A Line L Is Passing Through The Points On The Graph Of Y=x^2-2x+5 With Abscissas -1 And 2. What Is The

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Introduction

Understanding the relationship between quadratic functions and linear equations is fundamental in algebra and analytic geometry. When a line passes through specific points on a parabola, determining the equation of that line involves calculating its slope and y-intercept based on the given points. This article explores how to find the equation of such a line, given that it passes through points on the graph of $y = x^2 - 2x + 5$ with abscissas (x-coordinates) of -1 and 2.

Identifying the Points on the Parabola

Given the parabola $y = x^2 - 2x + 5$, find the points corresponding to the x-values -1 and 2.

To determine the specific points through which the line passes, substitute the given x-values into the parabola's equation:

1. For $x = -1$:
$$
y = (-1)^2 - 2(-1) + 5 = 1 + 2 + 5 = 8
$$
Therefore, the point is:
$$
P_{-1} = (-1, 8)
$$

2. For $x = 2$:
$$
y = (2)^2 - 2(2) + 5 = 4 - 4 + 5 = 5
$$
So, the point is:
$$
P_{2} = (2, 5)

\]

Summary of Points

- \(P_{-1} = (-1, 8) \)
- \(P_{2} = (2, 5) \)

Determining the Equation of Line L

Step 1: Calculate the Slope of Line L

The slope \(m \) of the line passing through points \((x_1, y_1) \) and \((x_2, y_2) \) is:
\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Substituting the points:
\[
m = \frac{5 - 8}{2 - (-1)} = \frac{-3}{3} = -1
\]
Thus, the slope of line L is:
\[
m = -1
\]

Step 2: Find the Equation of Line L

Using the point-slope form:
\[
y - y_1 = m (x - x_1)
\]
Choose either point; let's use \(P_{-1} = (-1, 8) \):
\[
y - 8 = -1 (x - (-1)) \Rightarrow y - 8 = -1 (x + 1)
\]
Simplify:
\[
y - 8 = -x - 1
\]
Add 8 to both sides:
\[

y = -x - 1 + 8
\]
\[
y = -x + 7
\]

Therefore, the equation of line L is:
\[
\boxed{y = -x + 7}
\]

Verifying the Line Passes Through Both Points on the Parabola

Check for \(P_{2} = (2, 5) \):

\[
y = -x + 7
\]
Substitute \(x = 2 \):
\[
y = -2 + 7 = 5
\]
which matches the y-coordinate of \(P_2 \). So, the line passes through this point.

Check for \(P_{-1} = (-1, 8) \):

Substitute \(x = -1 \):
\[
y = -(-1) + 7 = 1 + 7 = 8
\]
which matches the y-coordinate of \(P_{-1} \). Confirming that the line passes through both points.

Additional Insights and Applications

Why is this significant?

Finding the line passing through specific points on a parabola can help in various mathematical applications, such as:
- Determining tangent lines at certain points
- Approximating the parabola locally with a straight line

- Solving intersection problems between different curves

Implications for Further Study

This problem demonstrates key concepts:
- Calculating points on a parabola given x-values
- Computing slope between two points
- Deriving the equation of a line passing through two points
- Verifying the line's passage through the points

Conclusion

Given the points on the parabola \(y = x^2 - 2x + 5 \) corresponding to abscissas -1 and 2, we've calculated the points \((-1, 8) \) and \((2, 5) \). The line passing through these points has a slope of -1, and its equation is \(y = -x + 7 \). This line precisely intersects the parabola at those points, illustrating a classic problem of finding a line through specific points on a quadratic curve. Understanding such relationships is essential in advanced algebra and geometry, serving as a foundation for more complex topics like calculus and analytic geometry.

Further Exploration

- How would the line change if it passed through different points on the parabola?
- What is the geometric significance of the line's slope concerning the parabola's shape?
- How can this approach be extended to find tangents or normals to the parabola at given points?

This exploration underscores the interconnectedness of algebraic functions and their graphical representations, enriching our comprehension of geometric relationships in coordinate systems.

