A Particle Moves Along The X-axis So That The Position S Is Given As A Function Of Time T Byx(t)= 10t2

A Particle Moves Along The X-axis So That The Position S Is Given As A Function Of Time T By x(t) = 10t^2

Understanding the motion of particles is fundamental in the study of classical mechanics, a branch of physics that deals with the behavior of physical bodies under the influence of forces. One of the key concepts in this field is the analysis of how objects move along a straight line, often represented mathematically by position functions that depend on time. In this article, we will explore in detail the motion of a particle moving along the x-axis, where its position $x(t)$ is given as a quadratic function of time: $x(t) = 10t^2$.

This specific example provides a classic illustration of uniformly accelerated motion, as the quadratic dependence indicates acceleration is present. By examining this function, we can derive various important kinematic quantities such as velocity, acceleration, displacement, and the total distance traveled. We will also explore how these concepts fit into broader physics principles, including the equations of motion, and discuss their applications in real-world scenarios.

Understanding the Given Position Function: $x(t) = 10t^2$

The position function $x(t) = 10t^2$ describes how the particle's position along the x-axis evolves over time t. Here, t is typically measured in seconds (s), and $x(t)$ in meters (m). The key characteristics of this function include:

- Quadratic Nature: The second-degree polynomial implies acceleration.
- Initial Position: When $t = 0$, $x(0) = 0$, indicating the particle starts from the origin.
- Growth of Position: As time increases, the position increases quadratically, meaning the particle accelerates away from the origin.

This kind of function is typical in scenarios involving constant acceleration, such as free fall under gravity (ignoring air resistance), or an object pushed along a frictionless surface with a constant force.

Deriving Velocity and Acceleration from $x(t) = 10t^2$

To analyze the particle's motion comprehensively, we need to determine its velocity and acceleration at any given time.

Velocity as a Function of Time

Velocity $v(t)$ is the rate of change of position with respect to time. Mathematically, it is the first derivative of the position function:

$$
v(t) = \frac{dx(t)}{dt}
$$

Calculating the derivative:

$$
v(t) = \frac{d}{dt} (10t^2) = 20t
$$

Interpretation:

- The velocity increases linearly with time, starting from zero at $t=0$.
- At any time t, the particle's velocity is $20t$ meters per second (m/s).

Key points:

- When $t=0$, $v(0) = 0$, indicating the particle starts from rest.
- For $t > 0$, the particle accelerates with increasing speed.

Acceleration as a Function of Time

Acceleration $a(t)$ is the rate of change of velocity with respect to time. It is the derivative of velocity:

$$
a(t) = \frac{dv(t)}{dt}
$$

Calculating the derivative:

$$
a(t) = \frac{d}{dt} (20t) = 20
$$

Interpretation:

- The acceleration is constant at $20\, \text{m/s}^2$.
- This confirms the motion is uniformly accelerated, consistent with the quadratic position function.

Analyzing the Motion: Key Quantities

With the velocity and acceleration functions derived, we can analyze various aspects of the particle's motion:

1. Initial Conditions

- Initial position at $t=0$: $x(0) = 0$
- Initial velocity at $t=0$: $v(0) = 0$

2. Velocity at Time t

$$
v(t) = 20t
$$

- The velocity increases linearly with time.
- At $t=5\, \text{s}$, for instance, $v(5) = 20 \times 5 = 100\, \text{m/s}$.

3. Acceleration at Time t

$$
a(t) = 20\, \text{m/s}^2
$$

- The acceleration remains constant throughout the motion.

4. Displacement and Distance Traveled

- Displacement refers to the change in position $x(t) - x(0)$.
- Since initial position is zero, displacement at time t is simply $x(t)$.

Calculating Displacement and Total Distance Traveled

Given the position function $x(t) = 10t^2$, the displacement after time t is:

$$
\text{Displacement} = x(t) - x(0) = 10t^2
$$

Because the particle starts at the origin and moves forward along the x-axis, the total distance traveled at time t is also $10t^2$.

However, in cases where the particle might change direction, total distance traveled would require summing absolute displacements over each segment of motion. Since this particle's velocity $v(t) = 20t$ is positive for $t > 0$, the motion is unidirectional, and total distance equals displacement.

Graphical Representation of the Motion

Visualizing the particle's motion helps in understanding its behavior over time.

Position-Time Graph

- The graph of $x(t) = 10t^2$ is a parabola opening upwards.
- The vertex is at $t=0$, with $x=0$.

Velocity-Time Graph

- The graph of $v(t) = 20t$ is a straight line passing through the origin with slope 20.

Acceleration-Time Graph

- The acceleration is constant at $20\, \text{m/s}^2$, represented by a horizontal line.

These graphs visually confirm the uniform acceleration and increasing velocity over time.

