A Sample Of 4 Different Calculators Is Randomly Selected From A Group Containing 13 That Are Defective

A Sample Of 4 Different Calculators Is Randomly Selected From A Group Containing 13 That Are Defective is a classic problem in probability and statistics that involves understanding the likelihood of selecting defective items from a larger group. Such problems are foundational in quality control, manufacturing, and risk assessment, where the goal is to predict the probability of defective items in a sample and make informed decisions based on these predictions. This article explores the intricacies of this problem, providing a comprehensive overview, calculations, and real-world applications.

Understanding the Context: The Group of Calculators

Before delving into probabilities and calculations, it's important to understand the scenario at hand. Suppose there is a batch or group of calculators, some of which are defective. Specifically, the group contains a total of 13 defective calculators, but the total number of calculators in the entire group is not specified here. For the purpose of analysis, let's assume the total number of calculators in the group is known or is a typical batch size, such as 50 or 100 calculators.

The key questions that arise in such a scenario include:

- What is the probability that a randomly selected sample of 4 calculators contains a certain number of defective calculators?
- How does the size of the total group affect these probabilities?
- What implications do these probabilities have for quality control processes?

Understanding these questions is essential for quality assurance teams, procurement officers, and statisticians involved in sampling and testing procedures.

Basics of Hypergeometric Distribution

What Is the Hypergeometric Distribution?

The hypergeometric distribution models the probability of drawing a specific number of defective items when sampling without replacement from a finite population. It is particularly relevant when the population contains a known number of defective items, and the sample size is fixed.

The probability $P(X = k)$ of selecting exactly k defective calculators in a sample of size n from a population containing N total calculators with M defective calculators is given by:

$$
P(X = k) = \frac{\binom{M}{k} \binom{N - M}{n - k}}{\binom{N}{n}}
$$

Where:

- N = total number of calculators in the population
- M = total number of defective calculators in the population
- n = sample size (here, 4)
- k = number of defective calculators in the sample

This distribution is ideal for scenarios where sampling is done without replacement, meaning once an item is selected, it is not returned to the population.

Applying the Distribution to Our Scenario

Suppose the total group contains N calculators, with $M = 13$ defective ones. We are selecting $n = 4$ calculators at random. To analyze the probabilities, we need to consider all possible values of k (from 0 up to the minimum of 4 and 13), which represent the number of defective calculators in our sample.

The key calculations involve:

- The probability of selecting 0 defective calculators
- The probability of selecting 1 defective calculator
- The probability of selecting 2 defective calculators
- The probability of selecting 3 defective calculators
- The probability of selecting all 4 defective calculators

These probabilities help in assessing the quality of the batch and making decisions about acceptance or rejection of the sample.

Calculating Probabilities for Different Scenarios

Let’s consider specific calculations assuming a total of $N = 50$ calculators in the group, with 13 defective.

Probability of Selecting No Defective Calculators (k=0)

Using the hypergeometric formula:

$$
P(X=0) = \frac{\binom{13}{0} \binom{50-13}{4}}{\binom{50}{4}}
$$

\[

$$
P(X=0) = \frac{1 \times \binom{37}{4}}{\binom{50}{4}}
$$

Calculating the combinations:

$$
\binom{37}{4} = \frac{37 \times 36 \times 35 \times 34}{4 \times 3 \times 2 \times 1} = 66045
$$
$$
\binom{50}{4} = \frac{50 \times 49 \times 48 \times 47}{4 \times 3 \times 2 \times 1} = 230300
$$

Thus,

$$
P(X=0) = \frac{66045}{230300} \approx 0.287
$$

This indicates approximately a 28.7% chance that no defective calculators are found in a sample of four.

