A Locker Combination Has Three Nonzero Digits, And Digits Cannot Be Repeated. If The First Digit Is 3,

A Locker Combination Has Three Nonzero Digits, And Digits Cannot Be Repeated. If The First Digit Is 3, then understanding the possible combinations becomes an interesting mathematical problem that combines combinatorics with specific constraints. This scenario is a common example used in teaching basic principles of permutations and combinations, especially in contexts involving security systems, lock mechanisms, and combinatorial enumeration.

In many real-world situations, such as school lockers, safes, or password systems, the constraints placed on digit sequences help ensure security and complexity. When the rules specify that each digit must be nonzero, no repetitions are allowed, and the first digit is fixed as 3, the challenge becomes calculating the total number of possible combinations under these conditions. This article explores this problem in detail, providing a thorough understanding of the underlying principles, step-by-step calculations, and practical applications.

Understanding the Problem: Constraints and Conditions

Before diving into calculations, it’s essential to clearly understand the constraints and what they imply:

- The combination has three digits: The sequence is of length three.
- Digits are nonzero: Each digit can be any from 1 to 9.
- Digits cannot be repeated: No digit repeats within the sequence.
- The first digit is 3: The sequence starts with the digit 3.

Given these conditions, our goal is to determine how many different possible combinations (sequences) satisfy all these constraints.

Breaking Down the Constraints

Let’s analyze each constraint separately:

1. Fixed First Digit: The first digit is always 3. This reduces the problem because there’s

no choice for the first digit — it’s predetermined.

2. Remaining Digits: The second and third digits are chosen from the remaining nonzero digits, excluding 3.

3. No Repetition: The second and third digits must be distinct and different from 3 and each other.

Given these points, the problem simplifies to selecting the second and third digits from a subset of nonzero digits, with specific restrictions.

Step-by-Step Calculation

Let's proceed step by step to calculate the total number of possible combinations under the given constraints.

Step 1: Identify the available digits

- Total nonzero digits: 1, 2, 3, 4, 5, 6, 7, 8, 9
- Since the first digit is fixed as 3, the remaining digits for selection are: 1, 2, 4, 5, 6, 7, 8, 9

Number of remaining digits: 8

Step 2: Determine options for the second digit

- The second digit cannot be 3 (already used as the first digit).
- It can be any of the remaining 8 digits: 1, 2, 4, 5, 6, 7, 8, 9.

Number of choices for the second digit: 8

Step 3: Determine options for the third digit

- The third digit must be different from both the first digit (which is 3) and the second digit (chosen in the previous step).
- Since one digit has been used for the second position, the remaining options are:

Number of choices for the third digit: 7 (because we exclude the second digit and 3, which is fixed as the first digit and not available for the remaining positions)

Step 4: Calculate total combinations

- For each choice of the second digit (8 options), there are 7 choices for the third digit.
- Total combinations:

$$
\text{Total} = 8 \times 7 = 56
$$

Summary of the Calculation

- The first digit is fixed as 3.
- The second digit can be any of the remaining 8 nonzero digits.
- The third digit can be any of the remaining 7 digits after the second digit is chosen.

Therefore, the total number of valid locker combinations with these constraints is 56.

Practical Applications and Implications

Understanding how to calculate such combinations has practical significance in various fields:

- Security Systems: Designing lock combinations that are secure but follow specific rules can help prevent unauthorized access. For example, knowing how many combinations are possible with certain constraints helps assess security strength.

- Combinatorial Optimization: Many problems involve selecting sequences under constraints, such as password generation, code creation, or secure key distribution.

- Educational Contexts: Problems like this serve as excellent exercises in teaching fundamental principles of combinatorics, permutations, and probability.

Extensions and Variations of the Problem

While this article focuses on a specific case, variations can be explored to deepen understanding:

- Different Fixed Digits: What if the first digit is fixed as 5 instead of 3? How does that change the total combinations?

- Allowing Zero: How does the total change if zeros are allowed? This introduces additional digits (0-9) and alters the constraints.

- Repeating Digits: What if digits can be repeated? The calculation becomes simpler but less secure.

- Longer Sequences: Extending the sequence length to four or more digits involves more complex calculations but follows similar principles.

Conclusion

In summary, when a locker combination has three nonzero digits with no repeated digits, and the first digit is fixed as 3, the total number of possible combinations is 56. This calculation demonstrates fundamental combinatorial principles, including fixing positions, excluding certain options, and the multiplication principle for counting arrangements.

Understanding these principles not only helps in solving similar problems but also provides insight into designing secure systems, creating effective codes, and applying mathematical reasoning to real-world scenarios. Whether for academic, security, or practical purposes, mastering such combinatorial problems enhances analytical and problem-solving skills essential in many fields.

Frequently Asked Questions

What are the possible values for the second and third digits in the locker combination if the first digit is 3?

Since the digits cannot be repeated and are nonzero, the second and third digits can be any of the remaining digits from 1 to 9, excluding 3, so they can be any two different digits from 1, 2, 4, 5, 6, 7, 8, 9.

How many total combinations are possible if the first digit is fixed as 3?

There are 8 remaining digits (excluding 3), and since the second and third digits cannot be repeated, the total number of combinations is 8 options for the second digit times 7 options for the third digit, which equals 56 combinations.

Are repetitions allowed in the remaining two digits after fixing the first digit as 3?

No, repetitions are not allowed; each digit must be unique, so the second and third digits must be different from each other and from 3.

Can the second or third digit be 3 in this combination?

No, since the digits cannot be repeated and the first digit is 3, the second and third digits cannot be 3.

