# A Prism Is Completely Filled With 96 Cubes That Have Edge Lengths Of 1/2 Cm. What Is The Volume Of The

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Understanding the concept of volume is fundamental in mathematics, especially when dealing with three-dimensional shapes such as prisms. In this article, we will explore a practical problem involving a prism filled with smaller cubes, each with a specific edge length. The problem asks us to determine the total volume of the prism based on the number of cubes it contains and the size of each cube. This type of problem is common in geometry and helps develop spatial reasoning, measurement skills, and an understanding of volume calculations.

The scenario involves a prism that is completely filled with 96 smaller cubes, each measuring 1/2 cm on each edge. Our goal is to find the volume of the entire prism based on this information. To do so, we will analyze the problem step by step, starting from understanding the properties of individual cubes, calculating their volume, and then determining the overall volume of the prism.

This article will guide you through the process of solving such problems, including the necessary formulas, calculations, and logical reasoning. Whether you are a student preparing for geometry exams or a curious learner interested in understanding three-dimensional measurements, this comprehensive guide will provide clarity and insight.

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## Understanding the Basic Concepts

Before diving into the calculations, it is essential to review some fundamental concepts related to volume and shapes.

### What Is Volume?

Volume refers to the amount of space occupied by a three-dimensional object. It is measured in cubic units, such as cubic centimeters ($cm^3$), cubic meters ($m^3$), or cubic inches ($in^3$). In most geometry problems involving small objects, cubic centimeters are common units.

### Properties of a Cube

A cube is a three-dimensional shape with six equal square faces. The key property of a cube is that all its edges are of the same length.

- Edge length: The length of any side of the cube.
- Volume of a cube: Given by the formula $V = a^3$, where $a$ is the edge length.

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## Step 1: Analyzing the Dimensions of the Small Cubes

The problem states that each small cube has an edge length of 1/2 cm.

### Calculating the Volume of One Small Cube

Using the formula for the volume of a cube:

$$
V_{cube} = a^3
$$

where $a = \frac{1}{2}$ cm,

$$
V_{cube} = \left( \frac{1}{2} \right)^3 = \frac{1}{8} \text{ cm}^3
$$

Therefore, each small cube occupies $\frac{1}{8}$ cubic centimeters of space.

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## Step 2: Total Volume of All Small Cubes

Given that the prism contains 96 such cubes, the total volume occupied by all the small cubes is:

$$
V_{total\,cubes} = 96 \times V_{cube}
$$

Substituting the value of $V_{cube}$:

$$
V_{total\,cubes} = 96 \times \frac{1}{8} = \frac{96}{8} = 12 \text{ cm}^3
$$

This means the entire filled space inside the prism amounts to 12 cubic centimeters.

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# Step 3: Determining the Dimensions of the Prism

To find the volume of the prism itself, understanding its dimensions is crucial. Since the prism is completely filled with the 96 small cubes, the arrangement of the cubes determines the dimensions of the prism.

## Arranging the Cubes

Assuming the cubes are packed together without gaps, they form a rectangular prism with dimensions based on how many cubes fit along each length, width, and height.

- Total number of cubes: 96
- Each cube's edge length: 1/2 cm

The goal is to find the dimensions of the prism in terms of the number of cubes along each axis.

## Possible Arrangements

The total number of cubes can be factored into three integers representing the number of cubes along each dimension:

$$
\text{Number of cubes along length} \times \text{Number along width} \times \text{Number along height} = 96
$$

Common factorizations of 96 include:

- $1 \times 1 \times 96$
- $1 \times 2 \times 48$

- $1 \times 3 \times 32$
- $1 \times 4 \times 24$
- $1 \times 6 \times 16$
- $1 \times 8 \times 12$
- $2 \times 2 \times 24$
- $2 \times 3 \times 16$
- $2 \times 4 \times 12$
- $2 \times 6 \times 8$
- $3 \times 4 \times 8$
- $4 \times 4 \times 6$

Depending on the problem context or the physical arrangement, any of these could be valid. However, since the problem does not specify the arrangement, the most typical and balanced arrangement is the one that minimizes the differences between dimensions, such as $4 \times 4 \times 6$.

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## Step 4: Calculating the Dimensions of the Prism

Let's consider the arrangement where the small cubes are packed in a $4 \times 4 \times 6$ configuration.