Probability of Selecting Exactly One Defective Calculator (k=1)

$$
P(X=1) = \frac{\binom{13}{1} \binom{37}{3}}{\binom{50}{4}}
$$

Calculations:

$$
\binom{13}{1} = 13
$$
$$
\binom{37}{3} = \frac{37 \times 36 \times 35}{3 \times 2 \times 1} = 7770
$$

Therefore,

$$
P(X=1) = \frac{13 \times 7770}{230300} = \frac{101010}{230300} \approx 0.438
$$

This indicates about a 43.8% chance of selecting exactly one defective calculator.

Similarly, probabilities for $k=2, 3, 4$ can be computed, providing a full picture of the distribution of defective calculators in the sample.

Implications for Quality Control

Understanding these probabilities allows organizations to set acceptable quality levels and make informed decisions about batch acceptance. For example, if the probability of selecting 2 or more defective calculators exceeds a certain threshold, the batch might be rejected to prevent faulty products from reaching customers.

Acceptance Sampling Plans

Acceptance sampling involves pre-defined rules to accept or reject a batch based on sampled items. For instance:

- Single sampling plan: Accept the batch if the number of defective calculators in the sample does not exceed a certain limit.
- Double sampling plan: Take a second sample if the first sample's defect count falls within an ambiguous range.

Using the probabilities calculated, organizations can determine the likelihood of accepting defective batches and balance between quality assurance and production efficiency.

Factors Influencing the Probabilities

Several factors impact the probability calculations and the effectiveness of sampling:

- Population size (\(N \)): Larger populations affect the hypergeometric probabilities.
- Number of defective items (\(M \)): More defective calculators increase the likelihood of selecting defective items.
- Sample size (\(n \)): Larger samples provide more accurate estimates but are more resource-intensive.
- Sampling method: Sampling without replacement aligns with hypergeometric distribution assumptions, whereas sampling with replacement would require a different approach (binomial distribution).

Real-World Applications and Considerations

The problem of selecting defective calculators is representative of broader quality control processes across various industries, including electronics, automotive, pharmaceuticals, and food production. Companies rely on statistical sampling to:

- Ensure product quality
- Minimize costs associated with defective items
- Maintain customer satisfaction and safety standards

When applying these principles, organizations should consider:

- The acceptable level of risk (producer's risk and consumer's risk)
- The consequences of accepting a defective batch
- The costs associated with testing and sampling

Conclusion

A sample of 4 different calculators randomly selected from a group containing 13 that are defective exemplifies core concepts in probability theory, particularly the hypergeometric distribution. By calculating the probabilities of various defect counts within the sample, organizations can make informed decisions regarding quality control, batch acceptance, and process improvements. Understanding these statistical principles not only enhances operational efficiency but also helps maintain high standards of product reliability and customer satisfaction.

In practice, the choice of sampling size and acceptance criteria should be tailored to the specific context, balancing the risks of defective products reaching consumers with the costs and feasibility of testing. As industries continue to emphasize quality assurance, mastery of such probability problems remains a vital skill for professionals involved in manufacturing, quality management, and statistical analysis.

Frequently Asked Questions

What is the probability of selecting exactly 2 defective calculators out of 4 randomly chosen from a group of 13 defective and non-defective calculators?

The probability is calculated using the hypergeometric distribution: P = (C(13,2) C(10,2)) / C(23,4).

How many different samples of 4 calculators can be selected from the group of 23 calculators?

The total number of ways is C(23,4) = 8855.

What is the probability that all 4 selected calculators are non-defective?

Since there are 10 non-defective calculators, the probability is C(10,4) / C(23,4) = 210 / 8855 ≈ 0.0237.

If a sample of 4 calculators is selected, what is the probability that at least one is defective?

It is 1 minus the probability that none are defective: 1 - (C(10,4)/C(23,4)) ≈ 1 - (210/8855) ≈ 0.9763.

What is the expected number of defective calculators in a random sample of 4?

The expected number is (4) (13/23) ≈ 2.26, based on the proportion of defective calculators.

Is selecting 4 calculators from the group considered a hypergeometric experiment?

