# A Sample Of 4 Different Calculators Is Randomly Selected From A Group Containing 14 That Are Defective

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Understanding probability and statistical sampling is essential for quality control, decision-making, and assessing risk. In this context, consider a scenario where a sample of four calculators is randomly selected from a larger group of calculators, among which 14 are defective. This example illustrates key concepts such as hypergeometric probability, sampling without replacement, and the likelihood of selecting defective items. Whether you're a student, quality assurance professional, or someone interested in probability theory, analyzing this case provides valuable insights into how sample selection impacts the probability of encountering defective products.

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## Understanding the Scenario: Key Details

Before diving into calculations, it’s crucial to clarify the parameters of the problem:

- Total number of calculators in the group: N (unknown)
- Number of defective calculators: D = 14
- Number of non-defective calculators: N - D
- Sample size: n = 4

Key questions include:

- What is the probability that the sample contains a certain number of defective calculators?
- How does the proportion of defective calculators influence the probability?
- What are the chances that the sample contains no defective calculators?
- What is the probability that all selected calculators are defective?

To answer these questions, we need to understand the total size of the group and the distribution of defective calculators within it.

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## Assumptions and Clarifications

Given only the number of defective calculators, the total size of the group is not explicitly provided. For a comprehensive analysis, we consider possible scenarios:

1. Minimal Total Group Size: The total group could be as small as 14 (all calculators are defective).
2. Larger Total Group: The total number could be greater; for example, 20, 30, or more, with 14 defective and the rest non-defective.

For the purpose of this analysis, we will assume:

- The total number of calculators, N, is known or can be estimated.
- The selection is random and without replacement.
- The proportion of defective calculators affects the probability of sampling defective units.

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## Calculating Probabilities Using the Hypergeometric Distribution

The hypergeometric distribution is the appropriate model for this scenario because:

- Sampling is without replacement.
- The probability depends on the total population size, number of defective calculators, sample size, and the number of defective calculators in the sample.

The hypergeometric probability formula:

$$
P(X = k) = \frac{\binom{D}{k} \binom{N - D}{n - k}}{\binom{N}{n}}
$$

Where:

- $P(X = k)$: Probability that exactly $k$ defective calculators are in the sample.
- $D$: Total defective calculators (14).
- $N$: Total calculators in the group.
- $n$: Sample size (4).
- $k$: Number of defective calculators in the sample (can range from 0 to $\min(D, n)$).

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## Analyzing Different Probabilities

2.1 Probability of Selecting No Defective Calculators (k=0)

This scenario indicates the sample contains only non-defective calculators.

$$
P(0) = \frac{\binom{D}{0} \binom{N - D}{n}}{\binom{N}{n}}
$$

Implications:

- The probability depends heavily on the total size $N$.
- As $N$ increases, the probability of selecting only non-defective calculators decreases, assuming the proportion of defective calculators remains constant.

2.2 Probability of Selecting Exactly 1 Defective Calculator (k=1)

$$
P(1) = \frac{\binom{D}{1} \binom{N - D}{n - 1}}{\binom{N}{n}}
$$

2.3 Probability of Selecting All 4 as Defective (k=4)

$$
P(4) = \frac{\binom{D}{4} \binom{N - D}{0}}{\binom{N}{n}}
$$

Note: $\binom{D}{4}$ is only valid if $D \geq 4$.

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## Impact of Total Group Size on Probabilities

The total number of calculators $N$ significantly affects the probabilities:

- If $N = 14$: All calculators are defective; the probability of selecting only defective calculators is 100%.
- If $N > 14$: The probabilities decrease for all-defective samples because the non-defective calculators increase.
- If $N$ is large (e.g., 50 or 100): The chance of selecting only defective calculators diminishes, emphasizing the importance of understanding the

proportion of defective units in the population.