- Number of cubes along length: 4
- Number along width: 4
- Number along height: 6

Each small cube has an edge length of 1/2 cm, so:

- Length of the prism:

$$
L = 4 \times \frac{1}{2} = 2 \text{ cm}
$$

- Width of the prism:

$$
W = 4 \times \frac{1}{2} = 2 \text{ cm}
$$

- Height of the prism:

$$
H = 6 \times \frac{1}{2} = 3 \text{ cm}
$$

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## Step 5: Calculating the Volume of the Prism

The volume of a rectangular prism is given by:

$$
V_{prism} = L \times W \times H
$$

Substituting the dimensions:

$$
V_{prism} = 2 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^3
$$

This matches the total volume of the 96 small cubes, confirming the arrangement.

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## Final Summary and Conclusion

Based on the calculations:

- Each small cube has a volume of $\frac{1}{8}$ cm$^3$.
- Total volume of 96 cubes is 12 cm$^3$.
- An arrangement of $4 \times 4 \times 6$ cubes within the prism results in dimensions of 2 cm by 2 cm by 3 cm.
- The volume of the filled prism is therefore 12 cubic centimeters.

Key Takeaways:

- The total volume of the prism is directly related to the number of small cubes and the size of each cube.
- Understanding how to factor the total number of cubes helps determine the overall dimensions.
- The problem exemplifies the importance of spatial reasoning and the application of volume formulas in real-world and mathematical contexts.

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# Additional Related Topics

To deepen your understanding, consider exploring:

- Different arrangements of cubes within a prism
- How to calculate surface area alongside volume
- Volume formulas for other 3D shapes such as cylinders and cones
- Applications of volume calculations in real-world scenarios like packaging and storage

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# FAQs

Q1: Why is the volume of one cube $\frac{1}{8}$ $cm^3$?
Because the cube's edge length is 1/2 cm, and volume is calculated as $a^3$, so $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Q2: How do I know the arrangement of the cubes?
Without additional information, any factorization of 96 into three integers is possible. Common arrangements are based on symmetry and practicality.

Q3: Can the volume of the prism be different if arranged differently?
No. The total volume depends on the total space occupied, which remains the same regardless of arrangement. The shape's dimensions may change, but the total volume stays constant.

Q4: What if the edge length of the cubes was different?
You would recalculate the volume of each cube accordingly and multiply by the total number of cubes to find the total volume.

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By mastering these concepts and calculations, you'll be better equipped to handle diverse problems involving volume, measurement, and spatial reasoning in geometry.

# Frequently Asked Questions

## What is the total number of cubes filling the prism?

The prism is completely filled with 96 cubes.

## What is the edge length of each small cube?

Each cube has an edge length of 1/2 cm.

## How do you calculate the volume of one small cube?

The volume of one small cube is (1/2) cm × (1/2) cm × (1/2) cm = 1/8 cubic centimeters.

## What is the total volume of all 96 cubes?

Total volume = 96 × 1/8 $cm^3$ = 12 cubic centimeters.

## How does knowing the number of cubes help in finding the prism's volume?

Since the prism is completely filled with the cubes, the total volume of the prism equals the total volume of all the cubes, which is 12 $cm^3$.

## What assumptions are made in calculating the prism's volume?

It is assumed that the cubes are tightly packed without gaps and that they fill the entire volume of the prism.

## If each cube has an edge length of 1/2 cm, what are the dimensions of the prism?

The dimensions depend on how the cubes are arranged; for example, if arranged in a 8 × 6 × 2 configuration, the prism's dimensions would be 4 cm × 3 cm × 1 cm.

## What is the significance of the volume being 12 cubic centimeters?

It indicates the total space occupied by the 96 small cubes within the prism, confirming the overall volume of the prism itself.

# Additional Resources

A Prism Is Completely Filled With 96 Cubes That Have Edge Lengths Of 1/2 Cm. What Is The Volume Of The

In the world of geometry, understanding the volume of complex three-dimensional shapes often begins with examining their constituent parts. Imagine a prism perfectly filled with tiny cubes, each measuring half a centimeter along each edge. Such a scenario not only sparks curiosity but also provides a practical context for exploring volume calculations. In this article, we'll delve into the problem of determining the total volume of the prism based on the given information—namely, that it contains exactly 96 small cubes, each with an edge length of 1/2 cm. We'll unpack the steps involved, clarify key concepts, and demonstrate how to arrive at a precise answer.

# Understanding the Basic Components: The Small Cubes

Before calculating the volume of the entire prism, it's essential to understand the properties of the small cubes that fill it.

