# A Motorboat Cuts Its Engine When Its Speed Is 10.0m/s And Then Coasts To Rest. The Equation Describing

**A Motorboat Cuts Its Engine When Its Speed Is 10.0 m/s And Then Coasts To Rest. The Equation Describing**

Understanding the motion of a motorboat after its engine is turned off involves fundamental principles of physics, particularly the concepts of deceleration, friction, air resistance, and the equations governing motion under non-uniform forces. When a motorboat, initially traveling at a speed of 10.0 meters per second (m/s), cuts its engine, it no longer receives the propulsive force necessary to maintain its speed. Consequently, it begins to coast to a stop due to resistive forces like drag and friction acting opposite to its motion. This article provides a comprehensive overview of the physics governing this scenario, the mathematical equations involved, and practical implications.

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Understanding the Motion of a Coasting Motorboat

When analyzing the motion of a motorboat that slows down to rest after engine cutoff, several key concepts come into play:

- Deceleration (Negative Acceleration): The boat experiences a decrease in velocity over time due to resistive forces.
- Resistive Forces: Mainly water drag and air resistance, which depend on the boat's speed.
- Friction and Drag: These forces oppose the motion and are often modeled as proportional to some power of velocity.
- Kinematic Equations: Mathematical tools used to describe the velocity and position of the boat over time.

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Fundamental Principles and Assumptions

To derive the equations describing the boat's deceleration, certain assumptions are typically made:

- The resistive force is proportional to the boat's velocity $v$. This is valid for laminar flow regimes or when drag is approximately linear.
- The mass $m$ of the boat remains constant during the deceleration.
- The system is isolated, with no additional forces acting after the engine cutoff.
- The initial velocity $v_0$ is 10.0 m/s at the moment the engine is turned off.

- The deceleration is uniform or can be described by a known function.

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Deriving the Equation of Motion

1. Modeling Resistive Forces

The resistive force $F_{resist}$ opposing the boat's motion can often be modeled as:

$$
F_{resist} = -b v
$$

where:

- $b$ is the damping coefficient (a positive constant),
- $v$ is the velocity of the boat.

This linear model is suitable when drag is proportional to velocity, such as at low speeds or in laminar flow.

2. Applying Newton's Second Law

According to Newton's second law:

$$
m \frac{dv}{dt} = F_{resist} = -b v
$$

Rearranged:

$$
m \frac{dv}{dt} + b v = 0
$$

3. Solving the Differential Equation

This is a first-order linear differential equation:

$$
\frac{dv}{dt} + \frac{b}{m} v = 0
$$

The solution involves separation of variables:

$$
\frac{dv}{v} = - \frac{b}{m} dt
$$

Integrating both sides:

$$
\int \frac{1}{v} dv = - \frac{b}{m} \int dt
$$

$$
\ln |v| = - \frac{b}{m} t + C
$$

Exponentiating:

$$
|v| = e^{C} e^{ - \frac{b}{m} t }
$$

Let $v_0$ be the initial velocity at $t=0$:

$$
v(0) = v_0 = e^{C}
$$

Thus, the velocity as a function of time:

$$
v(t) = v_0 e^{ - \frac{b}{m} t }
$$

4. Final Velocity and Time to Rest

Since the exponential never truly reaches zero, in practice, the boat approaches rest asymptotically. To find the time $t_{rest}$ when the velocity drops below a certain threshold (e.g., effectively zero), we can use:

$$
v(t_{rest}) \approx 0
$$

or specify a small velocity $v_{threshold}$:

$$
t_{rest} = \frac{m}{b} \ln \left( \frac{v_0}{v_{threshold}} \right)
$$

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Practical Applications and Insights

1. Estimating Deceleration Rate

If the damping coefficient $b$ and the mass $m$ are known, the deceleration rate $a(t)$ can be expressed as:

$$
a(t) = \frac{dv}{dt} = - \frac{b}{m} v(t) = - \frac{b}{m} v_0 e^{ - \frac{b}{m} t }
$$

which shows that acceleration (or deceleration, in this case) decreases exponentially over time.

