A) Find The Slope Of The Curve Y=x^3 -12x At The Given Point P(1,-11) By Finding The Limiting Value Of

A) Find The Slope Of The Curve Y=x^3 -12x At The Given Point P(1,-11) By Finding The Limiting Value Of

Understanding how to find the slope of a curve at a specific point is fundamental in calculus. In this article, we will explore the process of determining the slope of the curve defined by the function $y = x^3 - 12x$ at the point $P(1, -11)$. This involves using the concept of limits, specifically by calculating the limiting value of the difference quotient as h approaches zero. This method not only provides the slope at the point but also deepens understanding of the derivative as a fundamental concept in calculus.

Introduction to the Derivative and Slope of a Curve

Before diving into the specific problem, it’s essential to understand the core concepts involved:

What Is the Derivative?

- The derivative of a function at a given point measures the rate at which the function's value changes with respect to changes in the input.
- Geometrically, the derivative at a point corresponds to the slope of the tangent line to the curve at that point.

Why Is the Slope Important?

- The slope indicates how steep the curve is at a particular point.
- It is crucial in various applications, including physics (velocity), engineering (stress analysis), and economics (marginal cost).

Understanding the Function $y = x^3 - 12x$

The function in question is a cubic polynomial:

$$
y = x^3 - 12x
$$

Key features include:
- The function's shape, which has inflection points and local extrema.
- Its derivative, which describes the slope at any point x.

Finding the Slope at Point P(1, -11): Step-by-Step Approach

To find the slope of the curve at $P(1, -11)$, we employ the limit definition of the derivative.

Step 1: Recall the Limit Definition of the Derivative

The slope m of the curve at a point $x = a$ is given by:

$$
m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
$$

Where:
- $f(x) = x^3 - 12x$
- $a = 1$

Step 2: Write the Difference Quotient

Calculate:

$$
\frac{f(1 + h) - f(1)}{h}
$$

which expands to:

$$
\frac{(1 + h)^3 - 12(1 + h) - [1^3 - 12(1)]}{h}
$$

Step 3: Simplify the Expression

Compute $f(1 + h)$:

$$

(1 + h)^3 - 12(1 + h) = (1 + 3h + 3h^2 + h^3) - 12 - 12h
\]

And \(f(1) \):

\[
1^3 - 12(1) = 1 - 12 = -11
\]

Now, the difference:

\[
[(1 + 3h + 3h^2 + h^3) - 12 - 12h] - (-11) = (1 + 3h + 3h^2 + h^3 - 12 - 12h) + 11
\]

Simplify numerator:

\[
(1 - 12 + 11) + 3h - 12h + 3h^2 + h^3 = 0 - 9h + 3h^2 + h^3
\]

Therefore, the difference quotient becomes:

\[
\frac{-9h + 3h^2 + h^3}{h}
\]

Step 4: Simplify the Quotient

Factor \(h \) out:

\[
\frac{h(-9 + 3h + h^2)}{h}
\]

Cancel \(h \) (for \(h \neq 0 \)):

\[
-9 + 3h + h^2
\]

Step 5: Take the Limit as \(h \to 0 \)

Evaluate:

\[
\lim_{h \to 0} (-9 + 3h + h^2) = -9 + 0 + 0 = -9

\]

Thus, the slope of the curve at $P(1, -11)$ is -9.

Interpreting the Result

The calculated derivative:

$$
\boxed{
\frac{dy}{dx} \bigg|_{x=1} = -9
}
$$

indicates that at the point $P(1, -11)$:

- The slope of the tangent line to the curve is -9.
- The negative sign signifies that the curve is decreasing at this point.
- The steepness of the slope reflects how rapidly the function's value changes around $x=1$.

Additional Insights: Derivative of $y = x^3 - 12x$

While the limit process provides the slope at a specific point, understanding the derivative function over all x offers broader insights.

Derivative Function

Using power rule:

$$
\frac{dy}{dx} = 3x^2 - 12
$$

At $x = 1$:

$$
3(1)^2 - 12 = 3 - 12 = -9
$$

which confirms our earlier calculation.

Implications of the Derivative Function

- The derivative $3x^2 - 12$ is a quadratic function.
- It determines where the function y is increasing or decreasing.
- Critical points occur where $3x^2 - 12 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$.

Visualizing the Curve and Tangent Line

Graphical representations help in understanding the slope at a point:

- The curve $y = x^3 - 12x$ has a shape typical of cubic functions.
- At $x = 1$, the tangent line's slope is -9.
- The tangent line equation at $P(1, -11)$:

$$
y - y_0 = m(x - x_0)
$$

$$
y + 11 = -9(x - 1)
$$

$$
y = -9x + 9 - 11 = -9x - 2
$$

This line touches the curve at $(1, -11)$ and has a slope of -9.