Applications and Real-World Contexts

Understanding this motion model is essential in various fields:

- Physics Education: Demonstrates basic principles of kinematics.
- Engineering: Design of systems involving acceleration, such as vehicles or robotic arms.
- Space Science: Modeling objects under constant acceleration, such as rockets during powered ascent.
- Sports Science: Analyzing acceleration phases in athletic movements.

Additional Calculations and Concepts

Beyond the fundamental derivations, several other calculations can provide deeper insights:

1. Time to Reach a Certain Position

Suppose we want to find out when the particle reaches $x = 200\, \text{m}$:

$$
x(t) = 10 t^2 = 200 \Rightarrow t^2 = 20 \Rightarrow t = \sqrt{20} \approx 4.47\, \text{s}
$$

At this time, the particle's velocity is:

$$
v(4.47) = 20 \times 4.47 \approx 89.4\, \text{m/s}
$$

2. Velocity at a Given Displacement

Using the velocity equation:

$$
v = \sqrt{2a x}
$$

This comes from the kinematic equation for constant acceleration:

$$
v^2 = v_0^2 + 2a x
$$

Since the particle starts from rest $(v_0=0)$:

$$
v = \sqrt{2 \times 20 \times x} = \sqrt{40x}
$$

Summary and Key Takeaways

- The position function $x(t) = 10t^2$ describes a particle undergoing constant acceleration along the x-axis.
- Its velocity increases linearly with time: $v(t) = 20t$.
- Its acceleration remains constant at $20\, \text{m/s}^2$.
- The motion starts from rest at the origin.
- The particle's displacement after time t is $10t^2$, and the total distance traveled equals this displacement due to unidirectional motion.
- These concepts are foundational in classical mechanics and have broad applications across physics and engineering disciplines.

Conclusion

Analyzing the motion of a particle with the position function $x(t) = 10t^2$ provides a comprehensive understanding of uniformly accelerated motion. By deriving velocity and acceleration, plotting the relevant graphs, and applying kinematic equations, we see how mathematical functions translate into physical behavior. Such analysis not only deepens our grasp of fundamental physics but also equips us with tools to approach real-world problems involving motion along a straight line.

Understanding these principles is essential for students, educators, and professionals working in fields ranging from mechanical engineering to

Frequently Asked Questions

What is the initial position of the particle when t = 0?

When t = 0, S(0) = 10 0^2 = 0. So, the initial position is at the origin.

How does the particle's velocity vary with time?

The velocity is the derivative of position S(t) with respect to time t, so v(t) = d/dt [10t^2] = 20t. It increases linearly with time.

At what time is the particle at rest?

The particle is at rest when its velocity v(t) = 20t = 0, which occurs at t = 0.

What is the acceleration of the particle, and how does it change over time?

Acceleration is the derivative of velocity: a(t) = d/dt [20t] = 20 m/s², which is constant.

How far does the particle travel from t = 0 to t = 5 seconds?

The total displacement is S(5) = 10 5^2 = 10 25 = 250 units.

What is the particle's velocity at t = 3 seconds?

v(3) = 20 3 = 60 units/sec.

Is the particle moving in the positive or negative x-direction?

Since the position S(t) = 10t^2 is always positive and increasing for t > 0, the particle moves in the positive x-direction.

How would the motion change if the position function

was $S(t) = 5t^3$?

The velocity would be $v(t) = 15t^2$, and the acceleration would be $a(t) = 30t$, indicating a non-constant acceleration that increases with time, leading to faster acceleration compared to the quadratic case.

Additional Resources

A particle moves along the x-axis so that the position s is given as a function of time t by $x(t) = 10t^2$. This fundamental kinematic equation provides a rich context for exploring the motion of a particle, analyzing its velocity, acceleration, and other key characteristics. Understanding how a particle behaves under such a quadratic position function not only deepens our grasp of basic physics but also offers insights into real-world applications, from vehicle acceleration to projectile motion.

Introduction to the Particle’s Motion

In classical mechanics, the position of a particle as a function of time is central to understanding its motion. When the position is given by a quadratic function, such as $x(t) = 10t^2$, it indicates that the particle experiences uniformly increasing velocity over time—an acceleration scenario. Analyzing this function allows us to derive important kinematic quantities like velocity and acceleration, which are crucial for predicting future positions, understanding the nature of the motion, and applying these principles to practical problems.

The Position Function: $x(t) = 10t^2$

What does the function tell us?

The given position function:

- $x(t) = 10t^2$

- Is a quadratic function of time, where:

- The coefficient 10 has units of meters per second squared (m/s^2), indicating the acceleration magnitude.

- The variable t is time in seconds.

This form suggests that the particle starts from the origin (assuming t=0, x=0) and accelerates along the x-axis as time progresses.

Initial Conditions

- At $t = 0$, $x(0) = 0$, so the particle starts at the origin.

- The initial velocity can be derived by taking the first derivative of $x(t)$.

