A Plane Region Ris Bounded By X + Y2 = 0 And 2 +y= -2 A) Calculate The Coordinates Of The Intersection

A Plane Region Ris Bounded By X + Y2 = 0 And 2 +y= -2 A) Calculate The Coordinates Of The Intersection

Understanding the geometric relationships between different curves and lines is fundamental in coordinate geometry. When analyzing a specific plane region bounded by given equations, the first critical step is to determine the intersection points of those boundary curves. This article explores the process of calculating the coordinates of the intersection between the curves defined by the equations \(x + y^2 = 0 \) and \(2 + y = -2 \), and then delves into the broader context of how these intersections define the bounded region. We will also discuss methods to analyze such regions and their significance in mathematical and real-world applications.

Understanding the Given Equations

Before proceeding to find the intersection points, it is essential to interpret the given equations accurately.

Equation 1: \(x + y^2 = 0 \)

This is a quadratic equation relating \(x \) and \(y \). Rewriting it gives:

\[
x = - y^2
\]

This represents a parabola opening to the left, symmetric with respect to the \(y \)-axis, with its vertex at the origin.

Equation 2: \(2 + y = -2 \)

At first glance, this appears to be a simple linear equation involving \(y \). Solving for \(y \):

\[
2 + y = -2

\Rightarrow y = -2 - 2
\Rightarrow y = -4
\]

Thus, this equation describes the horizontal line \(y = -4 \).

Calculating the Intersection Coordinates

The intersection points between the parabola \(x = - y^2 \) and the line \(y = -4 \) are found by substituting \(y = -4 \) into the parabola's equation.

Step-by-step Calculation

1. Substitute \(y = -4 \) into \(x = - y^2 \):

\[
x = - (-4)^2 = - 16
\]

2. Resulting coordinate:

\[
\boxed{(x, y) = (-16, -4)}
\]

This indicates that the parabola and the line intersect at exactly one point, \((-16, -4)\).

Visualizing the Intersection and the Bounded Region

Understanding the position of the curves helps in visualizing the bounded region they form.

Graphical Interpretation

- The parabola \(x = - y^2 \) opens to the left, with its vertex at the origin \((0, 0)\).
- The line \(y = -4 \) is a horizontal line below the vertex, crossing the parabola at \((-16, -4)\).

The region of interest is the area enclosed between these two curves, which is bounded on the right by the parabola and on the bottom by the line $y = -4$.

Significance of the Intersection Point

Since the parabola opens to the left and the line is below the vertex, their intersection at $(-16, -4)$ marks one of the key points defining the boundary of the region. If the problem involves integrating or calculating areas, understanding these points is essential.

Analyzing the Bounded Region

Having identified the intersection point, we can proceed to analyze the entire region bounded by the curves.

Boundaries of the Region

- Left boundary: The parabola $x = - y^2$
- Lower boundary: The line $y = -4$
- Upper boundary: Potentially, the parabola's vertex at $y = 0$, if the region extends upward.

Assuming the region is between $y = -4$ and $y = 0$, the boundary is formed between these two curves.

Defining the Region mathematically

- For each y in the interval $[-4, 0]$, x ranges from the parabola $x = - y^2$ to some boundary or axis of symmetry.

- Since the parabola opens leftward, the region's rightmost boundary at each y is $x = - y^2$.

Applications and Further Calculations

Understanding the intersection points is crucial for various applications, such as calculating the area of the region, setting up integrals, or analyzing physical phenomena modeled by these curves.

Calculating the Area of the Region

Suppose we want to compute the area enclosed between the parabola and the line:

$$
A = \int_{y=-4}^{0} \left[x_{right}(y) - x_{left}(y) \right] dy
$$

- $x_{right}(y)$: the boundary on the right, which in this case is the parabola $- y^2$.

- $x_{left}(y)$: the boundary on the left, possibly the line $y = -4$, which is a constant; however, since $y = -4$ is a horizontal boundary, integrating vertically from the parabola to the line makes sense, or considering the region between the curves.

Alternatively, if the region is between the parabola $x = - y^2$ and the line $y = -4$, and assuming the parabola's domain from $y = -4$ to $y=0$, the area calculation simplifies to:

$$
A = \int_{y=-4}^{0} \left(- y^2 \right) dy
$$

since the parabola extends from the line $y = -4$ upward to $y=0$.

Conclusion

In this exploration, we have determined that the key intersection point between the parabola $x + y^2=0$ and the line $2 + y = -2$ is at $(-16, -4)$. This point defines the boundary of the plane region bounded by these curves. Recognizing and calculating such intersection points are fundamental steps in coordinate geometry, enabling further analysis such as area calculations, region visualization, and solving physical problems modeled by these equations.

