A Particle Moves In The Xy Plane So That At Any Time -2 T 2, X = 2 Sint And Y=t-2 Cost +3. What Is The

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Introduction to Particle Motion in the XY Plane

Understanding the motion of a particle in the XY plane is a fundamental aspect of kinematics, a branch of physics that describes how objects move without considering the forces that cause the motion. When a particle's position is described parametrically—meaning its X and Y coordinates are expressed as functions of a parameter, usually time (t)—we can analyze various properties such as velocity, acceleration, trajectory, and other kinematic quantities.

In this article, we analyze the motion of a particle described by the parametric equations:

- $X(t) = 2 \sin t$
- $Y(t) = t - 2 \cos t + 3$

over the interval $t \in [-2, 2]$.

Our goal is to understand the path traced by the particle, find key quantities like velocity and acceleration, and interpret the physical significance of this motion.

Parametric Equations and Their Significance

Understanding the Given Equations

The parametric equations describe the position of the particle at any time t:

- X-coordinate: $X(t) = 2 \sin t$
- Y-coordinate: $Y(t) = t - 2 \cos t + 3$

These equations combine sinusoidal functions with a linear term in t, indicating that the particle's motion involves oscillations superimposed on a linear trend.

Interval of Motion

The interval for t is $-2 \leq t \leq 2$. This range limits the analysis to a finite segment of the particle’s path, which is particularly useful for understanding specific segments or behaviors within this timeframe.

Analyzing the Particle’s Trajectory

Plotting the Path

The trajectory is a curve in the XY plane traced out as t varies within the specified interval. To visualize this:

- For each value of t, compute $X(t)$ and $Y(t)$.
- Plot the points $(X(t), Y(t))$.

This path may exhibit oscillations in the X-direction due to the sine function and a combined linear and oscillatory trend in the Y-direction.

Key Features of the Trajectory

- Oscillatory Behavior in X: Since $X(t) = 2 \sin t$, the X-position oscillates between -2 and 2 as t varies.
- Combined Linear and Oscillatory in Y: $Y(t) = t - 2 \cos t + 3$. The term $t + 3$ indicates a linear trend, while $-2 \cos t$ adds oscillations.

Velocity and Acceleration of the Particle

Calculating Velocity Components

The velocity vector $\vec{v}(t)$ has components:

- $v_x(t) = \frac{dX}{dt}$
- $v_y(t) = \frac{dY}{dt}$

Calculating these derivatives:

$$
v_x(t) = \frac{d}{dt} (2 \sin t) = 2 \cos t
$$

$$
v_y(t) = \frac{d}{dt} (t - 2 \cos t + 3) = 1 + 2 \sin t
$$

Thus, the velocity vector is:

$$
\vec{v}(t) = (2 \cos t, 1 + 2 \sin t)
$$

Calculating Speed and Direction

- Speed (magnitude of velocity):

$$
|\vec{v}(t)| = \sqrt{(2 \cos t)^2 + (1 + 2 \sin t)^2}
$$

$$
= \sqrt{4 \cos^2 t + 1 + 4 \sin t + 4 \sin^2 t}
$$

Using the Pythagorean identity $\sin^2 t + \cos^2 t = 1$:

$$
|\vec{v}(t)| = \sqrt{4 \cos^2 t + 4 \sin^2 t + 4 \sin t + 1} = \sqrt{4 (\cos^2 t + \sin^2 t) + 4 \sin t + 1}
$$

$$
= \sqrt{4 \times 1 + 4 \sin t + 1} = \sqrt{5 + 4 \sin t}
$$

- Direction: The angle $\theta(t)$ of the velocity vector with respect to the x-axis:

$$
\theta(t) = \arctan \left(\frac{v_y(t)}{v_x(t)} \right) = \arctan \left(\frac{1 + 2 \sin t}{2 \cos t} \right)
$$

Note: The sign and magnitude of this angle vary with t, indicating changes in the direction of motion.

Calculating Acceleration Components

The acceleration vector $\vec{a}(t)$ components are derivatives of velocity components:

$$
a_x(t) = \frac{d v_x(t)}{dt} = -2 \sin t
$$

$$
a_y(t) = \frac{d v_y(t)}{dt} = 2 \cos t
$$

The acceleration vector:

$$
\vec{a}(t) = (-2 \sin t, 2 \cos t)
$$

Analyzing the Nature of the Motion

Velocity and Acceleration Relationship

- The acceleration components are negatives of the velocity components with respect to sine and cosine functions, indicating a circular or oscillatory nature in acceleration.
- Notably, the acceleration vector resembles a rotation of the velocity vector, hinting at harmonic motion.

Trajectory Curves and Their Interpretation

- The combination of sinusoidal and linear components produces a spiral-like or oscillating path.
- The particle exhibits oscillations in the X-direction while moving linearly in Y.
- The path's shape is influenced by both harmonic and linear components, leading to complex motion.