## Dimensions of Each Cube

- Edge Length: Each cube has an edge length of 1/2 cm, which can also be expressed as 0.5 cm.
- Shape: Being perfect cubes, all edges are equal, and angles between faces are right angles.

## Volume of a Single Cube

The volume $V_{cube}$ of a cube is calculated using the formula:

$$
V_{cube} = \text{edge length}^3
$$

Substituting the given edge length:

$$
V_{cube} = (0.5\, \text{cm})^3 = 0.5\, \text{cm} \times 0.5\, \text{cm} \times 0.5\, \text{cm}
$$

Calculating:

$$
V_{cube} = 0.125\, \text{cm}^3
$$

This means each small cube occupies a volume of 0.125 cubic centimeters.

# Determining the Total Volume of All Cubes

Given that the prism is filled with 96 such cubes, the total volume is simply the sum of the individual volumes:

$$
V_{total\_cubes} = \text{Number of cubes} \times V_{cube}
$$

$$
V_{total\_cubes} = 96 \times 0.125\, \text{cm}^3
$$

Calculating:

$$
V_{total\_cubes} = 12\, \text{cm}^3
$$

Thus, the combined volume of all the small cubes—and therefore the volume of the filled prism—is 12 cubic centimeters.

# Understanding the Dimensions of the Prism

While the total volume is straightforward once the volume of individual cubes is known, understanding the shape and dimensions of the prism itself provides additional context.

## Shape and Arrangement of the Cubes

- The prism is completely filled with these cubes, which suggests an arrangement where the cubes are tightly packed without gaps.
- The packing is likely a rectangular arrangement, with the cubes aligned along the length, width, and

height.

## Determining the Dimensions of the Prism

Since the total volume is 12 $\text{cm}^3$, and each small cube has a volume of 0.125 $\text{cm}^3$, the number of cubes (96) indicates the arrangement dimensions:

- Let's denote:
- $l$ = number of cubes along the length
- $w$ = number of cubes along the width
- $h$ = number of cubes along the height

- The total number of cubes:

$$
l \times w \times h = 96
$$

- The physical dimensions:

$$
L = l \times 0.5\, \text{cm}
$$
$$
W = w \times 0.5\, \text{cm}
$$
$$
H = h \times 0.5\, \text{cm}
$$

- The overall volume of the prism:

$$
V_{prism} = L \times W \times H = (l \times 0.5) \times (w \times 0.5) \times (h \times 0.5)
$$

which simplifies to:

$$
V_{prism} = (l \times w \times h) \times (0.5)^3
$$

Given $l \times w \times h = 96$:

$$
V_{prism} = 96 \times 0.125 = 12\, \text{cm}^3
$$

This confirms our earlier calculation and shows that the total volume of the prism matches the combined volume of the small cubes filling it.

## Implications and Practical Applications

Understanding the total volume of a filled prism has real-world implications in various fields—manufacturing, packaging, architecture, and more. For instance:

- Material Calculation: Knowing the total volume helps determine the amount of raw material needed to create a similar structure.
- Design Optimization: Engineers can use such calculations to optimize space usage and minimize waste.
- Educational Contexts: Demonstrating these concepts aids in teaching spatial reasoning and volume calculations.

## Additional Considerations: Variations and Assumptions

While the problem provides a straightforward scenario, several assumptions underlie the calculation:

- The cubes are perfectly packed without gaps or overlaps.
- All cubes are identical in size.
- The arrangement is a simple rectangular prism, not a more complex shape.

In more complex scenarios, factors such as packing density, irregular shapes, or gaps could influence the total volume calculations.

## Conclusion: The Final Answer

Bringing all these insights together, the key takeaway is clear: the volume of the prism, fully filled with 96 small cubes each measuring 1/2 cm on every edge, is 12 cubic centimeters. This result underscores how understanding the properties of individual components—small cubes in this case—can lead to accurate calculations of larger, more complex shapes. Whether in academic exercises or practical applications, such problem-solving approaches reinforce fundamental principles of geometry and volume measurement, equipping learners and professionals alike with essential analytical skills.

In summary:

- Volume of each small cube: 0.125 $cm^3$
- Number of cubes: 96
- Total volume of the prism: $96 \times 0.125 = 12$ $cm^3$

This clear, step-by-step breakdown demonstrates the power of basic geometric formulas and logical reasoning in solving spatial problems.

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