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# Real-world Applications and Relevance

Analyzing such sampling scenarios has practical applications:

- Quality Control: Estimating the likelihood of detecting defective units during sampling.
- Manufacturing: Determining inspection strategies based on defect rates.
- Supply Chain Management: Assessing risks associated with random sampling of products.
- Statistical Quality Assurance: Designing sampling plans that balance cost and defect detection probability.

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# Calculating Specific Probabilities: An Example

Suppose the total number of calculators in the group is N = 20.

Given:

- Total calculators: 20
- Defective calculators: 14
- Non-defective calculators: 6
- Sample size: 4

Let's compute some probabilities:

2.1 Probability that all 4 selected calculators are defective

$$
P(4) = \frac{\binom{14}{4} \binom{6}{0}}{\binom{20}{4}}
$$

Calculations:

- $\binom{14}{4} = \frac{14 \times 13 \times 12 \times 11}{4 \times 3 \times 2 \times 1} = 1001$
- $\binom{6}{0} = 1$
- $\binom{20}{4} = \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} = 4845$

Thus:

$$
P(4) = \frac{1001 \times 1}{4845} \approx 0.2065 \text{ or } 20.65\%
$$

2.2 Probability of selecting no defective calculators

$$
P(0) = \frac{\binom{14}{0} \binom{6}{4}}{\binom{20}{4}}
$$

Calculations:

- $\binom{14}{0} = 1$
- $\binom{6}{4} = \frac{6 \times 5}{2} = 15$
- $\binom{20}{4} = 4845$ (as above)

So:

$$
P(0) = \frac{1 \times 15}{4845} \approx 0.0031 \text{ or } 0.31\%
$$

2.3 Probability of exactly 2 defective calculators

$$
P(2) = \frac{\binom{14}{2} \binom{6}{2}}{\binom{20}{4}}
$$

Calculations:

- $\binom{14}{2} = \frac{14 \times 13}{2} = 91$
- $\binom{6}{2} = \frac{6 \times 5}{2} = 15$

Thus:

$$
P(2) = \frac{91 \times 15}{4845} \approx \frac{1365}{4845} \approx 0.2816 \text{ or } 28.16\%
$$

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## Strategies for Quality Inspection Based on Probabilities

Understanding these probabilities allows organizations to design effective inspection plans:

- Sampling Plan Adjustments: Based on the defect rate, decide the sample size needed to achieve a desired confidence level.
- Acceptance Sampling: Determine thresholds for accepting or rejecting batches based on the number of defective calculators found.
- Risk Management: Calculate the probability of passing a defective batch or rejecting a good batch, balancing risks of false acceptance or rejection.

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# Conclusion: Key Takeaways and Final Thoughts

The scenario of selecting 4 calculators from a group containing 14 defective units highlights critical principles in probability and sampling theory. The hypergeometric distribution provides the framework for calculating the likelihood of various outcomes, guiding decision-making in quality control and statistical analysis. The total size of the group significantly influences these probabilities, emphasizing the importance of accurate data and assumptions.

In practical applications, understanding these probabilities helps organizations optimize inspection strategies, minimize costs, and ensure product quality. Whether in manufacturing, supply chain management, or academic research, the principles illustrated by this example are foundational for effective sampling and quality assurance practices.

Remember:

- Always identify the total population size when applying hypergeometric probabilities.
- Consider the proportion of defective units in the population.
- Use appropriate formulas to evaluate the likelihood of

# Frequently Asked Questions

## What is the probability that the sample of 4 calculators contains exactly 2 defective ones?

The probability is calculated using the hypergeometric distribution: (Number of ways to choose 2 defective from 14) (Number of ways to choose 2 non-defective from remaining calculators) divided by the total ways to select 4 calculators. Specifically, (C(14,2) C(total calculators - 14, 2)) / C(total calculators, 4).

## How many different samples of 4 calculators can be selected from a group of 20 calculators?

The total number of combinations is C(20, 4) = 4845.

## What is the probability that all 4 selected calculators are defective?

The probability is C(14, 4) / C( total calculators, 4 ), assuming total calculators are 20, so it's C(14,4)/C(20,4).

