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This scenario presents a classic problem in electrostatics, involving the interaction of multiple point charges and the resulting electric fields and potentials. Understanding such configurations is essential for mastering concepts like Coulomb's law, superposition principle, and electric potential energy. In this article, we will explore the detailed physics behind the arrangement, analyze the forces and potentials involved, and discuss the implications of placing a test charge in this electric field.

Introduction to the Configuration

Imagine three point charges aligned along a straight line:

- A charge of +3q located at position $x = 0$.
- A charge of -q located at position $x = 2d$.
- A test charge of +q placed exactly midway between the two, at position $x = d$.

The key question is: What are the net electric field and potential experienced by the test charge +q at the midpoint? Additionally, we examine the forces acting on the test charge, the stability of its equilibrium position, and the nature of the interactions among these charges.

Understanding the Spatial Arrangement

Coordinates and Distances

- The fixed charges are positioned at:
- $Q_1 = +3q$ at $x = 0$,
- $Q_2 = -q$ at $x = 2d$.

- The test charge q is at:

- $x = d$, midway between the two.

Distances from the Test Charge to Fixed Charges

- Distance to Q_1: $r_{1} = d$,
- Distance to Q_2: $r_{2} = d$.

Since the test charge is equidistant from both fixed charges, symmetry plays a vital role in analyzing the electric field and potential.

Calculating the Electric Field at the Midpoint

The Principle of Superposition

Electric fields due to individual charges are vector quantities that add algebraically. The total electric field at the location of the test charge is the vector sum of the fields due to each fixed charge.

Electric Field Due to a Point Charge

The electric field $\vec{E}$ produced by a point charge Q at a distance r is given by Coulomb's law:

$$
\vec{E} = \frac{1}{4 \pi \varepsilon_0} \frac{Q}{r^2} \hat{r}
$$

where:
- ε_0 is the permittivity of free space,
- $\hat{r}$ is the unit vector pointing from the charge to the point of observation.

Calculating Fields from Fixed Charges

- From $Q_1 = +3q$:
- The field at the midpoint points away from Q_1 because the charge is positive.
- Magnitude:

$$
E_1 = \frac{1}{4 \pi \varepsilon_0} \frac{3q}{d^2}
$$

- From $Q_2 = -q$:
- The field at the midpoint points toward Q_2 because the charge is negative.
- Magnitude:

$$
E_2 = \frac{1}{4 \pi \varepsilon_0} \frac{q}{d^2}
$$

Direction of Electric Fields

- E_1: points from Q_1 to the midpoint, i.e., to the right.
- E_2: points from the midpoint toward Q_2, i.e., to the right (since negative charge attracts positive test charge).

Result: Both electric fields point in the same direction (to the right), so they add constructively.

Net Electric Field at the Midpoint

$$
E_{net} = E_1 + E_2 = \frac{1}{4 \pi \varepsilon_0} \left(\frac{3q}{d^2} + \frac{q}{d^2} \right) = \frac{1}{4 \pi \varepsilon_0} \frac{4q}{d^2}
$$

The direction is to the right, away from Q_1 and toward Q_2.

Calculating Electric Potential at the Midpoint

Since electric potential is a scalar quantity, contributions from individual charges add algebraically.

Potential Due to a Point Charge

$$
V = \frac{1}{4 \pi \varepsilon_0} \frac{Q}{r}
$$

Total Potential at the Midpoint

$$
V_{total} = V_{Q_1} + V_{Q_2} = \frac{1}{4 \pi \varepsilon_0} \left(\frac{3q}{d} - \frac{q}{d} \right) = \frac{1}{4 \pi \varepsilon_0}
$$

$$
\frac{2q}{d}
$$

Note the negative sign for Q_2 because it is negative.

Force on the Test Charge +q at the Midpoint

The force $\vec{F}$ on the test charge is given by:

$$
\vec{F} = q \vec{E}_{net}
$$

Using the earlier calculated electric field:

$$
F = q \times E_{net} = q \times \frac{1}{4 \pi \varepsilon_0} \frac{4q}{d^2} = \frac{1}{4 \pi \varepsilon_0} \frac{4q^2}{d^2}
$$

The force acts in the direction of the net electric field, which is to the right.

Analyzing the Nature of the Force and Equilibrium

Is the Midpoint a Point of Equilibrium?

- Since the electric field at the midpoint is non-zero, the test charge experiences a force.
- The force is directed to the right, indicating that the charge is pushed away from the fixed charges.

Stability of the Equilibrium

- To check for stability, consider displacing the test charge slightly:
- If displaced to the left: net force pushes it further left (unstable equilibrium).
- If displaced to the right: net force pushes it further right (unstable equilibrium).

Therefore, the midpoint is not a stable equilibrium point.

Electric Potential Energy of the System

The potential energy between two point charges Q_i and Q_j separated by distance r_{ij} is:

$$
U_{ij} = \frac{1}{4 \pi \varepsilon_0} \frac{Q_i Q_j}{r_{ij}}
$$

Total potential energy of the system:

$$
U_{total} = U_{13} + U_{23} + U_{q1} + U_{q2} + U_{q3}
$$

where:
- U_{13}: between $+3q$ and $-q$,
- U_{23}: between $-q$ and $+q$,
- U_{q1}: between test charge $+q$ and $+3q$,
- U_{q2}: between test charge $+q$ and $-q$,
- U_{q3}: between test charge and the other fixed charges (if any).

Since the test charge is a small probe, often the focus is on potential energy associated with fixed charges, but for completeness, all interactions can be considered.

Implications of Charge Placement and Electric Field Behavior

Electric Field Lines

- Lines originate from positive charges and terminate at negative charges.
- The combined field shows the influence of both fixed charges, with the highest intensity near the charges.

Force and Potential Profiles

- The force on the test charge increases with the magnitude of the fixed

charges and decreases with the square of the distance.
- The potential at the midpoint shows a positive value, indicating a net positive potential energy in the system.

Effects of Varying Parameters

- Increasing q or d: increases the electric field and force.
- Changing the fixed charges: alters the magnitude and direction of the electric field.
- Moving the test charge away from the midpoint: the symmetry breaks, leading to a different force and potential.

Applications and Further Considerations

- **Electrostatic Shielding:** Understanding how charges influence each other helps in designing shielding for sensitive electronic equipment.

- **Electric Field Mapping:** Analyzing such configurations aids in visualizing and calculating electric field distributions in complex systems.