Physical Interpretation

- The particle’s motion reflects a superposition of simple harmonic motion (in X) and uniform motion (in Y).
- The oscillations in X are symmetric and bounded between -2 and 2.
- The Y coordinate increases approximately linearly with t, modulated by oscillations.

Key Quantitative Results

- Velocity magnitude: $|\vec{v}(t)| = \sqrt{5 + 4 \sin t}$
- Velocity components: $v_x(t) = 2 \cos t$, $v_y(t) = 1 + 2 \sin t$
- Acceleration components: $a_x(t) = -2 \sin t$, $a_y(t) = 2 \cos t$
- Trajectory shape: A complex curve with oscillatory and linear features.

Conclusion and Summary

The motion of the particle as described by the equations:

$$
X(t) = 2 \sin t
$$
$$
Y(t) = t - 2 \cos t + 3
$$

over the interval $t \in [-2, 2]$, embodies a rich combination of harmonic oscillations and linear progression.

Key takeaways include:

- The particle oscillates horizontally with amplitude 2, governed by a sine function.
- Vertically, it exhibits a linear increase with superimposed oscillations.
- Velocity and acceleration components reveal harmonic behavior, with acceleration always directed in a manner consistent with simple harmonic motion.
- The path traced is complex but can be visualized as an oscillating trajectory with an overall upward trend.

Understanding such parametric motion is crucial in physics, engineering, and mathematics, providing insights into complex systems where oscillations and linear trends coexist.

Future explorations could involve:

- Plotting the trajectory for visualization.
- Determining points of maximum and minimum speed.

- Analyzing curvature and other geometric properties of the path.
- Extending the analysis to other intervals or including external forces.

This comprehensive analysis highlights the rich dynamics of particles governed by parametric equations and deepens our understanding of motion in a plane.

Frequently Asked Questions

What is the parametric equation of the particle's motion in the xy-plane?

The particle's position is given by $x = 2 \sin t$ and $y = t - 2 \cos t + 3$.

For what values of t does the particle move in the xy-plane according to the given equations?

The particle moves for t in the interval $[-2, 2]$, as specified in the problem statement.

How can we find the velocity components of the particle at any time t?

The velocity components are given by the derivatives dx/dt and dy/dt. Here, $dx/dt = 2 \cos t$ and $dy/dt = 1 + 2 \sin t$.

What is the expression for the acceleration of the particle in terms of t?

The acceleration components are the derivatives of the velocity components: $d^2x/dt^2 = -2 \sin t$ and $d^2y/dt^2 = 2 \cos t$.

Can we determine the trajectory equation of the particle in the xy-plane?

Yes. Since $x = 2 \sin t$ and $y = t - 2 \cos t + 3$, eliminating t involves expressing t in terms of x or y, but it may not simplify neatly. Alternatively, parametric plotting or numerical methods can be used to analyze the trajectory.

What is the significance of the interval $-2 \leq t \leq 2$ in analyzing the particle's motion?

This interval specifies the time during which the particle's position is considered, affecting the shape and extent of its trajectory in the xy-plane for the given equations.

Additional Resources

Exploring the Motion of a Particle in the XY Plane: An In-Depth Analysis of Its Trajectory and Characteristics

Understanding the motion of particles in a plane is fundamental in physics and engineering, providing insights into trajectories, velocities, accelerations, and the underlying mathematical relationships. In this comprehensive review, we will analyze the given parametric equations of a particle's motion in the XY plane:

$$
\begin{cases}
X(t) = 2 \sin t \\
Y(t) = t - 2 \cos t + 3
\end{cases}
\quad \text{for} \quad t \in [-2, 2]
$$

Our goal is to interpret these equations thoroughly, explore the particle's trajectory, derive key kinematic quantities, and understand the motion's nature.

Understanding the Given Parametric Equations

1. The Equations in Context

The particle's position at any time t is described by the functions:

- X-coordinate: $X(t) = 2 \sin t$
- Y-coordinate: $Y(t) = t - 2 \cos t + 3$

These equations are parametric, meaning the particle's position depends on a parameter t, which can be interpreted as time within the interval $[-2, 2]$.

Key observations:

- The $X(t)$ component is a scaled sine function, oscillating between -2 and 2.
- The $Y(t)$ component combines a linear term t, a scaled cosine $-2 \cos t$, and a constant shift $+3$.

Range and Behavior of the Coordinates

2. Range of the X-coordinate

Since $X(t) = 2 \sin t$:

- Maximum value: $2 \times 1 = 2$
- Minimum value: $2 \times (-1) = -2$

For $t \in [-2, 2]$, $\sin t$ varies smoothly within $[-1, 1]$. So, X oscillates within $[-2, 2]$.

3. Range of the Y-coordinate

Given $Y(t) = t - 2 \cos t + 3$:

- The t term varies linearly between -2 and 2.
- $-2 \cos t$ varies between $-2 \times 1 = -2$ and $-2 \times (-1) = 2$.