Frequently Asked Questions

What is the equation of the line passing through the points on the graph of y = x^2 - 2x + 5 with abscissas -1 and 2?

First, find the points: for x = -1, y = (-1)^2 - 2(-1) + 5 = 1 + 2 + 5 = 8. For x = 2, y = 2^2 - 2(2) + 5 = 4 - 4 + 5 = 5. The points are (-1, 8) and (2, 5). The slope m = (5 - 8) / (2 - (-1)) = (-3) / 3 = -1. Using point-slope form with point (-1,8): y - 8 = -1(x + 1). Simplify: y = -x + 7. So, the line's equation is y = -x + 7.

How do you determine the equation of a line passing through two points on a parabola?

To find the line's equation, first compute the coordinates of both points, then calculate the slope using (y2 - y1) / (x2 - x1). Finally, use point-slope form with either point to write the line's equation.

What is the significance of the points with abscissas -1 and 2 on the parabola y = x^2 - 2x + 5?

These points help determine the specific line passing through them. They also provide insight into how the line intersects or relates to the parabola at those x-values.

Can the line passing through the points on the parabola intersect the parabola at other points?

Yes, depending on the line's slope and position, it may intersect the parabola at additional points. In this case, since the line passes through two points on the parabola, it is tangent or intersects at those points, but further analysis is needed to check for other intersections.

What is the general method to find the equation of a line passing through points on a quadratic graph?

Identify the points, calculate the slope between them, then use the point-slope form to write the line's equation. This method applies generally to any two points on a quadratic graph.

How does knowing points on a parabola help in understanding its tangent lines?

Points on the parabola can serve as points of tangency for certain lines. By analyzing the slope at those points, you can determine the equations of tangent lines, which touch the parabola at exactly one point.

Additional Resources

A Line L Is Passing Through The Points On The Graph Of Y=x^2-2x+5 With Abscissas -1 And 2. What Is The — this intriguing problem explores the intersection of algebraic functions and geometric interpretations, serving as a fundamental example of how analytic geometry helps in understanding the properties of curves and lines. At its core, this problem asks us to determine the equation of a line passing through two specific points on a parabola described by the quadratic function $y = x^2 - 2x + 5$, with the points corresponding to the abscissas (x-coordinates) of -1 and 2. This scenario provides a comprehensive platform to analyze concepts such as point plotting, the slope of a line, and the equation of a line, all within the context of quadratic functions, which are central to algebra and calculus.

Understanding the Problem Statement

The Function $y = x^2 - 2x + 5$

The quadratic function $y = x^2 - 2x + 5$ describes a parabola opening upwards, with its vertex and roots providing geometric insight into its shape. This function can be rewritten in vertex form to better understand its properties:

$$
y = (x - 1)^2 + 4
$$

which indicates the parabola's vertex is at $(1, 4)$. The coefficient of x^2 is positive, confirming the parabola opens upward. The parabola's minimum point is at the vertex, and it is symmetric about the vertical line $x=1$.

Points on the Parabola at Abscissas -1 and 2

Given the points are on the parabola at $x = -1$ and $x = 2$:

- For $x = -1$:

$$
y = (-1)^2 - 2(-1) + 5 = 1 + 2 + 5 = 8
$$

So, the point is $(-1, 8)$.

- For $x = 2$:

$$
y = (2)^2 - 2(2) + 5 = 4 - 4 + 5 = 5
$$

So, the point is $(2, 5)$.

These two points are crucial because they define the path through which the line L passes.

Finding the Equation of Line L

Step 1: Calculate the Slope (m)

The slope of the line passing through points $(x_1, y_1) = (-1, 8)$ and $(x_2, y_2) = (2, 5)$ is:

$$
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 8}{2 - (-1)} = \frac{-3}{3} = -1
$$

A negative slope indicates the line is decreasing as x increases.