Conclusion: Mastering the Limit Method to Find Slopes

Calculating the slope of a curve at a specific point using the limit definition involves systematic steps:

1. Write the difference quotient.
2. Simplify algebraically.
3. Take the limit as $h \to 0$.

In the case of $y = x^3 - 12x$, we confirmed that the slope at $P(1, -11)$ is -9. This process illuminates the foundational concept of derivatives in calculus, bridging algebra and geometric intuition. Whether for academic purposes or practical applications, mastering the limit approach to derivatives empowers learners to analyze curves deeply and accurately.

SEO Keywords and Phrases:

- Find the slope of the curve at a point
- Derivative of $y = x^3 - 12x$
- Limit definition of derivative
- How to calculate slope using limits
- Slope of a tangent line at a point
- Calculus derivatives tutorial
- Mathematical explanation of derivatives
- Derivative at $x=1$
- Slope of cubic functions
- Limit process in calculus

If you want to learn more about derivatives, calculus principles, and how to apply limits to find slopes at various points on different functions, exploring online tutorials, educational videos, and calculus textbooks can further enhance your understanding.

Frequently Asked Questions

How do you find the slope of the curve y = x^3 - 12x at a specific point P(1, -11)?

You find the slope by calculating the derivative of y with respect to x and then evaluating it at x = 1. Alternatively, you can use the limit definition of the derivative to find the slope by computing the limiting value of the difference quotient as h approaches zero.

What is the difference between using the derivative formula and the limit definition to find the slope at point P(1, -11)?

Using the derivative formula involves directly differentiating the function and substituting x = 1, which is quicker. The limit definition involves computing the limit of the difference quotient as h approaches zero, providing a more fundamental understanding of the derivative.

How do you set up the limit for finding the slope of y = x^3 - 12x at x = 1 using the limit definition?

The limit is set up as: lim_{h→0} [f(1 + h) - f(1)] / h, where f(x) = x^3 - 12x. You evaluate f(1 + h) and f(1), then compute the limit as h approaches zero.

What is the value of f(1 + h) for the function y = x^3 - 12x?

f(1 + h) = (1 + h)^3 - 12(1 + h) = (1 + 3h + 3h^2 + h^3) - 12 - 12h.

How do you evaluate the limit $\lim_{h \to 0} [f(1 + h) - f(1)] / h$ for this function?

First compute $f(1 + h)$ and $f(1)$, then subtract: $[f(1 + h) - f(1)] / h = [(1 + 3h + 3h^2 + h^3 - 12 - 12h) - (-11)] / h$. Simplify numerator and then take the limit as h approaches zero.

What is the simplified form of the difference quotient before taking the limit?

The simplified form is $[3h + 3h^2 + h^3 - 12h + 11] / h$, which simplifies further to $3 + 3h + h^2 - 12$, as h approaches zero.

What is the limiting value of the difference quotient as h approaches zero, and what does it represent?

The limit is 3, which represents the slope of the curve $y = x^3 - 12x$ at the point P(1, -11).

How is the derivative of $y = x^3 - 12x$ related to the slope at a specific point?

The derivative y' gives the instantaneous rate of change of y with respect to x at any point. Evaluating y' at x = 1 gives the slope of the tangent to the curve at P(1, -11).

What is the final value of the slope of the curve at P(1, -11) using the limit approach?

The slope at P(1, -11) is 3, obtained by evaluating the limit of the difference quotient as h approaches zero.

Additional Resources

Find The Slope Of The Curve $Y = x^3 - 12x$ At The Given Point P(1, -11) By Finding The Limiting Value Of

Understanding how to determine the slope of a curve at a specific point is a fundamental aspect of calculus, particularly in the study of derivatives. The process involves examining the limiting behavior of the slope of secant lines as they approach the tangent line at the point of interest. In this case, we are tasked with finding the slope of the curve $y = x^3 - 12x$ at the point P(1, -11) by evaluating the limiting value of the difference quotient. This comprehensive exploration will walk you through the concepts, methods, and calculations involved, clarifying each step and its significance in the broader context of differential calculus.

Introduction to the Concept of Slope of a Curve

The slope of a curve at a particular point signifies the rate of change of the function at that point. Unlike straight lines where the slope remains constant, curves have slopes that vary from point to point. To determine the slope at a specific point, calculus introduces the concept of derivatives, which can be understood as the limiting value of average rates of change (secant slopes) as the two points used to measure the change get infinitely close.

Key ideas:

- The derivative at a point gives the instantaneous rate of change.
- It is the limit of the difference quotient as the change in x approaches zero.
- Geometrically, the derivative corresponds to the slope of the tangent line to the curve at that point.

Understanding the Function and the Point P(1, -11)

The given function is:

$$ y = x^3 - 12x $$

and the point P is:

$$ P(1, -11) $$

which lies on the curve because plugging $x = 1$ into the function:

$$ y = (1)^3 - 12(1) = 1 - 12 = -11 $$

confirms that P is indeed on the curve. Our goal is to find the slope of the tangent line to the curve at this point.

