

A Line Passes Through The Points P(4,7,7) And Q(1,1,1). Find The Standard Parametric Equations For The

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A Line Passes Through The Points P(4,7,7) And Q(1,1,1). Find The Standard Parametric Equations For The Line is a common problem in three-dimensional coordinate geometry. Understanding how to derive the parametric equations of a line passing through two given points is essential for students and professionals working in mathematics, physics, engineering, and related fields. In this article, we will explore the step-by-step process of finding these equations, delve into the concepts involved, and provide helpful tips to master this fundamental skill.

Understanding the Basics of Line Equations in 3D Space

What Is a Line in Three Dimensions?

A line in three-dimensional space is a straight one-dimensional figure extending infinitely in both directions. It is uniquely determined by two points through which it passes or by a point and a direction vector.

Parametric Equations of a Line

The parametric form of a line expresses the coordinates of any point on the line as functions of a parameter, usually denoted as t . In 3D space, the general form is:

- $x = x_0 + at$
- $y = y_0 + bt$
- $z = z_0 + ct$

where (x_0, y_0, z_0) is a point on the line, and (a, b, c) is the direction vector indicating the

line's direction.

Given Points and Their Significance

In our problem, the points are:

- $P(4, 7, 7)$
- $Q(1, 1, 1)$

These points serve as references to determine the line's position and direction.

Step-by-Step Solution to Find the Parametric Equations

Step 1: Find the Direction Vector

The direction vector d is obtained by subtracting the coordinates of P from Q (or vice versa). It gives the direction in which the line extends from one point to the other.

1. Calculate the difference in x-coordinates: $1 - 4 = -3$
2. Calculate the difference in y-coordinates: $1 - 7 = -6$
3. Calculate the difference in z-coordinates: $1 - 7 = -6$

Thus, the direction vector d is:

$d = \langle -3, -6, -6 \rangle$ (Note: Since subtracting Q from P gives negative components, you can choose either vector or its negative for the direction; both describe the same line.)

Step 2: Choose a Point on the Line

Either point P or Q can be used as a reference point. For simplicity, let's select point $P(4, 7, 7)$.

Step 3: Write the Parametric Equations

Using point P and the direction vector d , the parametric equations are:

- $x = 4 + (-3)t$

- $y = 7 + (-6)t$

- $z = 7 + (-6)t$

Or, simplified:

- $x = 4 - 3t$

- $y = 7 - 6t$

- $z = 7 - 6t$

Interpreting the Parametric Equations

The parametric equations describe every point on the line as t varies over all real numbers. When $t=0$, the point is at $P(4,7,7)$; when $t=1$, the point shifts along the direction vector to $(4 - 3, 7 - 6, 7 - 6) = (1, 1, 1)$, which is point Q .

Alternative Forms and Notations

Vector Equation of the Line

The vector form combines the position vector of a point on the line with the direction vector:

$$r(t) = r_0 + td$$

where r_0 is the position vector of point P , and d is the direction vector:

$$r(t) = (4, 7, 7) + t(-3, -6, -6)$$

Symmetric Equations of the Line

From the parametric equations, the symmetric form can be written as:

$$\frac{x - 4}{-3} = \frac{y - 7}{-6} = \frac{z - 7}{-6}$$

or equivalently:

$$\frac{x - 4}{3} = \frac{y - 7}{6} = \frac{z - 7}{6}$$

Common Mistakes to Avoid

- Using the wrong order when subtracting points, leading to an incorrect direction vector.
- Forgetting to include the parameter t in all three equations.
- Mixing up the points P and Q when choosing the reference point.
- Neglecting to simplify the parametric equations for clarity.

Practical Applications of Line Equations in 3D

Understanding and deriving the parametric equations of a line is vital in various fields:

- Computer graphics and 3D modeling
- Physics simulations involving trajectories
- Engineering design and CAD software
- Robotics and motion planning
- Navigation and GPS technology

Summary

To summarize, given two points in 3D space, $P(4,7,7)$ and $Q(1,1,1)$, the steps to find the standard parametric equations of the line passing through them are:

1. Compute the direction vector by subtracting the coordinates of P from Q .
2. Select either P or Q as the point on the line.
3. Write the parametric equations using the chosen point and the direction vector.