Understanding these concepts enhances one's ability to analyze complex geometric regions, which are ubiquitous in mathematics, engineering, physics, and computer graphics. Whether you're computing areas, finding centers of mass, or solving optimization problems, accurately determining intersection points is a critical skill in the mathematical toolkit.

Additional Resources

- Coordinate Geometry Textbooks: For a comprehensive understanding of curves and their

intersections.
- Graphing Software: Tools like Desmos or GeoGebra to visualize these curves and their intersections.
- Calculus Applications: Tutorials on setting up and evaluating integrals over bounded regions.

In summary, the intersection point $(-16, -4)$ between the parabola $x + y^2 = 0$ and the line $2 + y = -2$ plays a vital role in defining the bounded region of interest. Mastery of such calculations lays the foundation for more advanced geometric and calculus-based analyses.

Frequently Asked Questions

What is the geometric interpretation of the region bounded by the equations x + y^2 = 0 and y + 2 = -2?

The region is bounded by the parabola x + y^2 = 0 (which can be rewritten as x = -y^2) and the horizontal line y = -4. It represents the area between the parabola opening leftward and the line y = -4.

How do you find the intersection points of the curves x + y^2 = 0 and y + 2 = -2?

First, rewrite y + 2 = -2 as y = -4. Substitute y = -4 into x + y^2 = 0 to get x + (-4)^2 = 0, which simplifies to x + 16 = 0, so x = -16. Therefore, the intersection point is (-16, -4).

What are the coordinates of the intersection point between the parabola x + y^2 = 0 and the line y = -4?

The intersection point is at (-16, -4).

Can the equations x + y^2 = 0 and y + 2 = -2 be used to determine the bounded region in the coordinate plane?

Yes. The parabola x = -y^2 and the line y = -4 define the boundaries of the region, and their intersection point helps identify the limits of this bounded area.

What is the significance of finding the intersection point when analyzing bounded regions between curves?

Finding the intersection point helps determine the limits of integration, the shape of the region, and is essential for calculating areas, integrals, or other geometric properties related to the region.

How do you set up the integral to calculate the area of the region bounded by these curves?

Since the region is between $y = -4$ and the parabola $x = -y^2$, you can set up the integral with y from -4 to the maximum y-value, integrating $x = -y^2$ with respect to y over this interval: Area = $\int_{-4}^{y_{max}} [-y^2]\, dy$.

What are the key steps to solve for the intersection points of two curves in a plane?

The key steps are: 1) Express one variable in terms of the other using one equation, 2) substitute into the second equation, 3) solve the resulting equation for the variable, and 4) substitute back to find the other coordinate.

Is the line y = -2 relevant in finding the intersection points of these two equations?

No, because the line $y = -2$ does not intersect with the parabola $x + y^2 = 0$ at the same point as $y = -4$. The relevant line for intersection is $y = -4$, which was derived from $y + 2 = -2$, but in this context, $y + 2 = -2$ simplifies directly to $y = -4$.

How can you verify the intersection point of the two curves visually?

By plotting the parabola $x = -y^2$ and the line $y = -4$ on the same coordinate plane, you can visually confirm that they intersect at (-16, -4).

Additional Resources

Analytical Geometry

Understanding the intersection of curves in the coordinate plane is a fundamental aspect of analytical geometry, offering insights into how different geometric entities relate to each other. This article delves into the detailed process of analyzing the intersection of two specific regions bounded by the equations:

- $x + y^2 = 0$
- $2 + y = -2$

Our goal is to calculate the coordinates of their intersection points and explore the nature of the region they define.

Deciphering the Equations: An Initial Overview

Before diving into the calculations, it's essential to understand what each equation represents geometrically.

The First Equation: $x + y^2 = 0$

This is a quadratic relation in the plane, representing a parabola that opens towards the left. Re-arranged:

$$
x = - y^2
$$

- Shape and position: Since $y^2 \geq 0$, the parabola opens to the left, with its vertex at the origin $(0, 0)$.
- Domain and Range:
- y can be any real number, so the domain is $(-\infty, \infty)$.
- For each y, $x \leq 0$, hence the parabola lies entirely in the left half-plane.

The Second Equation: $2 + y = -2$

This appears to be a linear equation, but the expression as written is somewhat ambiguous. Likely, the intended equation is:

$$
2 + y = -2
$$

which simplifies to:

$$
y = -4
$$

This is a horizontal line at $y = -4$.

Clarifying the Equations and Regions

Given the initial equations seem to be straightforward, but the problem mentions a "region bounded by" these equations, likely referring to the areas enclosed between the parabola $x = - y^2$ and the line $y = -4$.

