. HH (Heads on the first toss and Heads on the second)
2. HT (Heads on the first toss and Tails on the second)
3. TH (Tails on the first toss and Heads on the second)
4. TT (Tails on the first toss and Tails on the second)

Calculating Basic Probabilities

Probability of Each Outcome

Since the coin is fair and each toss is independent, each of the four outcomes has an equal probability:

- P(HH) = 1/4
- P(HT) = 1/4
- P(TH) = 1/4
- P(TT) = 1/4

These probabilities are derived from the fact that each individual toss has a probability of 1/2, and the outcomes are independent:

P(H on first toss) = 1/2
P(H on second toss) = 1/2
Therefore, for HH: P(H) × P(H) = (1/2) × (1/2) = 1/4

Probability of Specific Events

Beyond individual outcomes, we often want to calculate the probability of more general events, such as "getting at least one Heads" or "getting exactly one Heads."

Calculating Probabilities of Specific Events

Event A: Both Tosses Result in Heads (HH)

- This is a single outcome with probability 1/4.
- So, P(HH) = 1/4.

Event B: Both Tosses Result in Tails (TT)

- This is a single outcome with probability 1/4.
- So, P(TT) = 1/4.

Event C: Exactly One Heads (HT or TH)

- These are two outcomes: HT and TH.
- Each has probability 1/4.
- The combined probability is: P(HT) + P(TH) = 1/4 + 1/4 = 2/4 = 1/2.

Event D: At Least One Heads

This includes outcomes with one or two Heads: HH, HT, and TH.

- P(HH) = 1/4
- P(HT) = 1/4
- P(TH) = 1/4

Sum these probabilities:

P(at least one Heads) = 1/4 + 1/4 + 1/4 = 3/4.

Event E: No Heads (i.e., Both Tails)

- This is the outcome TT, with probability 1/4.

Complementary Probabilities

Understanding Complements

In probability, the complement of an event is the set of all outcomes not in the event. The probability of an event plus the probability of its complement equals 1.

Calculating the Probability of the Complement

- For example, the probability of "no Heads" (both Tails) is 1/4.

- Thus, the probability of "at least one Heads" is 1 - P(no Heads) = 1 - 1/4 = 3/4.

Summary of Probabilities

Event	Outcome(s)	Probability (as fractions)
Both Heads (HH)	HH	1/4
Both Tails (TT)	TT	1/4
Exactly One Heads (HT or TH)	HT, TH	2/4 = 1/2
At Least One Heads	HH, HT, TH	3/4
No Heads (both Tails)	TT	1/4

Conclusion

In summary, when tossing a fair coin twice, each of the four possible outcomes (HH, HT, TH, TT) has an equal probability of 1/4. By understanding the basic principles of probability, independence, and the sample space, we can compute the likelihood of various combined events as fractions. These calculations demonstrate fundamental concepts in probability theory, such as the use of the sample space, the addition rule for mutually exclusive events, and the complement rule. Mastery of these fundamental calculations forms the basis for analyzing more complex probabilistic scenarios.

Frequently Asked Questions

What is the probability of getting exactly one head when a fair coin is tossed twice?

The probability is 2/4, which simplifies to 1/2.

What is the probability of getting two tails when a fair coin is tossed twice?

The probability is 1/4.

What is the probability of getting at least one head in two tosses of a fair coin?

The probability is 3/4.

What is the probability of getting no heads (i.e., two tails) in two coin tosses?

The probability is 1/4.

What is the probability of getting two heads in two tosses of a fair coin?

The probability is 1/4.

Are the outcomes of each coin toss independent in this experiment?

Yes, each toss is independent, meaning the outcome of one does not affect the other.

What is the total number of equally likely outcomes when tossing a fair coin twice?

There are 4 outcomes: HH, HT, TH, TT.

Additional Resources

A Fair Coin Is Tossed Twice. Calculate The Probabilities. (Enter Your Probabilities As Fractions.) (a)

When exploring basic probability concepts, few examples are as instructive and intuitive as tossing a fair coin twice. This simple experiment helps illustrate fundamental principles such as sample spaces, outcomes, and calculating probabilities using fractions. Whether you're a student studying probability theory or a curious mind exploring randomness, understanding how to compute these probabilities provides a solid foundation for more complex statistical analyses.

In this article, we will delve into the problem: A fair coin is tossed twice. Calculate the probabilities. (Enter your probabilities as fractions.) We will break down the problem systematically, define key concepts, and walk through the calculations step-by-step. By the end, you'll have a clear understanding of how to approach similar probability problems with confidence.