Yes, because the selection is without replacement, and the probability depends on the composition of defective and non-defective calculators.

How does the probability change if the group contains more defective calculators, say 15 instead of 13?

The probability of selecting defective calculators increases proportionally, and calculations should be updated using C(15, k) for defectives and C(8, 4 - k) for non-defectives accordingly.

Additional Resources

A Sample Of 4 Different Calculators Is Randomly Selected From A Group Containing 13 That Are Defective

In the realm of quality control and manufacturing, understanding the probability of selecting defective items from a large lot is crucial. Recently, analysts and engineers have turned to probabilistic models to assess the likelihood of encountering defective units in random samples. One such scenario involves selecting a sample of four calculators from a batch of computers, where 13 out of the total are known to be defective. This seemingly straightforward problem offers a rich ground for exploring concepts such as hypergeometric distribution, probability calculations, and their practical implications in quality assurance processes.

Understanding the Scenario: The Context of the Problem

Imagine a manufacturing company that produces calculators, and during quality inspection, a random sample of calculators is taken to estimate the overall defect rate. The scenario specifies:

- Total calculators in the batch: N (unknown in this context, but typically total is known)
- Number of defective calculators in the batch: D = 13
- Sample size: n = 4 calculators

The core question is: What is the probability that the sample contains a certain number of defective calculators? For instance, what is the probability that the sample contains exactly 0, 1, 2, 3, or all 4 defective calculators?

Understanding these probabilities aids in decision-making processes such as accepting or rejecting the entire batch, estimating the defect rate, and improving quality control protocols.

The Hypergeometric Distribution: The Mathematical Backbone

The problem at hand is a classic example of the hypergeometric distribution, which models the probability of drawing a specific number of successes (in this case, defective calculators) in a fixed number of draws without replacement from a finite population.

Key Elements of the Hypergeometric Distribution:

- Population size (N): Total calculators in the batch
- Number of successes in the population (D): Number of defective calculators (13)
- Sample size (n): Number of calculators selected (4)
- Number of observed successes (k): Number of defective calculators in the sample

The probability of selecting exactly k defective calculators in the sample is given by:

$$
P(X = k) = \frac{\binom{D}{k} \times \binom{N - D}{n - k}}{\binom{N}{n}}
$$

where:

- $\binom{a}{b}$ is the binomial coefficient, representing the number of ways to choose b items from a options.

This formula calculates the ratio of favorable outcomes (ways to select k defective and n - k non-defective calculators) to the total possible ways to select n calculators from the entire batch.

Calculating Probabilities: Step-by-Step

To analyze the scenario comprehensively, we need to compute the probabilities for all feasible values of k (0 through 4). Here, the total population N must be specified to perform exact calculations. Assuming the total batch size is N = 50 (for illustration), the calculations proceed as follows:

Note: If the total number N is unknown, the calculations can be generalized or performed for specific values based on additional data.

Step 1: Define the parameters

- $N = 50$
- $D = 13$
- $n = 4$

Step 2: Compute probabilities for each k (0 to 4):

- k=0 (no defective calculators):

\[
P(0) = \frac{\binom{13}{0} \times \binom{37}{4}}{\binom{50}{4}}
\]

- k=1 (one defective calculator):

\[
P(1) = \frac{\binom{13}{1} \times \binom{37}{3}}{\binom{50}{4}}
\]

- k=2 (two defective calculators):

\[
P(2) = \frac{\binom{13}{2} \times \binom{37}{2}}{\binom{50}{4}}
\]

- k=3 (three defective calculators):

\[
P(3) = \frac{\binom{13}{3} \times \binom{37}{1}}{\binom{50}{4}}
\]

- k=4 (all four are defective):

\[
P(4) = \frac{\binom{13}{4} \times \binom{37}{0}}{\binom{50}{4}}
\]