2. Calculating Distance Traveled While Coasting

The distance $s(t)$ traveled during deceleration is obtained by integrating velocity:

$$
s(t) = \int_0^t v(t') dt' = v_0 \int_0^t e^{ - \frac{b}{m} t' } dt'
$$

$$
s(t) = v_0 \left[ - \frac{m}{b} e^{ - \frac{b}{m} t' } \right]_0^t = \frac{m v_0}{b} \left( 1 - e^{ - \frac{b}{m} t } \right)
$$

As $t \to \infty$, the total stopping distance approaches:

$$
s_{total} = \frac{m v_0}{b}
$$

3. Real-World Factors Affecting Deceleration

In reality, other factors influence the deceleration:

- Variable Drag: At higher speeds, drag may become quadratic with velocity.
- Water Conditions: Turbulence, waves, and currents can alter resistance.
- Boat Design: Shape and hull material affect drag coefficient.
- Environmental Conditions: Wind and water temperature influence resistance.

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Extending the Model: Non-Linear Drag

In many practical situations, resistive forces are proportional to the square of velocity:

$$
F_{resist} = - c v^2
$$

where $c$ is a drag coefficient.

The equation of motion becomes:

$$
m \frac{dv}{dt} = - c v^2
$$

which leads to a different solution:

$$
\frac{dv}{v^2} = - \frac{c}{m} dt
$$

Integrating:

$$
- \frac{1}{v} = - \frac{c}{m} t + C
$$

Applying initial condition $v(0) = v_0$:

$$
- \frac{1}{v_0} = C
$$

Thus:

$$
\frac{1}{v} = \frac{c}{m} t + \frac{1}{v_0}
$$

and

$$
v(t) = \frac{1}{ \frac{c}{m} t + \frac{1}{v_0} }
$$

This model predicts deceleration that slows more quickly compared to linear drag at high velocities.

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Summary of Key Equations

| Scenario | Velocity as a function of time | Total stopping distance |
|---|---|---|
| Linear drag $F_{resist} = -b v$ | $v(t) = v_0 e^{ - \frac{b}{m} t }$ | $s_{total} = \frac{m v_0}{b}$ |
| Quadratic drag $F_{resist} = - c v^2$ | $v(t) = \frac{1}{ \frac{c}{m} t + \frac{1}{v_0} }$ | $s_{total} = \frac{m}{c} v_0$ |

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Practical Implications for Boat Operators

- Estimating Deceleration Time: Knowing the damping coefficient allows operators to predict how long it takes for the boat to stop.
- Calculating Stopping Distance: Critical for safe navigation, especially near docks or shallow areas.
- Design Considerations: Boat hull design and materials influence resistance and, consequently, coasting behavior.
- Safety Measures: Understanding deceleration helps in planning safe distances and maneuvering strategies.

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Conclusion

The physics behind a motorboat coasting to rest after cutting its engine revolves around the principles of deceleration due to resistive forces. The key to modeling this behavior lies in applying Newton's second law and selecting an appropriate resistive force model—linear or quadratic in velocity. The resulting exponential decay of velocity under linear drag provides a clear mathematical framework to predict how quickly a boat slows down and how far it travels before coming to a stop. Recognizing these equations and concepts is essential for boat designers, operators, and safety planners to ensure efficient and safe navigation.

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References

- Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers. Cengage Learning.
- Tipler, P. A., & Mosca, G. (2008). Physics for Scientists and Engineers. W. H

# Frequently Asked Questions

## What is the typical equation used to describe the deceleration of a motorboat coasting to rest after cutting its engine?

The motion can be described by the equation of motion under constant deceleration: v = v0 - at, where v is the velocity at time t, v0 is the initial velocity (10.0 m/s), and a is the deceleration due to resistive forces.

## How is the deceleration of the motorboat determined from the given information?

Deceleration can be found using the equation $a = (v0 - v) / t$, where v0 is 10.0 m/s and v eventually becomes zero as the boat comes to rest. If the time to stop is known, the deceleration can be calculated directly.

## What assumptions are typically made when deriving the equation for the boat's motion as it coasts to rest?

Assumptions include constant deceleration due to resistive forces (like water drag), no additional forces acting on the boat after the engine is cut, and neglecting any external influences such as wind or waves.

## If the deceleration is constant, how long does it take for the motorboat to come to rest?

The time to stop can be calculated using $t = v0 / a$, where v0 is 10.0 m/s and a is the constant deceleration. Without the value of a, the exact time cannot be determined.

