A Random Sample Of Size N = 25 Yield A Sample Mean Of 50 Andstandard Deviation Of 8. Calculate The 95%

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Introduction

In statistical analysis, understanding how to interpret sample data and calculate confidence intervals is crucial for making informed decisions. When working with a random sample, especially when estimating population parameters such as the mean, it's essential to know how to determine the confidence interval—specifically, the 95% confidence interval in this context. This article explores the process of calculating the 95% confidence interval for a population mean based on a sample, using the provided data: a sample size of N=25, a sample mean of 50, and a sample standard deviation of 8.

Understanding the Key Concepts

Before diving into the calculations, it's important to clarify some fundamental statistical concepts involved in estimating population parameters:

Sample Mean (\(\bar{x}\))

- The average value calculated from the sample data.
- In this case, \(\bar{x} = 50\).

Sample Standard Deviation (s)

- Measures the dispersion or variability within the sample data.
- Given as \(s = 8\).

Sample Size (N)

- Total number of observations in the sample.
- Here, \(N=25\).

Confidence Interval (CI)

- A range of values within which the true population parameter (mean) is estimated to lie with a certain level of confidence.
- The 95% confidence interval means there is a 95% probability that the interval contains the true population mean.

Why Confidence Intervals Matter

- They provide a range of plausible values for the population parameter.
- Useful in decision-making, quality control, and research to quantify uncertainty.

Assumptions and Conditions

Calculating a confidence interval for the population mean involves certain assumptions:

- The sample data are randomly selected from the population.
- The data are approximately normally distributed, especially critical for small samples ($n<30$).
- The population standard deviation is unknown (common in practice), so the sample standard deviation is used.

Since the sample size is 25, which is relatively small, the t-distribution is appropriate for the calculation rather than the normal distribution.

Step-by-Step Calculation of the 95% Confidence Interval

Let's now proceed with the detailed steps to compute the 95% confidence interval for the population mean.

Step 1: Identify the Sample Data

- Sample size, $n = 25$
- Sample mean, $\bar{x} = 50$
- Sample standard deviation, $s = 8$

Step 2: Determine the Appropriate Distribution and Critical Value

- Since the population standard deviation is unknown and the sample size is small, use the Student's t-distribution.
- Degrees of freedom (df) = $n - 1 = 24$.

Using a t-table or calculator, find the t-value for a 95% confidence level with 24 degrees of freedom:

Confidence Level	Degrees of Freedom	t-value (approximate)
95%	24	2.064

(Note: The t-value may vary slightly depending on the table or calculator used.)

Step 3: Calculate the Standard Error of the Mean (SE)

$$
SE = \frac{s}{\sqrt{n}} = \frac{8}{\sqrt{25}} = \frac{8}{5} = 1.6
$$

Step 4: Calculate the Margin of Error (ME)

$$
ME = t \times SE = 2.064 \times 1.6 = 3.3024
$$

Step 5: Determine the Confidence Interval

- Lower bound:
$$
\bar{x} - ME = 50 - 3.3024 = 46.6976
$$
- Upper bound:
$$
\bar{x} + ME = 50 + 3.3024 = 53.3024
$$

Therefore, the 95% confidence interval for the population mean is approximately:

$$
\boxed{
(46.70, 53.30)
}
$$

Interpreting the Results

The calculated confidence interval suggests that, based on the sample data, we are 95% confident that the true population mean lies between approximately 46.70 and 53.30.

Key points to consider:

- The interval provides an estimate that accounts for sampling variability.
- A wider interval indicates more uncertainty, while a narrower interval suggests greater precision.
- If multiple samples are taken, approximately 95% of such calculated intervals would contain the true population mean.

Additional Considerations

Effect of Sample Size

- Larger sample sizes tend to produce narrower confidence intervals, increasing estimate precision.
- With N=25, the interval is reasonably precise but could be improved with more data.

Normality Assumption

- For small samples, it's important that data are approximately normally distributed.
- If data are skewed or contain outliers, alternative methods or transformations may be necessary.

Using Z-Distribution Instead of t-Distribution

- When the population standard deviation is known or the sample size is large (typically N>30), the Z-distribution can be used.
- In this case, since σ is unknown and N is small, the t-distribution is appropriate.

Practical Applications of Confidence Intervals

- Quality control in manufacturing.
- Estimating average customer satisfaction scores.
- Assessing average test scores in educational research.
- Comparing treatment effects in clinical trials.

Summary

Calculating a confidence interval is a fundamental skill in statistics that enables researchers and analysts to infer population parameters from sample data. In this scenario, with a sample size of 25, a mean of 50, and a standard deviation of 8, the 95% confidence interval for the population mean is approximately (46.70, 53.30). This interval indicates that we can be 95% confident the true mean falls within this range, which is crucial for making informed decisions based on the sample data.

Understanding and correctly applying the concepts of sample mean, standard deviation, standard error, and the t-distribution are essential for accurate statistical inference. Whether for academic research, business analytics, or scientific studies, mastering the calculation of confidence intervals enhances the reliability and credibility of your conclusions.

References and Further Reading

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By understanding how to compute and interpret confidence intervals, you can confidently analyze sample data and estimate population parameters with quantifiable certainty.

Frequently Asked Questions

What is the 95% confidence interval for the population mean given a sample size of 25, sample mean of 50, and standard deviation of 8?

The 95% confidence interval is calculated as 50 ± (t × (8/√25)). With df=24, t ≈ 2.064. So, the margin of error is 2.064 × 1.6 = 3.3024. Therefore, the confidence interval is approximately (46.6976, 53.3024).

How do you interpret the 95% confidence interval derived from this sample?

It means that we are 95% confident that the true population mean lies between approximately 46.70 and 53.30 based on this sample data.