Step 2: Write the Equation of Line L

Using point-slope form:

$$
y - y_1 = m (x - x_1)
$$

Substituting $(x_1, y_1) = (-1, 8)$ and $m = -1$:

$$
y - 8 = -1 (x + 1)
$$

Simplify:

$$
y - 8 = -x - 1
$$
$$
y = -x + 7
$$

Thus, the equation of line L is:

$$
\boxed{ y = -x + 7 }
$$

Analysis and Geometric Significance

Features of Line L

- Slope: -1, indicating a 45-degree downward inclination.
- Y-intercept: 7, where the line crosses the y-axis.
- Points on the line: It passes through $(-1, 8)$ and $(2, 5)$, both on the parabola.

Intersection with the Parabola

Given the line's equation and the parabola's equation, the intersection points can be verified by solving:

$$
x^2 - 2x + 5 = -x + 7
$$

Bring all to one side:

$$
x^2 - 2x + 5 + x - 7 = 0
$$

Simplify:

$$
x^2 - x - 2 = 0
$$

Factor:

$$
(x - 2)(x + 1) = 0
$$

Solutions:

$$
x = 2 \quad \text{or} \quad x = -1
$$

Corresponding y-values:

- For $x=2$:

\[

$$
y = -2 + 7 = 5
$$

- For $x=-1$:

$$
y = 1 + 7 = 8
$$

These match the points initially identified, confirming that line L intersects the parabola exactly at the points $(-1,8)$ and $(2,5)$.

Implications and Applications

Understanding the Geometric Relationship

The analysis reveals a key insight: the line $y = -x + 7$ is precisely the secant line passing through two points on the parabola. This scenario demonstrates how linear functions can be used to approximate or analyze the parabola's behavior between points.

Pros:

- Helps in understanding tangent and secant lines.
- Useful in calculus for estimating slopes and derivatives.
- Demonstrates the relationship between quadratic functions and linear approximations.

Cons:

- It doesn't describe the tangent at a specific point unless the line is tangent (which requires the slope to equal the derivative at that point).
- Focusing solely on secants may overlook local curvature properties.

Real-World Applications

- Physics: Trajectories often modeled by quadratic functions; lines passing through points can represent paths or constraints.
- Engineering: Structural analysis involving parabolic arches and linear supports.
- Economics: Cost functions with quadratic models intersected by linear constraints.

Further Exploration: Derivatives and Tangents

While the problem centers on a secant line passing through two points, it naturally leads to exploring tangents. The derivative of the quadratic function:

$$
\frac{dy}{dx} = 2x - 2
$$

evaluates the slope of the tangent at any point x.

- At $x = -1$:

$$
m_{\text{tangent}} = 2(-1) - 2 = -2 - 2 = -4
$$

The tangent at $(-1, 8)$ has a slope of -4, which differs from the secant's slope of -1.

- At $x = 2$:

$$
m_{\text{tangent}} = 2(2) - 2 = 4 - 2 = 2
$$

The tangent at $(2, 5)$ has a slope of 2, again different from the secant slope.

This distinction emphasizes how secants approximate the behavior of the parabola between points, while tangents provide the instantaneous rate of change.

Conclusion and Final Remarks

The problem of determining the line passing through points on the parabola $y = x^2 - 2x + 5$ at specific abscissas exemplifies fundamental concepts in algebra and geometry. By computing the points, the slope, and the line's equation, we not only arrive at a concrete solution but also deepen our understanding of the relationships between quadratic functions and linear elements. This exercise underscores the importance of visualization, algebraic manipulation, and geometric interpretation in problem-solving.

Key takeaways include:

- The line passing through $(-1,8)$ and $(2,5)$ has the equation $y = -x + 7$.
- It acts as a secant line intersecting the parabola at two points.
- The derivative provides insight into tangent slopes, contrasting with secant slopes.
- Such problems reinforce the interconnectedness of algebra, calculus, and geometry.

In educational contexts, this problem is an excellent example for students to practice combining multiple mathematical concepts and for teachers to illustrate the practical applications of theoretical principles. Whether for academic purposes or real-world modeling, understanding how lines and parabolas interact is a cornerstone of mathematical literacy.

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