If the first digit is 3, what is the probability that the second digit is even?

There are 4 even digits remaining (2, 4, 6, 8) out of the 8 possible remaining digits. Since the second digit is chosen uniformly at random from these 8, the probability is 4/8 or 1/2.

What is the probability that the last digit is a prime number, given the first digit is 3?

Prime digits among 1-9 are 2, 3, 5, 7. Since 3 is already used as the first digit, the remaining prime options are 2, 5, and 7, totaling 3 out of 8 remaining digits. The probability that the third digit is prime is therefore 3/8.

How many combinations have the second digit as 4 if the first digit is 3?

If the second digit is fixed as 4, then the third digit can be any of the remaining 7 digits (excluding 3 and 4), so there are 7 possible combinations.

Additional Resources

A Locker Combination Has Three Nonzero Digits, And Digits Cannot Be Repeated. If The First Digit Is 3, this problem presents an engaging scenario that combines combinatorial reasoning with logical deduction. Whether you're a student preparing for a math contest, a teacher designing a problem set, or simply a puzzle enthusiast, understanding how to approach such problems enhances your analytical skills and deepens your grasp of permutations and constraints. In this guide, we will explore the nuances of this specific problem, providing a step-by-step breakdown that covers all possible cases, the reasoning behind each choice, and the final count of valid combinations.

Understanding the Problem

Before diving into calculations, let's clarify the key components of the problem:

- The locker combination consists of three digits.
- All digits are nonzero, which means each digit is from 1 to 9.
- The digits cannot be repeated within the combination.
- The first digit is fixed as 3.

Given these constraints, our main goal is to determine how many different locker combinations satisfy all the conditions when the first digit is 3.

Step 1: Fixing the First Digit

Since the first digit is fixed as 3, this choice removes the variability for the first position. The problem then reduces to selecting the remaining two digits under the given constraints.

- First digit: 3 (fixed)
- Second digit: ? (from 1–9, but not 3)
- Third digit: ? (from 1–9, but not equal to the second digit or 3)

The key is to count the number of valid choices for the second and third digits.

Step 2: Choosing the Second Digit

The second digit must be:

- Nonzero (from 1 to 9)
- Not equal to 3 (to avoid repeated digits)

This leaves us with:

- Total options: 9 (digits from 1 to 9)
- Excluding 3: 8 options

Number of choices for the second digit: 8

Step 3: Choosing the Third Digit

After selecting the second digit, the third digit must:

- Be from 1 to 9
- Not be equal to 3 (already used as the first digit)
- Not be equal to the second digit (already chosen)

Since the second digit has been fixed, the remaining options for the third digit are:

- Total digits: 9 (digits 1–9)

- Exclude: the first digit (3) and the second digit

Therefore, the number of choices for the third digit depends on which digit was chosen as the second digit.

Number of options for the third digit:

- Total remaining digits: 9
- Excluding 2 digits: 3 (the first digit 3 and the second digit)
- Remaining options: 9 - 2 = 7

Note: Since the second digit is fixed after each choice, in each case, there are always 7 options for the third digit.

Step 4: Total Number of Valid Combinations

To find the total number of valid combinations:

- For each choice of the second digit (8 options), there are 7 choices for the third digit.

Thus, total combinations:

Total = 8 (second digit options) × 7 (third digit options) = 56

Summary of the Calculation

Step	Description	Number of options
Fix first digit	First digit is 3	1 (fixed)
Choose second digit	From 1–9, excluding 3	8
Choose third digit	From remaining 7 options, excluding 3 and second digit	7
Total combinations	Multiply options for second and third digits	8 × 7 = 56

Final answer: There are 56 valid locker combinations where the first digit is 3, all digits are nonzero, and no digits are repeated.

Additional Considerations and Variations

While this specific scenario is straightforward, similar problems often involve additional layers of complexity. Here are some common variations and how to approach them:

1. If the First Digit Is Not Fixed

- When the first digit can be any nonzero digit, the total number of combinations would be:

- First digit: 9 options (1–9)
- Second digit: 8 options (excluding the first digit)
- Third digit: 7 options (excluding the first two chosen digits)

Total combinations: $9 \times 8 \times 7 = 504$

2. Allowing Zero as a Digit

- If zero is permitted, the set of digits expands to 0–9, but the problem states nonzero digits, so zero is excluded here.

3. Repetition Allowed

- If digits can repeat, the counting would be different, often simpler, but that’s not the case here.

Practical Applications and Tips for Similar Problems

- Identify fixed elements: Fixing one element simplifies the counting process.
- Break down the problem into steps: First choose the fixed part, then move to variable parts.
- Use permutations when order matters: Each position is distinct, so permutations are often relevant.
- Account for constraints systematically: List out what is excluded and what remains at each step.

Final Thoughts

Understanding problems involving constraints on digits, such as non-repetition and fixed positions, enhances problem-solving skills in combinatorics. The key is to carefully analyze each condition, break down the problem step-by-step, and systematically count the options at each stage. In this particular case, fixing the first digit as 3 simplifies the problem, leading to a straightforward calculation of 56 valid combinations.

By mastering these approaches, you'll be better equipped to tackle a wide range of combinatorial problems—whether in exams, coding challenges, or real-world scenarios involving arrangements and selections.

In conclusion, when the first digit of a three-digit locker combination is fixed as 3, with all digits being nonzero and non-repeating, there are 56 possible valid combinations. This approach exemplifies the power of systematic counting and logical deduction in combinatorial mathematics, providing a solid foundation for more complex problems in the future.

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