The Methodology: Finding the Slope via Limiting Value

To find the slope at P, we utilize the limit of the difference quotient:

$$ m = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} $$

For the point $x = 1$, this becomes:

$$ m = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} $$

This approach involves:

1. Calculating $f(1 + h)$
2. Substituting into the difference quotient
3. Simplifying the expression
4. Taking the limit as $h \to 0$

Step-by-Step Calculation

1. Compute $f(1 + h)$

Using the function:

$$ f(1 + h) = (1 + h)^3 - 12(1 + h) $$

Expanding:

$$ (1 + h)^3 = 1 + 3h + 3h^2 + h^3 $$

and

$$ -12(1 + h) = -12 - 12h $$

Therefore,

$$ f(1 + h) = 1 + 3h + 3h^2 + h^3 - 12 - 12h $$

Simplify:

$$ f(1 + h) = (1 - 12) + (3h - 12h) + 3h^2 + h^3 $$

$$ f(1 + h) = -11 - 9h + 3h^2 + h^3 $$

2. Write the difference quotient

Recall $f(1) = -11$. Thus,

$$ \frac{f(1 + h) - f(1)}{h} = \frac{(-11 - 9h + 3h^2 + h^3) - (-11)}{h} $$

$$ = \frac{-11 - 9h + 3h^2 + h^3 + 11}{h} $$

$$ = \frac{-9h + 3h^2 + h^3}{h} $$

3. Simplify the expression

Factor h from numerator:

$$ \frac{h(-9 + 3h + h^2)}{h} $$

Cancel h:

$$ -9 + 3h + h^2 $$

4. Take the limit as $h \to 0$

$$ m = \lim_{h \to 0} (-9 + 3h + h^2) = -9 + 3(0) + 0^2 = -9 $$

Thus, the slope of the curve at P(1, -11) is -9.

Interpreting the Result and Final Thoughts

The calculated derivative reveals that the tangent line to the curve at the point P has a slope of -9. This negative value indicates that the curve is decreasing at that point, meaning as x increases near 1, y decreases.

Implications:

- The tangent line at P is inclined downward with a steepness of 9 units per unit increase in x.
- The derivative at a point provides crucial insights into the behavior of the function: whether it is increasing, decreasing, or changing concavity.

Additional Concepts Related to the Derivative Calculation

Geometric Interpretation

The limit process essentially measures the slope of the secant line between the point P(1, -11) and a nearby point (1 + h, f(1+h)). As h approaches zero, the secant line approaches the tangent line, and the limit of the slopes of these secants gives the exact slope of the tangent.

Alternative Methods

While the limit of the difference quotient is the fundamental method, derivatives can also be obtained directly through rules such as power rule, product rule, or chain rule. For the given function:

$$ y = x^3 - 12x $$

the derivative can be quickly computed as:

$$ y' = 3x^2 - 12 $$

and evaluating at $x = 1$:

$$ y' = 3(1)^2 - 12 = 3 - 12 = -9 $$

which confirms our earlier result through a more straightforward approach.

Pros and Cons of the Limit Method

Pros:

- Provides a rigorous foundation for the derivative concept.
- Deepens understanding of the behavior of functions at specific points.
- Can be applied when derivative rules are not readily available.

Cons:

- Can be algebraically intensive and time-consuming.
- More prone to computational errors, especially in complex functions.
- Less efficient than applying derivative rules for simple functions.

Features of Derivative Calculation via Limits

- Fundamental to calculus: Builds the foundation for understanding rates of change.
- Versatility: Applicable to any function, regardless of differentiability rules.
- Conceptual clarity: Offers insight into the nature of instantaneous change.

Summary and Key Takeaways

In conclusion, finding the slope of the curve $y = x^3 - 12x$ at P(1, -11) via the limit of the difference quotient is a classical and instructive process. It illustrates how the concept of limits bridges the gap between average and instantaneous rates of change. The calculated slope of \(-9\) not only provides the tangent line's inclination but also exemplifies the power of calculus in analyzing the behavior of curves. Whether approached through the limit process or by applying derivative rules, understanding both methods enhances one's grasp of differential calculus and its applications in mathematics and science.

Final Note: Mastery of derivative concepts through limit processes enriches your mathematical toolkit, enabling you to tackle more complex functions and deepen your comprehension of change and motion in various contexts.

A Find The Slope Of The Curve Y X 3 12x At The Given Point P 1 11 By Finding The Limiting Value Of

A Find The Slope Of The Curve Y X 3 12x At The Given Point P 1 11 By Finding The Limiting Value Of

Related Articles

- A. What Is Your Response To Thom Hartmann's Comment "free Trade Has Crippled Our Economy So Badly"? B.
- 96 Nios En Un Campamento De Verano Han De Ser Repartidos En Varios Grupos De Modo Que Cada Grupo Tenga
- 38 ML Of A Gas Was Collected At 581 Mm Hg And 25 Degrees Celsius. What Would The Volume Be At Standard

Back to Home