## If a sample of 4 calculators is randomly selected, what is the probability that at least one is defective?

Calculate the complement: 1 minus the probability that none are defective. That is, 1 - [Number of non-defective calculators choose 4] / total combinations.

## What is the expected number of defective calculators in a random sample of 4?

Using the hypergeometric expectation, the expected number is 4 (Number of defective calculators / total calculators).

## If 14 out of 20 calculators are defective, what is the probability that exactly 3 of the selected 4 are defective?

The probability is C(14, 3) C(6, 1) / C(20, 4).

## How does increasing the number of defective calculators in the group affect the probability of selecting a defective calculator in the sample?

As the number of defective calculators increases, the probability of selecting at least one defective calculator in the sample also increases.

## What is the variance in the number of defective calculators in a sample of 4?

The variance can be computed using the hypergeometric distribution formula: variance = n (K/N) (1 - K/N) (N - n) / (N - 1), where N is total calculators, K is defective calculators, and n is sample size.

## Can we determine the probability of selecting exactly 1 defective calculator in the sample?

Yes, the probability is C(14, 1) C(6, 3) / C(20, 4).

# Additional Resources

A Sample Of 4 Different Calculators Is Randomly Selected From A Group Containing 14 That Are Defective

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Introduction

In the realm of quality control and reliability testing, understanding the likelihood of selecting defective units from a batch is crucial. This is especially important in manufacturing environments where the integrity of products directly impacts consumer safety and satisfaction. One common scenario involves randomly selecting a sample of items from a larger population to assess defect rates or to estimate overall quality.

This article explores a specific case: A sample of 4 different calculators is randomly selected from a group containing 14 that are defective. Through a detailed analysis, we examine the probabilistic implications, the potential for quality estimation, and how these insights can inform decision-making processes in quality assurance.

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Background and Context

Suppose a manufacturer produces a batch of calculators, with some units defective and others functioning properly. For the purpose of this analysis, the entire batch contains a total of N calculators, of which D are defective. In our specific scenario, the focus is on a subgroup of 14 defective calculators, but the total batch size N is not explicitly specified.

However, for meaningful probabilistic analysis, it's essential to specify or assume the total batch size. Typically, in such studies, the total batch size is known or estimated, and the goal is to determine the probability of selecting a certain number of defective calculators in a sample of size 4.

Assumptions

To proceed with a thorough analysis, the following assumptions are made:

- The total number of calculators in the batch is N.
- The number of defective calculators is D = 14.
- The selection of calculators is random and without replacement.

- The sample size is n = 4.
- The remaining calculators (non-defective) are N - D.

Given that the sample is drawn randomly without replacement, we are dealing with the hypergeometric distribution, which models the probability of drawing a specific number of defective calculators within the sample.

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Clarifying the Population Parameters

The key missing piece in this puzzle is the total batch size, N. Since only the number of defective calculators (14) is specified, the total batch size could vary. To analyze different scenarios, we consider several plausible values for N:

- Scenario 1: The total batch size N equals 20 (i.e., 14 defective, 6 functional).
- Scenario 2: The total batch size N equals 30 (i.e., 14 defective, 16 functional).
- Scenario 3: The total batch size N equals 50 (i.e., 14 defective, 36 functional).

This approach allows us to evaluate how the probability outcomes vary with different batch sizes.

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The Hypergeometric Distribution: A Primer

The hypergeometric distribution provides the probability of k defective calculators in a sample of size n drawn without replacement from a population with D defective and N - D non-defective units. The probability mass function (PMF) is given by:

$$ P(X = k) = \frac{\binom{D}{k} \binom{N - D}{n - k}}{\binom{N}{n}} $$

where:

- $\binom{a}{b}$ is the binomial coefficient, representing the number of ways to choose b items from a.