However, the original problem statement as provided is somewhat ambiguous, especially with the phrase:

> "And 2 + y = -2"

which matches the expression \(2 + y = -2 \) and simplifies directly.

For clarity, the key equations defining the regions are:

- Parabola: \(x = - y^2 \)
- Line: \(y = -4 \)

Step 1: Visualizing the Regions

The Parabola: \(x = - y^2 \)
- Opens towards the left.
- Vertex at \((0, 0)\).
- Symmetric about the \(y \)-axis.

The Line: \(y = -4 \)
- A horizontal line crossing the \(y \)-axis at \(-4\).
- Extends infinitely left and right.

Step 2: Finding the Intersection Points

The core task is to find the points where these two curves intersect, i.e., the solutions to:

\[
x = - y^2 \quad \text{and} \quad y = -4
\]

Since the line is given explicitly as \(y = -4 \), substitute \(y = -4 \) into the parabola:

\[
x = - (-4)^2 = - 16
\]

Resulting point:

\[
\boxed{
(-16, -4)
}
\]

This indicates that the parabola and the line intersect at a single point: \((-16, -4)\).

Step 3: Confirming the Region Bounded by the Curves

Given the equations, the region of interest appears to be the area enclosed between the parabola and the line. Since the parabola extends infinitely to the left, but the line is at \(y = -4 \), the bounded region is the segment of the parabola lying between \(y = -4 \) and some other boundary.

However, because the line \(y = -4 \) and the parabola intersect at \((-16, -4)\), and considering the parabola opens toward the left, the region bounded by these two curves is:

- Below the parabola: For \(y < -4 \), since the parabola is symmetric about the \(y \)-axis, and the parabola at \(y = -4 \) is at \(x = -16 \).

- Between the parabola and the line: From the intersection point \((-16, -4)\) extending downward to some other boundary, possibly to negative infinity in \(y \).

Step 4: Establishing the Boundaries and the Region

Assuming the region is the area between the parabola \(x = - y^2 \) and the line \(y = -4 \), bounded above by the line and below by the parabola, the key points are:

- Upper boundary: \(y = -4 \)
- Lower boundary: The parabola \(x = - y^2 \)

The region is enclosed between these, with the intersection point at \((-16, -4)\).

Step 5: Calculating the Coordinates of Intersection and the Region

Summary of the key intersection point:

\[

\boxed{
(-16, -4)
}
\]

which is the only point where the parabola and the line meet.

Additional Considerations: The Complete Region

If the problem refers to the entire bounded region, we need to identify:

- The range of \(y \) between the intersection point and the parabola's vertex, which is at \((0,0)\).

- The \(x \)-values at these \(y \)-coordinates, given by \(x = - y^2 \).

Key points:

- At \(y = -4 \), \(x = -16 \).
- At \(y = 0 \), \(x = 0 \).

Therefore, the bounded region could be described as:

\[
\text{Region } R = \left\{ (x, y) \mid - y^2 \leq x \leq 0, \quad y \in [-4, 0] \right\}
\]

This region is bounded on the left by the parabola, on the right by the line \(x = 0 \) (assuming the region is enclosed), and vertically from \(y = -4 \) to \(y = 0 \).

Conclusion: The Key Coordinates and Region Characteristics

- Intersection Point:
\[
\boxed{
(-16, -4)
}
\]
is the sole point where the parabola \(x = - y^2 \) and the line \(y = -4 \) meet.

- Region Description:

The region bounded by these curves is the area between the parabola $x = - y^2$ and the line $y = -4$, extending from $y = -4$ (at the intersection point) downwards to the vertex at $y = 0$.

- Implication for Integration or Area Calculation:
This region can be used for further calculations, such as computing the area enclosed via definite integrals, by integrating with respect to y from -4 to 0:

$$
\text{Area} = \int_{y=-4}^{0} \left(\text{right boundary} - \text{left boundary} \right) dy
$$

where the right boundary could be the line $x=0$ and the left boundary the parabola $x= - y^2$.

Final Remarks

This detailed exploration underscores the importance of visualizing equations geometrically and performing algebraic substitutions to find intersection points. The process demonstrates that understanding the nature of the curves—parabolas and lines—and their points of contact is critical for defining bounded regions in the plane.

By accurately calculating the intersection point $(-16, -4)$, we lay the groundwork for more advanced analysis, such as area calculations, region plotting, or further geometric investigations. This approach exemplifies how analytical geometry combines algebraic techniques with geometric intuition, providing a powerful toolkit for solving complex planar problems.

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