## How does resistive force affect the equation describing the boat's motion after the engine is cut?

Resistive force introduces a negative acceleration (deceleration) in the equation, causing the velocity to decrease over time until the boat comes to rest, modeled by the equation $v = v0 - at$.

## Can the equation describing the boat's deceleration be used to find the distance traveled before stopping?

Yes, using the equation $s = v0\ t - (1/2)\ a\ t^2$, where t is the time to stop, or by integrating velocity over time; knowing a and v0 allows calculation of the stopping distance.

## What physical factors influence the deceleration of the motorboat after the engine is cut?

Factors include water resistance (drag), the boat's shape and size, surface roughness, and any external forces like wind or waves acting on the boat.

## How can the equation describing the boat's motion be modified if the deceleration is not constant?

If deceleration varies with velocity or other factors, differential equations such as dv/dt = -f(v) must be used, where f(v) describes the velocity-dependent resistive forces, requiring more complex modeling.

# Additional Resources

**A Motorboat Cuts Its Engine When Its Speed Is 10.0 m/s And Then Coasts To Rest. The Equation Describing**

Understanding the dynamics of a motorboat that transitions from powered motion to coasting involves delving into fundamental principles of physics, notably the concepts of motion, forces, and energy dissipation. Such an analysis not only offers insights into the behavior of vehicles moving through fluids but also exemplifies the practical application of kinematic and dynamic equations. This article aims to explore the scenario comprehensively, detailing the physical principles, deriving the relevant equations, and analyzing the factors influencing the boat's deceleration.

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# Introduction: The Scenario in Context

Imagine a motorboat cruising across a calm lake at a velocity of 10.0 meters per second (m/s). At a certain point, the operator decides to cut the engine, allowing the boat to coast to a stop purely under the influence of resistive forces such as water drag and possibly air resistance. This situation encapsulates a common real-world scenario where vehicles or objects in motion gradually come to rest due to dissipative forces.

The core question that arises is: How can we mathematically describe the deceleration of the boat? To answer this, we need to develop an equation that relates its velocity over time, considering the forces acting upon it.

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# Fundamental Principles Involved

Before deriving the specific equation, it's essential to understand the physical principles at play:

1. Newton's Second Law of Motion
- The law states that the net force acting on an object equals its mass times

its acceleration ($F_{net} = m a$).
- When the engine is cut, the net force is primarily resistive, acting opposite to the direction of motion, leading to negative acceleration (deceleration).

2. Resistive Forces (Drag)
- In fluids, objects experience drag forces that oppose their motion.
- For relatively moderate speeds, this drag often follows a quadratic relationship with velocity, but in many practical cases, especially at lower speeds or for simplicity, a linear approximation suffices.

3. Energy Dissipation
- As the boat slows, kinetic energy is converted into heat and other forms of energy due to resistive forces.
- The deceleration process can be analyzed by considering the work done by resistive forces and the resulting change in kinetic energy.

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## Modeling the Deceleration: Deriving the Equation

To describe the boat's slowing down quantitatively, we need to establish a mathematical model.

Assumption: Linear Drag Force

While real-world drag often has a quadratic dependence on velocity, for simplicity and analytical tractability, assume a linear resistive force:

$$
F_{drag} = -b v
$$

where:
- $v$ is the velocity of the boat at time $t$,
- $b$ is a positive damping coefficient that encapsulates the effects of water resistance and other resistive forces,
- The negative sign indicates that the force opposes the direction of motion.

Applying Newton's Second Law:

$$
m \frac{dv}{dt} = -b v
$$

This is a first-order linear differential equation describing the velocity as a function of time.

Solving the Differential Equation:

Rearranged as:

$$
\frac{dv}{dt} = - \frac{b}{m} v
$$

This equation can be solved via separation of variables:

$$
\frac{dv}{v} = - \frac{b}{m} dt
$$

Integrating both sides:

$$
\int \frac{1}{v} dv = - \frac{b}{m} \int dt
$$

$$
\ln |v| = - \frac{b}{m} t + C
$$

Exponentiating:

$$
|v| = e^{C} e^{-\frac{b}{m} t}
$$

Define the integration constant $C' = e^{C}$, which corresponds to the initial velocity:

$$
v(t) = v_0 e^{-\frac{b}{m} t}
$$

where:
- $v_0$ is the velocity at $t = 0$, the moment when the engine is cut.