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Calculating Probabilities for Different Scenarios

Scenario 1: Total Batch Size N=20

- Defective calculators: D=14
- Non-defective calculators: 6
- Sample size: n=4

Possible values of k (number of defective calculators in the sample): 0, 1, 2, 3, 4

Calculations:

- Probability of 0 defective:

$$ P(0) = \frac{\binom{14}{0} \binom{6}{4}}{\binom{20}{4}} = \frac{1 \times 15}{4845} \approx 0.0031 $$

- Probability of 1 defective:

$$ P(1) = \frac{\binom{14}{1} \binom{6}{3}}{\binom{20}{4}} = \frac{14 \times 20}{4845} \approx 0.0578 $$

- Probability of 2 defective:

$$ P(2) = \frac{\binom{14}{2} \binom{6}{2}}{\binom{20}{4}} = \frac{91 \times 15}{4845} \approx 0.281 $$

- Probability of 3 defective:

$$ P(3) = \frac{\binom{14}{3} \binom{6}{1}}{\binom{20}{4}} = \frac{364 \times 6}{4845} \approx 0.45 $$

- Probability of 4 defective:

$$ P(4) = \frac{\binom{14}{4} \binom{6}{0}}{\binom{20}{4}} = \frac{1001 \times 1}{4845} \approx 0.206 $$

Summary for N=20:

| k | Probability | Explanation |
|---|-------------|---------------------------------------------|
| 0 | 0.0031 | No defective calculators in the sample |
| 1 | 0.0578 | Exactly one defective calculator |
| 2 | 0.281 | Exactly two defective calculators |
| 3 | 0.45 | Exactly three defective calculators |
| 4 | 0.206 | All four calculators are defective |

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Scenario 2: Total Batch Size N=30

- D=14, non-defective=16, n=4

Calculations:

- P(0):

\[ \frac{\binom{14}{0} \binom{16}{4}}{\binom{30}{4}} = \frac{1 \times

1820}{27405} \approx 0.0664 \]

- P(1):

$$ \frac{\binom{14}{1} \binom{16}{3}}{\binom{30}{4}} = \frac{14 \times 560}{27405} \approx 0.286 $$

- P(2):

$$ \frac{\binom{14}{2} \binom{16}{2}}{\binom{30}{4}} = \frac{91 \times 120}{27405} \approx 0.398 $$

- P(3):

$$ \frac{\binom{14}{3} \binom{16}{1}}{\binom{30}{4}} = \frac{364 \times 16}{27405} \approx 0.213 $$

- P(4):

$$ \frac{\binom{14}{4} \binom{16}{0}}{\binom{30}{4}} = \frac{1001 \times 1}{27405} \approx 0.0365 $$

Summary for N=30:

| k | Probability | Explanation |
|---|-------------|--------------------------------------------------|
| 0 | 0.0664 | No defective in the sample |
| 1 | 0.286 | Exactly one defective |
| 2 | 0.398 | Exactly two defective |
| 3 | 0.213 | Exactly three defective |
| 4 | 0.0365 | All four are defective |

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Scenario 3: Total Batch Size N=50

- D=14, non-defective=36, n=4

Calculations:

- P(0):

$$ \frac{\binom{14}{0} \binom{36}{4}}{\binom{50}{4}} = \frac{1 \times 58905}{230300} \approx 0.255 $$

- P(1):

$$ \frac{\binom{14}{1} \binom{36}{3}}{\binom{50}{4}} = \frac{14 \times 7140}{230300} \approx 0.434 $$

- P(2):

$$ \frac{\binom{14}{2} \binom{36}{2}}{\binom{50}{4}} = \frac{91 \times 630}{230300} \approx 0.249 $$

- P(3):

$$ \frac{\binom{14}{3} \binom{36}{1}}{\binom{50}{4}} = \frac{364 \times 36}{230300} \approx 0.057 $$

- P(4):

$$ \frac{\binom{14}{4} \binom{36}{0}}{\binom{50}{4}} = \frac{1001 \times 1}{230300} \approx 0.0043 $$

Summary for N=50:

| k | Probability | Explanation |
|---|---|---|
| 0 | 0.255 | No defective in the sample |
| 1 | 0.434 | Exactly one defective |

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