

A Lot Consisting Of 50 Bulbs Is Inspected By Taking At Random 10 Bulbs And Testing Them. If The Number

A Lot Consisting Of 50 Bulbs Is Inspected By Taking At Random 10 Bulbs And Testing Them. If The Number of defective bulbs found in the sample exceeds a certain threshold, the entire lot may be rejected. This process epitomizes the principles of statistical sampling and quality control, which are essential in manufacturing and industrial inspections to ensure product reliability while minimizing testing costs. In this comprehensive guide, we will explore the concepts, methods, and applications of such sampling inspection procedures, emphasizing their importance in quality assurance, decision-making, and process improvement.

Understanding Sampling Inspection in Quality Control

What Is Sampling Inspection?

Sampling inspection is a statistical quality control method where a subset of items from a larger lot is tested to infer the overall quality of the entire batch. Instead of testing every item, which can be costly and time-consuming, a carefully selected sample provides sufficient information to decide whether the lot meets quality standards.

Why Use Sampling Inspection?

- Cost Efficiency: Testing every item can be expensive; sampling reduces costs.
- Time Savings: Faster decision-making processes.

- Resource Optimization: Less labor and testing materials needed.
- Risk Management: Balances the risk of accepting defective lots against the cost of inspection.

Scenario: Inspecting a Lot of 50 Bulbs

Sample Size and Testing Procedure

In our scenario, a lot consists of 50 bulbs. A sample of 10 bulbs is randomly selected and tested. The key question is: How does the number of defective bulbs in this sample influence the decision to accept or reject the entire lot?

Acceptance Criteria and Decision Rules

Typically, the quality inspector sets a threshold value for the maximum number of defective bulbs allowed in the sample. For example, the lot may be accepted if no more than 2 defective bulbs are found in the sample of 10; otherwise, it is rejected.

Statistical Foundations of Sampling Inspection

Hypergeometric Distribution

The hypergeometric distribution models the probability of finding a certain number of defective bulbs in a sample drawn without replacement from a finite population.

Key parameters:

- Total bulbs in lot: $N = 50$
- Number of defective bulbs in lot: D (unknown, but can be estimated)
- Sample size: $n = 10$
- Number of defective bulbs in sample: k

Probability formula:

$$P(k) = \frac{\binom{D}{k} \binom{N-D}{n-k}}{\binom{N}{n}}$$

This formula calculates the probability of observing exactly k defective bulbs in the sample, given D defective bulbs in the lot.

Decision-Making Using Sampling Plans

Sampling plans specify:

- Acceptance number (c): maximum number of defective items allowed in the sample.
- Rejection number: if the number of defective bulbs exceeds c , the lot is rejected.

For example, a plan might state:

- Accept the lot if ≤ 2 defective bulbs are found in the sample.
- Reject if > 2 defective bulbs are found.

Types of Sampling Inspection Plans

Single Sampling Plan

A simple plan where one sample is taken, and the decision is made based on the number of defectives found.

Advantages:

- Easy to implement.
- Quick decision-making.

Disadvantages:

- Less flexible in balancing risk levels.

Double and Multiple Sampling Plans

More complex plans involve taking additional samples if the initial sample is inconclusive, providing more nuanced control over acceptance and rejection probabilities.

Attributes of an Effective Sampling Plan

- Producer's risk (α): probability of rejecting a good lot.
- Consumer's risk (β): probability of accepting a bad lot.
- Lot tolerance percentage defectives (LTPD): the defect level that should be rejected with high probability.

Optimizing Inspection: Balancing Risks and Costs

Acceptance Quality Level (AQL)

AQL is the maximum percentage of defective items that is considered acceptable during inspection. For example, an AQL of 1% means that a lot with up to 1% defective bulbs is deemed acceptable.

Designing an Inspection Plan

To optimize the sampling plan:

- Decide on acceptable risks (α and β).
- Choose an appropriate sample size.
- Set acceptance and rejection thresholds.

Example:

- Sample size: 10
- Accept if ≤ 2 defectives
- Reject if > 2 defectives

This plan strikes a balance between quality assurance and inspection costs.

Applications of Sampling Inspection in Industry

Manufacturing Quality Control

Ensuring product reliability in industries like electronics, pharmaceuticals, and consumer goods.

Supply Chain Management

Verifying supplier quality before acceptance.

Regulatory Compliance

Meeting safety and quality standards through statistical sampling.

Improving Processes

Identifying defect patterns to enhance manufacturing processes.

Advantages and Limitations of Sampling Inspection

Advantages

- Reduced testing time and costs.
- Quick decision-making.
- Effective for large lots.

Limitations

- Risk of accepting defective lots (consumer's risk).
- Possible rejection of good lots (producer's risk).
- Requires proper plan design and statistical understanding.