For our case, the parametric equations are:

$$x = 4 - 3t,$$

$$y = 7 - 6t,$$

$$z = 7 - 6t$$

This comprehensive approach enables you to describe any line in three-dimensional space accurately and efficiently.

Conclusion

Mastering the derivation of parametric equations for lines passing through two points is a fundamental skill in vector geometry. Whether you're analyzing geometric problems or applying these concepts in real-world scenarios, understanding the process enhances your spatial reasoning and problem-solving capabilities. Remember to carefully compute the direction vector, choose an appropriate point, and then formulate the equations clearly. Practice with different points to strengthen your understanding and become proficient in 3D line equations.

Frequently Asked Questions

What is the direction vector of the line passing through points P(4,7,7) and Q(1,1,1)?

The direction vector is obtained by subtracting the coordinates of Q from P: $(4-1, 7-1, 7-1) = (3, 6, 6)$.

How do you find the parametric equations of a line passing through a point with a given direction vector?

The parametric equations are written as $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$, where (x_0, y_0, z_0) is a point on the line and (a, b, c) is the direction vector.

What point should be used as the initial point in the parametric equations for this line?

You can choose either point P(4,7,7) or Q(1,1,1). Using P(4,7,7) simplifies calculations.

What are the standard parametric equations for the line passing through P(4,7,7) with direction vector (3,6,6)?

The equations are $x = 4 + 3t$, $y = 7 + 6t$, $z = 7 + 6t$, where t is a real parameter.

Why is the direction vector (3,6,6) simplified from the points P and Q?

Because subtracting Q from P gives the vector connecting them, which indicates the line's direction; simplifying is unnecessary as (3,6,6) is already in its simplest form.

How can the parametric equations be written using point Q(1,1,1) instead of P?

Using Q as the point, the parametric equations become $x = 1 + 3t$, $y = 1 + 6t$, $z = 1 + 6t$.

What is the importance of the parameter t in the parametric equations?

The parameter t determines the position of any point on the line; changing t moves along the line in the direction of the vector.

Can the parametric equations for the line be written in symmetric form?

Yes, by solving each parametric equation for t, the symmetric form is $(x - 4)/3 = (y - 7)/6 = (z - 7)/6$.

How do I verify that the parametric equations correctly represent the line passing through points P and Q?

Substitute the coordinates of P and Q into the equations to ensure they satisfy the equations for some values of t; for P, $t = 0$, and for Q, $t = -1$.

Additional Resources

A Line Passes Through The Points P(4,7,7) And Q(1,1,1): Find The Standard Parametric Equations For The Line

Understanding how to find the parametric equations of a line passing through two points in three-dimensional space is fundamental in vector calculus and analytic geometry. This problem involves translating the geometric information—specifically two points—into algebraic equations that describe the line's path precisely. When given points P(4,7,7) and Q(1,1,1), the goal is to determine the standard parametric equations that represent all points on the line passing through these points. This task is not only an essential skill for students of mathematics but also a practical tool in fields like engineering, physics, and computer graphics.

Understanding the Geometry of the Line

Before diving into the equations, it's critical to visualize what the problem entails. In three-dimensional space, a line is a one-dimensional object extending infinitely in both directions. The line passing through points P and Q is uniquely determined because any two distinct points define exactly one line.

Key Concepts:

- Points in 3D Space: Each point is represented by an ordered triplet (x, y, z). In this case, P(4,7,7) and Q(1,1,1).
- Direction Vector: The vector pointing from one point to the other, defining the line's direction.
- Parametric Equations: Expressions that describe the coordinates of any point on the line as functions of a parameter, usually denoted as t.

By understanding these concepts, one can translate geometric data into algebraic representations seamlessly.

Calculating the Direction Vector

A crucial step in deriving the parametric equations is to find the direction vector of the line. This vector indicates the line's orientation in space and is obtained by subtracting the coordinates of Q from P.