Understanding the Basics of Probability

Before diving into calculations, it is essential to understand some basic terminology:

- Experiment: An action that results in one or more outcomes (e.g., tossing a coin twice).
- Sample Space (S): The set of all possible outcomes of an experiment.
- Event: A subset of the sample space, representing outcomes of interest.
- Probability of an event (P): A measure of the likelihood that an event occurs, calculated as the ratio of favorable outcomes to total outcomes, assuming each outcome is equally likely.

In the case of tossing a fair coin (which has an equal chance of landing heads (H) or tails (T)), each toss is independent, and the probability of heads or tails on a single toss is 1/2.

Step 1: Define the Sample Space

The first step in solving probability problems involving multiple coin tosses is to identify the sample space. Since the coin is tossed twice, each toss can result in either heads or tails. The total number of possible outcomes can be determined using the multiplication principle:

- Number of outcomes for the first toss: 2 (H or T)
- Number of outcomes for the second toss: 2 (H or T)

Total outcomes: 2 × 2 = 4

Listing All Outcomes

Let's write out all possible outcomes explicitly:

1. HH (first toss Heads, second toss Heads)
2. HT (first toss Heads, second toss Tails)
3. TH (first toss Tails, second toss Heads)
4. TT (first toss Tails, second toss Tails)

Thus, the sample space S = {HH, HT, TH, TT}.

Step 2: Assign Probabilities to Outcomes

Since the coin is fair, each individual outcome in the sample space is equally likely. The probability of each outcome is:

$$P(\text{each outcome}) = \frac{1}{\text{total number of outcomes}} = \frac{1}{4}$$

This is a key assumption: each of the four outcomes has an equal chance of occurring.

Step 3: Identify the Event of Interest

The problem asks us to calculate the probabilities of various outcomes or events based on tossing the coin twice. Since the prompt specifies (a), let's assume the event of interest is:

Example Event (a): "Getting exactly one head in two tosses"

This is a common event to analyze, and it helps illustrate how to calculate probabilities for specific outcomes.

The outcomes where exactly one head appears are:

- HT
- TH

Step 4: Calculate the Probability of the Event

The probability of an event is calculated using the formula:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total outcomes in the sample space}}$$

In our case:

- Favorable outcomes for getting exactly one head: HT, TH
- Total outcomes: 4 (from the sample space)

Therefore:

$$ P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2} $$

Answer for (a): The probability of getting exactly one head in two tosses is 1/2.

Step 5: Explore Other Possible Events

To deepen understanding, let's consider additional events and their probabilities. You can extend this approach to any event within the sample space.

Event (b): "Getting two heads"

Favorable outcomes: HH

Total outcomes: 4

$$ P(\text{two heads}) = \frac{1}{4} $$

Event (c): "Getting no heads" (i.e., two tails)

Favorable outcomes: TT

Total outcomes: 4

$$ P(\text{no heads}) = \frac{1}{4} $$

Event (d): "Getting at least one head"

Favorable outcomes: HH, HT, TH

Total outcomes: 4

$$ P(\text{at least one head}) = \frac{3}{4} $$

Important Concepts Illustrated

- Mutually exclusive outcomes: Outcomes like HH and HT cannot occur simultaneously.
- Complementary events: The probability of "getting at least one head" (event A) is complemented by "getting no heads" (event A'):

$$ P(\text{at least one head}) + P(\text{no heads}) = 1 $$

- Addition rule: For mutually exclusive events, the probability of either event occurring is the sum of individual probabilities.

Summary of Probabilities as Fractions

Event	Probability as Fraction
Exactly one head in two tosses	1/2
Two heads	1/4
No heads (both tails)	1/4
At least one head	3/4

Extending to Other Scenarios

While this example focused on two coin tosses, the principles extend to more complex situations:

- Multiple tosses: The sample space grows exponentially, but the same logic applies.
- Biased coins: If the coin isn't fair, probabilities for heads and tails would differ, requiring adjusted calculations.
- Multiple events: Calculating joint probabilities, conditional probabilities, and using rules like Bayes' theorem.

Final Remarks

Calculating probabilities for tossing a fair coin twice is a fundamental exercise that builds intuition around randomness and outcomes. By systematically listing outcomes, assigning equal probabilities, and identifying favorable events, you can accurately compute the likelihood of various scenarios. Remember to always define your sample space precisely, consider whether events are mutually exclusive, and use fractions to express probabilities clearly.

This foundational understanding paves the way for tackling more advanced probability problems encountered in statistics, gaming, decision theory, and beyond.

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