Therefore, $Y(t)$ is a combination of a linear trend and oscillatory behavior.

To find the range of $Y(t)$:

- For a fixed t, $Y(t)$ depends on t and $\cos t$.
- The maximum of $Y(t)$:

$$
Y_{\max} = t_{\max} - 2 \cos t_{\max} + 3
$$

- The minimum of $Y(t)$:

$$
Y_{\min} = t_{\min} - 2 \cos t_{\min} + 3
$$

Because t is within $[-2, 2]$, the extreme values occur at the endpoints and potentially at critical points where the derivative of $Y(t)$ is zero.

Analyzing the Particle's Trajectory

4. Deriving the Path Equation (Eliminating t)

A key step in understanding the motion is to eliminate t to find the Cartesian equation of the trajectory.

Since $X(t) = 2 \sin t$, then:

$$
\sin t = \frac{X}{2}
$$

The inverse sine function gives:

$$
t = \arcsin \left(\frac{X}{2} \right)
$$

But because $t \in [-2, 2]$, and t appears explicitly in $Y(t)$, it’s more straightforward to express Y in terms of X and t, then eliminate t.

Recall:

$$
Y(t) = t - 2 \cos t + 3
$$

Using the identity:

$$
\cos t = \pm \sqrt{1 - \sin^2 t} = \pm \sqrt{1 - \left(\frac{X}{2} \right)^2}
$$

Given $t \in [-2, 2]$, the sign of $\cos t$ depends on t. But for a thorough analysis, it's common to consider both branches or analyze the particle's path segment-wise.

Expressing Y in terms of X:

$$
Y = t - 2 \sqrt{1 - \left(\frac{X}{2} \right)^2} + 3
$$

with $t = \arcsin \left(\frac{X}{2} \right)$, but the sign ambiguity remains. Alternatively, for a complete picture, plotting the parametric equations directly is preferable to avoid ambiguities.

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5. Graphical Interpretation and Shape of Trajectory

Plotting $X(t)$ against $Y(t)$ for $t \in [-2, 2]$:

- X-coordinate: oscillates smoothly between -2 and 2.
- Y-coordinate: increases approximately linearly with t, with superimposed oscillations due to $-2 \cos t$.

The combined effect suggests a wave-like path with a drift in the Y-direction, effectively creating a distorted sinusoidal trajectory with an upward trend.

Kinematic Quantities: Velocity and Acceleration

6. Deriving Velocity Components

The particle's velocity vector $\vec{v}(t)$ has components:

$$
v_x(t) = \frac{dX}{dt} = 2 \cos t
$$
$$
v_y(t) = \frac{dY}{dt} = 1 + 2 \sin t
$$

This derivation uses the derivatives:

- $\frac{d}{dt} \sin t = \cos t$
- $\frac{d}{dt} (t) = 1$
- $\frac{d}{dt} (-2 \cos t) = 2 \sin t$

Key observations:

- The $v_x(t)$ oscillates between -2 and 2.
- The $v_y(t)$ oscillates between $1 - 2 = -1$ and $1 + 2 = 3$.

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7. Speed and Direction

The magnitude of velocity:

$$
|\vec{v}(t)| = \sqrt{v_x^2 + v_y^2} = \sqrt{(2 \cos t)^2 + (1 + 2 \sin t)^2}
$$

Simplify:

$$
|\vec{v}(t)| = \sqrt{4 \cos^2 t + (1 + 2 \sin t)^2}
$$

$$
= \sqrt{4 \cos^2 t + 1 + 4 \sin t + 4 \sin^2 t}
$$

Using the Pythagorean identity $\sin^2 t + \cos^2 t = 1$:

$$
|\vec{v}(t)| = \sqrt{4 (1 - \sin^2 t) + 1 + 4 \sin t + 4 \sin^2 t}
$$

$$
= \sqrt{4 - 4 \sin^2 t + 1 + 4 \sin t + 4 \sin^2 t}
$$

Notice $-4 \sin^2 t + 4 \sin^2 t = 0$, so:

$$
|\vec{v}(t)| = \sqrt{4 + 1 + 4 \sin t} = \sqrt{5 + 4 \sin t}
$$

Range of velocity magnitude:

- Since $\sin t \in [-1, 1]$:

$$
|\vec{v}(t)| \in [\sqrt{5 - 4}, \sqrt{5 + 4}] = [\sqrt{1}, \sqrt{9}] = [1, 3]
$$

Interpretation:

- The particle's speed varies smoothly between 1 and 3 units.
- The maximum speed occurs when $\sin t = 1$, i.e., at $t = \frac{\pi}{2}$ (within the interval).

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8. Acceleration Components

The acceleration vector components:

$$
a_x(t) = \frac{d v_x}{dt} = -2 \sin t
$$

\[

a_y(t) = \frac{d v_y}{dt} = 2 \cos t
\]

These are derivatives of velocity components, representing how the particle's velocity changes over time.

Acceleration magnitude:

\[

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