Conclusion: The Importance of Proper Sampling in Quality

Assurance

Sampling inspection, such as testing 10 bulbs out of a lot of 50, is an indispensable tool in modern quality control. By understanding statistical principles like the hypergeometric distribution and carefully designing sampling plans, industries can efficiently maintain product quality, ensure customer satisfaction, and optimize operational costs. Whether in manufacturing, supply chain management, or

regulatory compliance, the principles of sampling inspection help strike the right balance between quality assurance and resource utilization.

Key Takeaways

- Sampling inspection relies on statistical models to infer lot quality.
- Properly designed sampling plans reduce costs while managing risks.
- The hypergeometric distribution models the probability of defective items in samples drawn without replacement.
- Acceptance criteria are based on the maximum allowable defective bulbs in the sample.
- Balancing producer's and consumer's risks ensures fair and effective quality control.
- Industry applications span manufacturing, supply chain, and regulatory sectors.

By mastering these concepts, quality professionals can implement robust inspection strategies that uphold high standards without unnecessary expenditure, ensuring products meet customer expectations and regulatory requirements.

Meta Description:

Learn how sampling inspection of 50 bulbs with a random sample of 10 aids quality control. Discover statistical methods, sampling plans, and industry applications for effective quality assurance.

Frequently Asked Questions

What is the purpose of inspecting a sample of 10 bulbs from a lot of 50?

The purpose is to estimate the quality or defect rate of the entire lot by testing a representative sample, which helps in quality control decisions.

How is the sample of 10 bulbs selected in the inspection process?

The 10 bulbs are selected randomly from the lot of 50 to ensure that each bulb has an equal chance of being chosen, reducing sampling bias.

What statistical method is typically used to analyze the results of such bulb inspections?

Statistical methods like binomial or hypergeometric distribution are used to analyze the probability of defects and to make inferences about the entire lot.

If a certain number of defective bulbs are found in the sample, how can this inform decisions about the whole lot?

The number of defective bulbs in the sample can be used to estimate the defect rate in the entire lot, guiding whether to accept, reject, or inspect further.

What is the importance of random sampling in quality inspection?

Random sampling ensures an unbiased representation of the lot, making the inspection results more reliable and statistically valid.

Can the inspection of 10 bulbs from 50 provide an accurate estimate of the total defect rate?

Yes, if the sampling is random and the sample size is appropriate, it can provide a reasonable

estimate of the overall defect rate with a certain confidence level.

What factors influence the reliability of the inspection results in this scenario?

Factors include the randomness of sampling, the sample size, the defect prevalence, and the consistency of the testing process.

How does the hypergeometric distribution relate to the inspection of bulbs in this context?

The hypergeometric distribution models the probability of finding a specific number of defective bulbs in the sample without replacement, which is relevant here.

What actions might be taken if the sample inspection indicates a high defect rate?

If a high defect rate is detected, actions may include rejecting the entire lot, re-inspecting, or investigating the manufacturing process for issues.

Additional Resources

A Lot Consisting Of 50 Bulbs Is Inspected By Taking At Random 10 Bulbs And Testing Them. If The Number of Defective Bulbs Found Among These Sampled Bulbs Is Known, What Can We Infer About The Quality of the Entire Batch? This question touches on a fundamental principle in quality control and statistical inference, which plays a pivotal role in manufacturing, supply chain management, and product assurance. In this article, we delve into the nuances of sampling methods, the statistical tools used to analyze inspection results, and the practical implications for industries committed to maintaining high standards without incurring excessive testing costs.

Understanding the Context: Why Sampling Matters in Quality Control

In many manufacturing settings, inspecting every single item for defects is often impractical, time-consuming, and costly. Instead, companies rely on sampling techniques—selecting a subset of items from the batch to infer the overall quality. The premise is straightforward: if the sample is representative, then the findings can reliably estimate the condition of the entire lot.

Key reasons for adopting sampling include:

- Cost Efficiency: Testing all items might be prohibitively expensive; sampling reduces costs significantly.
- Speed: Faster inspections allow for quicker decision-making regarding product release or rejection.
- Resource Allocation: Resources can be allocated more effectively when only a sample is tested thoroughly.

In the scenario under consideration, a batch of 50 bulbs is inspected by randomly selecting 10 bulbs and testing them. The critical question is: based on the number of defective bulbs identified in this sample, what inferences can be made about the entire lot?

Fundamental Concepts in Sampling and Inference

Before analyzing the example, it's essential to understand some foundational concepts:

Sample and Population

- Population (Lot): The entire group of items—in this case, 50 bulbs.
- Sample: The subset selected from the population—in this case, 10 bulbs.

Defects and Quality Metrics

- Defective Bulbs: Bulbs that do not meet quality standards.
- Defect Proportion: The ratio of defective bulbs in the entire lot, which we aim to estimate.

Sampling Methods

- Simple Random Sampling: Every bulb has an equal chance of being selected.
- Systematic Sampling: Selecting every n th bulb.
- Stratified Sampling: Dividing the lot into subgroups before sampling.

In our context, the sampling is random, and each bulb has an equal probability of selection, which simplifies the statistical analysis.