Using binomial coefficient calculations:

\[
\binom{13}{0} = 1,\quad \binom{13}{1} = 13,\quad \binom{13}{2} = 78,\quad \binom{13}{3} = 286,\quad \binom{13}{4} = 715
\]

\[
\binom{37}{4} = 66045,\quad \binom{37}{3} = 7770,\quad \binom{37}{2} = 666,\quad \binom{37}{1} = 37,\quad \binom{37}{0} = 1
\]

And the total:

\[
\binom{50}{4} = 230300
\]

Calculating each probability:

- P(0):

\[

$$
P(0) = \frac{1 \times 66045}{230300} \approx 0.287
$$

- P(1):

$$
P(1) = \frac{13 \times 7770}{230300} \approx 0.438
$$

- P(2):

$$
P(2) = \frac{78 \times 666}{230300} \approx 0.226
$$

- P(3):

$$
P(3) = \frac{286 \times 37}{230300} \approx 0.046
$$

- P(4):

$$
P(4) = \frac{715 \times 1}{230300} \approx 0.003
$$

These probabilities sum to 1, confirming the calculations.

Practical Implications of the Probabilities

Understanding these probabilities yields significant insights for quality control teams:

- Likelihood of No Defective Items in the Sample (≈28.7%)
Indicates a substantial chance that a small sample might not reveal any defect, potentially leading to false assurance about the batch quality if based solely on such a sample.

- Probability of Finding Exactly One Defective (≈43.8%)
The most probable outcome, suggesting that in nearly half of the samples, only a single defective calculator might be detected.

- Risk of Multiple Defects (≈22.6% for two, 4.6% for three, 0.3% for all four)
Highlights the chance of detecting higher defect counts, which could trigger batch rejection.

These probabilities help define inspection policies — for example, deciding how large a sample should be to confidently assess the overall defect rate.

Limitations and Assumptions in the Model

While the hypergeometric model provides a robust framework for the problem, several assumptions underpin its validity:

- Random Sampling:
The sample must be truly random, ensuring each calculator has an equal chance of selection.

- Known Total Population and Defective Count:
Accurate counts of total calculators and defectives are essential. Any misreporting affects probability estimates.

- No Replacement:
The calculations assume sampling without replacement, which matches typical inspection procedures.

- Homogeneity:
It assumes that all items are equally likely to be defective or non-defective, and that defectiveness is randomly distributed.

In real-world scenarios, deviations from these assumptions can occur, influencing the accuracy of probability estimates.

Broader Context: Quality Control and Decision Making

The probabilistic analysis of selecting defective calculators is not merely an academic exercise; it impacts practical decisions in manufacturing and quality assurance:

- Acceptance Sampling:
Companies may adopt sampling plans based on such probability models to decide whether to accept or reject entire batches.

- Process Improvement:
High probabilities of defect detection in small samples can signal a need for process improvements.

- Cost-Benefit Analysis:
Balancing the costs of inspection versus the risks of passing defective units involves understanding these probabilities.

- Regulatory Compliance:
In industries with strict quality standards, statistical sampling ensures compliance without exhaustive testing of every unit.

Extending the Analysis: Variations and Additional Considerations

While the current scenario provides foundational understanding, more complex situations can be modeled:

- Different Sample Sizes:
Analyzing the probabilities for larger or smaller samples.

- Multiple Quality Levels:
Considering degrees of defectiveness or varying defect severities.

- Batch Size Variations:
Adjusting calculations for different total batch sizes.

- Sequential Sampling:
Implementing stepwise sampling procedures with decision rules based on interim results.

Such extensions enhance the robustness of quality assurance strategies and allow for tailored decision frameworks.

Conclusion: The Significance of Probabilistic Analysis in Quality Control

The example of selecting a sample of four calculators from a batch with 13 defectives illustrates the power of probability theory in real-world manufacturing contexts. By leveraging the hypergeometric distribution, quality engineers can quantify the likelihood of detecting

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