Calculation:

$$\vec{d} = \overrightarrow{PQ} = (x_P - x_Q, y_P - y_Q, z_P - z_Q)$$

Plugging in the points:

$$\vec{d} = (4 - 1, 7 - 1, 7 - 1) = (3, 6, 6)$$

Features of the Direction Vector:

- It points from Q to P, but the same line can be described using $\vec{d}$ or $-\vec{d}$.
- It encapsulates the line's orientation, indicating how the coordinates change as one moves along the line.

Optional Simplification:

- The vector can be scaled down to its simplest form by dividing each component by their greatest common divisor, which is 3:

$$\vec{d} = (1, 2, 2)$$

This simplified vector makes the equations easier to interpret and work with.

Formulating the Standard Parametric Equations

The parametric equations of a line in space are generally written as:

$$\begin{aligned}x &= x_0 + a t \\y &= y_0 + b t \\z &= z_0 + c t\end{aligned}$$

where $((x_0, y_0, z_0))$ is a point on the line, $(\vec{d} = (a, b, c))$ is the direction vector, and (t) is a real number parameter.

Choosing a Point:

- Either point P or Q can be used as the base point. Here, we'll use point Q(1,1,1) for simplicity.

Using the Direction Vector:

- Using the simplified vector $(\vec{d} = (1, 2, 2))$.

Parametric Equations:

$$\boxed{\begin{aligned}x &= 1 + 1 \cdot t = 1 + t \\y &= 1 + 2 \cdot t = 1 + 2t \\z &= 1 + 2 \cdot t = 1 + 2t\end{aligned}}$$

These equations describe all points $((x, y, z))$ on the line passing through P and Q.

Features and Variations of the Parametric

Equations

The derived parametric equations serve as a powerful and flexible way to represent the line. They have several notable features:

Advantages:

- Clarity: Clearly shows how each coordinate depends on the parameter t .
- Versatility: Easily used to find points at specific parameters or to check if a point lies on the line.
- Ease of Graphing: Facilitates plotting the line in 3D space.
- Extension to Equations of Planes: Serves as a foundational concept for more complex geometric entities.

Limitations:

- Parameter Range: The parameter t spans all real numbers; interpreting specific segments requires setting bounds.
- No Direct Geometric Intuition: Without visualization, the equations might be less intuitive than the geometric representation.

Alternative Forms and Related Equations

While the standard parametric form is most common, other forms can be derived or used depending on the application:

Symmetric Equations:

Expressed by eliminating the parameter t :

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Using the base point $Q(1,1,1)$ and the direction vector $(1, 2, 2)$:

$$\frac{x - 1}{1} = \frac{y - 1}{2} = \frac{z - 1}{2}$$

This form is often favored for solving intersection problems with other geometric entities.

Features:

- Provides a concise way to express the line.
- Useful for solving systems involving lines and planes.

Applications and Practical Implications

Understanding how to derive and interpret the parametric equations of a line through two points has numerous practical applications:

- Computer Graphics: Rendering lines and trajectories in 3D models.
- Physics: Describing particle trajectories, projectile motion, or electromagnetic field lines.
- Engineering: Designing components with specific directional features.
- Navigation: Calculating the shortest path or intersection points in 3D space.

The ability to transition seamlessly from geometric points to algebraic equations enables professionals and students to solve complex spatial problems efficiently.

Conclusion: Mastery of Line Equations in 3D Space

The process of deriving the standard parametric equations for a line passing through two points — in this case, $P(4,7,7)$ and $Q(1,1,1)$ — is a fundamental skill that bridges geometry and algebra. By calculating the direction vector and choosing an appropriate point, one can formulate versatile equations that describe the line comprehensively. This task not only reinforces understanding of spatial relationships but also equips learners with tools applicable in advanced fields such as computer-aided design, robotics, and physics.

Understanding the nuances of these equations, their features, and their applications enhances one's geometric intuition and analytical prowess. Whether for academic purposes or real-world problem-solving, mastering the derivation and interpretation of line equations in three dimensions is a cornerstone of spatial reasoning in mathematics.

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