Hypergeometric Distribution: The Core of Sampling Without Replacement

Since the sampling is without replacement—once a bulb is tested, it isn't replaced—the appropriate probability model is the hypergeometric distribution.

What Is the Hypergeometric Distribution?

The hypergeometric distribution describes the probability of drawing a specific number of successes (defective bulbs) in a fixed number of draws (10 bulbs), from a finite population without replacement.

Parameters:

- Population size, $(N = 50)$
- Number of defectives in the lot, (D) (unknown)
- Sample size, $(n = 10)$
- Number of defectives found in the sample, (k)

The probability of finding exactly (k) defective bulbs in the sample, given (D) , is:

$$P(k | D) = \frac{\binom{D}{k} \binom{N - D}{n - k}}{\binom{N}{n}}$$

This formula accounts for all possible combinations where (k) defective bulbs are selected from the (D) defective ones, and the remaining $(n - k)$ bulbs are non-defective.

Estimating the Number of Defective Bulbs in the Lot

Suppose you tested 10 bulbs and found (k) defective bulbs. How does this information inform your estimate of the total defective bulbs in the entire lot?

Point Estimation

The most straightforward estimate of the total defect count, based on the sample, is:

$$\hat{D} = \frac{k}{n} \times N$$

This is called the proportionate extrapolation—assuming the sample reflects the lot's defect rate.

For example:

- If 2 defective bulbs were found in the sample:

$$\hat{D} = \frac{2}{10} \times 50 = 10$$

- If 5 defective bulbs were found:

$$\hat{D} = \frac{5}{10} \times 50 = 25$$

Confidence Intervals for the Number of Defectives

Since the sample provides a range of possible defect counts, statistical methods allow constructing confidence intervals, giving a range within which the true number of defectives likely falls with a specified probability (e.g., 95%).

One common approach is to use the hypergeometric distribution to find bounds on (D) based on the observed (k) . Alternatively, approximate methods such as the binomial approximation or Bayesian inference can be employed for larger samples.

Practical Scenarios and Decision-Making

The implications of the sample inspection depend heavily on the observed defect count:

Scenario 1: No Defects Found ($k=0$)

- Interpretation: The lot is likely of high quality.
- Action: Accept the lot with confidence, possibly reducing the need for further testing.
- Risks: Small chance of missing defective bulbs; statistical methods can quantify this risk.

Scenario 2: Few Defects Found ($k=1$ or 2)

- Interpretation: The defect rate is low, but not negligible.
- Action: Consider additional testing or accepting with a certain quality assurance threshold.
- Statistical Inference: Estimation can suggest the maximum probable number of defectives with high confidence.

Scenario 3: Many Defects Found ($k \geq 5$)

- Interpretation: The lot likely has a high defect rate.
- Action: Reject or rework the lot, or investigate manufacturing processes.
- Quality Control: Statistical evidence supports making decisive quality decisions.

Balancing Sampling Size and Confidence

While testing 10 out of 50 bulbs offers a snapshot, increasing the sample size improves the precision of the estimate but also increases testing costs. The trade-off involves:

- Sampling error: Larger samples reduce uncertainty.
- Cost considerations: Smaller samples save resources but may lead to less reliable conclusions.

Industry standards often specify acceptable sample sizes based on lot size and quality requirements.

Advanced Statistical Techniques for Better Inference

Beyond basic estimates, industry professionals employ more sophisticated tools:

Bayesian Inference

- Incorporates prior beliefs about defect rates.
- Updates the probability distribution based on observed data.
- Provides a full posterior distribution of the total defect count, enabling more nuanced decision-making.

Sequential Sampling Plans

- Sampling proceeds in stages, with evaluation after each stage.
- Sampling stops once sufficient evidence is collected.
- Balances inspection costs with confidence in the decision.

Use of Quality Control Charts

- Monitor defect rates over multiple batches.
- Detect trends or shifts in manufacturing quality.

Implications for Industry and Quality Assurance

The practice of inspecting a random subset of bulbs exemplifies a broader principle: statistical sampling empowers industries to make informed decisions with minimal resource expenditure.

Accurate interpretation of sample results influences:

- Product release decisions
- Customer satisfaction
- Cost management
- Regulatory compliance

By understanding the underlying statistical models, manufacturers can optimize their sampling plans, balance risk and cost, and uphold rigorous quality standards.

Conclusion

Inspecting a lot of 50 bulbs by testing 10 at random involves more than just counting defective units. It requires a solid grasp of probability models like the hypergeometric distribution, estimation techniques, and decision-making frameworks. Whether the goal is to accept a batch, reject it, or decide on further testing, the insights gained from sampling are invaluable. They serve as a bridge between practical constraints and statistical rigor, ensuring that quality assurance is both efficient and reliable.

In an era where consumer trust and regulatory standards are paramount, leveraging statistical principles in quality control is not just best practice—it's essential. The art lies in designing sampling plans that are statistically sound, cost-effective, and aligned with industry quality goals, ultimately safeguarding both manufacturers' interests and consumer satisfaction.

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