Deriving Velocity and Acceleration

Velocity as a Function of Time

Velocity, denoted as $v(t)$, is the rate of change of position with respect to time:

- $v(t) = dx/dt$

Given:

$x(t) = 10t^2$

Differentiate with respect to t:

$v(t) = d/dt\,(10t^2) = 20t$

Interpretation:

- The velocity increases linearly with time.

- At $t=0$, $v(0) = 0$ m/s, indicating the particle starts from rest.

- For any $t > 0$, the velocity is positive, showing motion along the positive x-axis.

Acceleration as a Function of Time

Acceleration, denoted as $a(t)$, is the rate of change of velocity:

- $a(t) = dv/dt$

Differentiate $v(t)$:

$a(t) = d/dt\,(20t) = 20$

Interpretation:

- The acceleration is constant at 20 m/s^2.

- This confirms the particle is undergoing uniform acceleration.

Graphical Representation of the Motion

Visualizing the motion helps in better understanding the particle's behavior over time.

Position vs. Time ($x(t)$)

- Plotting $x(t) = 10t^2$ yields a parabola opening upwards.

- The particle starts at the origin and moves increasingly faster along the x-axis.

Velocity vs. Time ($v(t)$)

- Plotting $v(t) = 20t$ results in a straight line passing through the origin with slope 20.

- The velocity increases linearly with time, reflecting the constant acceleration.

Acceleration vs. Time ($a(t)$)

- Since $a(t) = 20$ is constant, the graph is a horizontal line at 20 m/s^2.

Key Kinematic Quantities

Initial and Final Velocities

- Initial velocity (v_0): At $t=0$, $v(0) = 0$ m/s.

- Velocity at any time t: $v(t) = 20t$.

- Velocity after t seconds: The particle gains 20 m/s every second.

Displacement over Time

- The position function $x(t) = 10t^2$ directly gives the displacement at any time t.

- For example, after 5 seconds:

$x(5) = 10 (5)^2 = 10 25 = 250$ meters.

Time to reach a certain position

Suppose you want to find the time it takes for the particle to reach a position s:

- $s = 10t^2$

- $t = \sqrt{(s/10)}$

For a position of 100 meters:

$t = \sqrt{(100/10)} = \sqrt{10} \approx 3.16$ seconds.

Practical Applications and Real-World Examples

Understanding the motion described by $x(t) = 10t^2$ is applicable in a variety of contexts:

1. Vehicle Acceleration: A car accelerating at 20 m/s^2 from rest will have its position over time modeled similarly.

2. Object in Free Fall (Vertical Motion): Although the gravitational acceleration ($\approx 9.8\ m/s^2$) differs, the quadratic form is similar in projectile motion.

3. Physics Education: Demonstrates uniformly accelerated motion and the derivation of key kinematic equations.

Advanced Analysis

Calculating Displacement over Specific Intervals

Suppose you want the displacement between t_1 and t_2:

- $\Delta x = x(t_2) - x(t_1) = 10t_2^2 - 10t_1^2$

For example, from t=2 seconds to t=4 seconds:

$\Delta x = 10(4)^2 - 10(2)^2 = 10(16) - 10(4) = 160 - 40 = 120$ meters.

Average and Instantaneous Quantities

- Average velocity over $[t_1, t_2]$:

$v_{avg} = \Delta x / \Delta t = (x(t_2) - x(t_1)) / (t_2 - t_1)$

- Instantaneous velocity at any t:

$v(t) = 20t$

Acceleration’s Role in Motion

Given constant acceleration, the particle's velocity and position can be predicted for any future time:

- $v(t) = v_0 + a\ t$

- $x(t) = x_0 + v_0 t + 0.5 a t^2$

In our case, initial velocity $v_0=0$, $x_0=0$, and $a=20$ m/s², leading back to the original functions.

Summary of Key Equations

Quantity	Expression	Description
Position	$x(t) = 10t^2$	Position along x-axis over time
Velocity	$v(t) = 20t$	Instantaneous velocity over time
Acceleration	$a(t) = 20$	Constant acceleration
Displacement between t_1 and t_2	$\Delta x = 10(t_2^2 - t_1^2)$	Change in position over interval
Time to reach position s	$t = \sqrt{s/10}$	Time taken to reach a specific position

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Final Remarks

The analysis of $x(t) = 10t^2$ exemplifies fundamental principles of kinematics, illustrating how a quadratic position function corresponds to uniformly accelerated motion. By deriving velocity and acceleration from the position function, we gain a comprehensive understanding of the particle's behavior. Such an understanding is essential not only in theoretical physics but also in practical engineering and technology, where predicting motion accurately is crucial.

Whether modeling the acceleration of a vehicle, the trajectory of a projectile, or the dynamics of a falling object, the principles demonstrated here form the foundation for more complex motion analysis. Mastery of these concepts empowers physicists, engineers, and students to analyze, predict, and optimize motion in diverse real-world scenarios.

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