Can we use the standard deviation directly to compute the confidence interval for the population mean in this case?

Yes, since the sample size is 25 (n>30 is ideal for normal approximation), but ideally, if the population standard deviation is unknown, the sample standard deviation is used with the t-distribution, which is appropriate here.

What is the significance of using the t-distribution instead of the normal distribution in this calculation?

Because the sample size is small (n=25), the t-distribution accounts for additional variability and provides a more accurate confidence interval than the normal distribution would in this scenario.

How would increasing the sample size to 100 affect the confidence interval calculation?

A larger sample size would decrease the standard error (8/√N), resulting in a narrower confidence interval, thus providing a more precise estimate of the population mean.

Additional Resources

A Random Sample Of Size N = 25 Yield A Sample Mean Of 50 And Standard Deviation Of 8. Calculate The 95% Confidence Interval.

Understanding how to accurately estimate a population parameter based on a sample is a fundamental skill in statistics. When dealing with a random sample of size N = 25 that yields a sample mean of 50 and a standard deviation of 8, calculating a 95% confidence interval allows us to infer the likely range within which the true population mean lies with a high degree of certainty. This guide will walk you through the steps involved in computing this confidence interval, emphasizing the underlying concepts, formulas, and interpretation.

Introduction to Confidence Intervals

Before diving into the calculations, it's essential to grasp what a confidence interval (CI) represents. A confidence interval provides a range of plausible values for a population parameter—here, the mean—based on sample data. The "95%" indicates that if we were to take many such samples and compute their confidence intervals, approximately 95% of those intervals would contain the true population mean.

The Data at a Glance

Let's summarize the provided data:

- Sample size, $N = 25$
- Sample mean, $\bar{x} = 50$
- Sample standard deviation, $s = 8$
- Confidence level, 95%

Step 1: Recognize the Distribution and Appropriate Method

Since the sample size is relatively small ($N < 30$), and the population standard deviation is unknown, the appropriate method to compute the confidence interval is based on the t-distribution rather than the normal distribution.

Key points:

- Use the t-distribution for small samples.
- Degrees of freedom (df) = $N - 1 = 24$.

Step 2: Find the Critical t-Value

The critical t-value corresponds to the confidence level and degrees of freedom. For a 95% confidence interval with df = 24, you look up the t-value in a t-distribution table or use statistical software.

Standard t-value for 95% CI with df=24: approximately 2.064.

(Note: Exact value may vary slightly depending on the source, but 2.064 is standard.)

Step 3: Calculate the Standard Error (SE)

The standard error of the mean quantifies the variability of the sample mean estimate:

$$
SE = \frac{s}{\sqrt{N}} = \frac{8}{\sqrt{25}} = \frac{8}{5} = 1.6
$$

Step 4: Compute the Margin of Error (ME)

The margin of error indicates how much the sample mean could vary from the true population mean at the given confidence level:

$$
ME = t_{(df, 1 - \alpha/2)} \times SE = 2.064 \times 1.6 = 3.3024
$$

(Round to four decimal places for precision: 3.3024)

Step 5: Calculate the Confidence Interval

Finally, the confidence interval is:

$$
\text{Lower bound} = \bar{x} - ME = 50 - 3.3024 = 46.6976
$$

$$
\text{Upper bound} = \bar{x} + ME = 50 + 3.3024 = 53.3024
$$

Result:

The 95% confidence interval for the population mean is approximately (46.70, 53.30).

Interpretation of the Confidence Interval

This interval suggests that, based on our sample, we are 95% confident that the true population mean lies between 46.70 and 53.30. It's important to recognize that this does not mean there's a 95% probability that the true mean is within this range; rather, it indicates that if we repeated the sampling process many times, approximately 95% of those intervals would contain the true mean.

Additional Considerations

1. Assumption of Normality

The t-distribution-based confidence interval assumes the underlying data are approximately normally distributed, especially critical with small sample sizes. If the data are skewed or have outliers, the interval may not be reliable. Tools such as histograms or normality tests can help assess this assumption.

2. Effect of Sample Size

While n=25 is somewhat small, it's generally acceptable for the t-distribution approach, especially if data are approximately normal. Larger samples tend to produce narrower confidence intervals, reflecting increased precision.

3. Variability and Standard Deviation

The sample standard deviation (s=8) provides an estimate of variability within the data. Larger variability results in wider confidence intervals, indicating less precision in estimating the true mean.

Practical Applications

Calculating confidence intervals is vital across numerous fields:

- Business: Estimating average customer satisfaction scores.
- Medicine: Determining mean blood pressure levels in a population.
- Social Sciences: Assessing average income levels or educational attainment.
- Quality Control: Estimating the mean defect rate in manufacturing.

Understanding how to correctly compute and interpret these intervals ensures informed decision-making based on sample data.

Summary: Step-by-Step Recap

1. Identify the sample data: N=25, x̄ =50, s=8.
2. Determine the distribution: Use the t-distribution due to small N.
3. Find the critical t-value: Approximately 2.064 for 95% confidence and df=24.
4. Calculate the standard error: $SE = 8/5 = 1.6$.
5. Compute the margin of error: $ME = 2.064 \times 1.6 \approx 3.3024$.
6. Construct the interval:
- Lower bound: $50 - 3.3024 = 46.70$.
- Upper bound: $50 + 3.3024 = 53.30$.

Final Thoughts

Calculating a 95% confidence interval provides a powerful way to infer the true population mean from a sample. By carefully applying the t-distribution, considering assumptions, and interpreting the results accurately, statisticians and analysts can make meaningful, data-driven conclusions. Remember, the key lies in understanding the data, choosing the correct method, and communicating the findings clearly.

Ready to analyze your own data? Follow these steps and ensure your conclusions are both accurate and insightful!

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