Initial Condition:

Given in the problem:

$$
v(0) = v_0 = 10.0\, \text{m/s}
$$

Thus, the velocity decay is described by the exponential function:

$$
\boxed{
v(t) = v_0 e^{-\frac{b}{m} t}
}
$$

This expression captures how the boat's speed diminishes over time due to resistive forces.

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## Understanding the Decay Equation

The exponential decay model indicates that the velocity decreases continuously, approaching zero asymptotically as $t \to \infty$. The key parameters influencing this decay are:

- Mass of the Boat ($m$): Heavier boats have more inertia, resisting deceleration.
- Damping Coefficient ($b$): Represents the combined effects of water and air resistance; higher $b$ leads to faster deceleration.
- Initial Velocity ($v_0$): Starting point of the decay process.

The characteristic time constant $\tau = \frac{m}{b}$ signifies how quickly the velocity diminishes. Smaller $\tau$ (large $b$) implies rapid deceleration, while larger $\tau$ indicates a slow slowdown.

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## Additional Relations and Practical Implications

1. Time to Rest:

To determine how long it takes for the boat to come to a complete stop (theoretically asymptotic), we note:

$$
v(t) \to 0 \quad \text{as} \quad t \to \infty
$$

In practice, the boat approaches rest within a finite time, say when $v(t)$ falls below a certain small threshold.

2. Distance Traveled During Deceleration:

The displacement $s(t)$ during coasting is obtained by integrating velocity over time:

$$
s(t) = \int_0^t v(t') dt' = v_0 \int_0^t e^{-\frac{b}{m} t'} dt'
$$

$$
s(t) = v_0 \left[ - \frac{m}{b} e^{-\frac{b}{m} t'} \right]_0^t = \frac{m v_0}{b} \left( 1 - e^{-\frac{b}{m} t} \right)
$$

As $t \to \infty$:

$$
s_{total} = \frac{m v_0}{b}
$$

This represents the total distance traveled during the deceleration process.

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# Real-World Factors and Model Limitations

While the exponential decay model offers a neat analytical solution, real-world scenarios involve complexities:

- Nonlinear Drag: At higher speeds, drag often scales with the square of velocity ($F_{drag} \propto v^2$), leading to more complex equations.

- Variable Resistance: Water conditions, boat hull design, and additional forces (like wind) can influence deceleration.

- Energy Losses and Turbulence: Turbulent water flow and wave-making can alter resistive forces unpredictably.

- Engine and Mechanical Factors: If the engine's cut-off isn't instantaneous or if other mechanical factors come into play, the deceleration process may differ.

Despite these complexities, the exponential model provides a robust first approximation, especially in controlled environments or for educational purposes.

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# Advanced Considerations: Beyond Linear Drag

For more precise modeling, especially at higher velocities, the quadratic drag law is more accurate:

$$
F_{drag} = - c v^2
$$

Leading to the differential equation:

$$
m \frac{dv}{dt} = - c v^2
$$

which integrates to:

$$
\frac{1}{v} = \frac{c}{m} t + \frac{1}{v_0}
$$

and the velocity as a function of time:

$$
v(t) = \frac{v_0}{1 + \frac{c v_0}{m} t}
$$

This rational function indicates a different deceleration profile, with the velocity decreasing hyperbolically rather than exponentially.

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# Conclusion: The Significance of the Equation

The derived exponential decay equation $v(t) = v_0 e^{-\frac{b}{m} t}$ encapsulates the fundamental physics of a motorboat coasting to rest under resistive forces. It highlights the interplay between inertia (mass), resistive forces (through $b$), and initial conditions.

Understanding this relationship is crucial for various applications:

- Design Optimization: Engineers can optimize hull design and propulsion to minimize deceleration times.
- Navigation and Safety: Predicting stopping distances aids in safe navigation planning.
- Educational Insight: The model serves as an accessible illustration of differential equations and exponential decay concepts.

While real-world complexities necessitate more sophisticated models, the core principles outlined here form the foundation for analyzing motion and deceleration in fluid environments. The equation provides both a practical tool and a window into the elegant mathematics governing dynamical systems.

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References:

- Serway, R. A., & Jewett, J. W.

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