A Small, Square Loop Carries A 41A Current. The On-axis Magnetic Field Strength 48 Cm From The Loop Is

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Understanding magnetic fields generated by current-carrying conductors is fundamental in electromagnetism, with applications ranging from electric motors and transformers to wireless communication and medical imaging. In this article, we explore a specific scenario involving a small, square loop carrying a substantial current and analyze the magnetic field strength at a point located 48 centimeters along its axis. We will delve into the principles governing magnetic fields produced by loops, derive the relevant formulas, perform detailed calculations, and discuss the implications of the results.

Introduction to Magnetic Fields of Current-Carrying Loops

When an electric current flows through a conductor, it produces a magnetic field around it—a phenomenon described by Ampère's Law and encapsulated in the Biot-Savart Law. The magnetic field's shape and strength depend on the geometry of the conductor and the current's magnitude.

A current-carrying loop is a fundamental configuration in electromagnetism, often used to study magnetic field distributions. The characteristics of the magnetic field along the axis of the loop are particularly important because they simplify calculations and are relevant in many practical applications, such as MRI machines, inductors, and inductive sensors.

Fundamentals of Magnetic Field Calculation for a Square Loop

The Biot-Savart Law

The Biot-Savart Law relates the magnetic field $\mathbf{B}$ at a point in space to the current I flowing through a conductor:

\[

$$
\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\mathbf{l} \times \mathbf{\hat{r}}}{r^2}
$$

where:
- μ_0 is the permeability of free space ($4\pi \times 10^{-7} \, \mathrm{T \cdot m/A}$)
- $d\mathbf{l}$ is an element of the current-carrying conductor
- $\mathbf{\hat{r}}$ is the unit vector from $d\mathbf{l}$ to the observation point
- r is the distance between $d\mathbf{l}$ and the observation point

Calculating the magnetic field directly from the Biot-Savart Law for a square loop involves integrating over each segment, which can be complex. However, along the axis of a symmetric loop (whether circular or square), the problem simplifies because of symmetry.

Magnetic Field Along the Axis of a Square Loop

For a square loop of side length a, carrying a current I, the magnetic field at a point along the axis, a distance x from the center, can be derived by summing contributions from each side. Due to symmetry, the field contributions from opposite sides add up along the axis.

The magnetic field at a point on the axis of a square loop is given by:

$$
B = \frac{2 \mu_0 I}{\pi a} \times \left(\frac{a/2}{\sqrt{(a/2)^2 + x^2}} \right)
$$

This formula accounts for the contribution of each side, considering the geometry involved.

Applying the Formula to Our Scenario

Given data:
- Current $I = 41\, \mathrm{A}$
- Distance from the loop $x = 48\, \mathrm{cm} = 0.48\, \mathrm{m}$

To proceed, we need the side length a of the square loop. Since the problem states the loop is small but does not specify a, we can assume a typical small size, or for the sake of calculation, consider a specific value.

Assumption: Let's assume $a = 10\, \mathrm{cm} = 0.10\, \mathrm{m}$

This assumption is reasonable given the description of a "small" loop.

Calculating the Magnetic Field

Step 1: Compute the parameters

- Side length $a = 0.10\, \mathrm{m}$
- Observation point $x = 0.48\, \mathrm{m}$

- Half side length $a/2 = 0.05\, \mathrm{m}$

Step 2: Calculate the denominator term $\sqrt{(a/2)^2 + x^2}$

$$
\sqrt{(0.05)^2 + (0.48)^2} = \sqrt{0.0025 + 0.2304} = \sqrt{0.2329} \approx 0.4828\, \mathrm{m}
$$

Step 3: Compute the ratio $\frac{a/2}{\sqrt{(a/2)^2 + x^2}}$

$$
\frac{0.05}{0.4828} \approx 0.1035
$$

Step 4: Plug into the magnetic field formula

$$
B = \frac{2 \times \mu_0 \times I}{\pi \times a} \times 0.1035
$$

where

$$
\mu_0 = 4\pi \times 10^{-7}\, \mathrm{T \cdot m/A}
$$

Calculating numerator:

$$
2 \times 4\pi \times 10^{-7} \times 41 = 8\pi \times 10^{-7} \times 41
$$

\[

8\pi \times 41 \approx 8 \times 3.1416 \times 41 \approx 8 \times 128.74 \approx 1029.9
\]

Thus,

\[
\text{Numerator} = 1029.9 \times 10^{-7} = 1.0299 \times 10^{-4}
\]

Denominator:

\[
\pi \times a = 3.1416 \times 0.10 = 0.31416
\]

Now, compute:

\[
B = \frac{1.0299 \times 10^{-4}}{0.31416} \times 0.1035
\]

First, divide:

\[
\frac{1.0299 \times 10^{-4}}{0.31416} \approx 3.278 \times 10^{-4}
\]

Then multiply by 0.1035:

\[
B \approx 3.278 \times 10^{-4} \times 0.1035 \approx 3.39 \times 10^{-5}\, \mathrm{T}
\]

Final result:

\[
\boxed{
B \approx 3.39 \times 10^{-5}\, \mathrm{Tesla} = 33.9\, \mu\mathrm{T}
}
\]

Discussion of the Results

The calculated magnetic field strength at 48 cm along the axis of a small square loop carrying 41 A current is approximately 33.9 microteslas (μT). This magnitude is comparable to the Earth's magnetic field, which ranges roughly from 25 to 65 μT, indicating the magnetic influence of the loop at this distance is relatively modest but measurable with sensitive instruments.

Factors Influencing Magnetic Field Strength:

- Current magnitude: Increasing I linearly increases B.
- Loop size: Larger loops produce stronger magnetic fields at the same distance, given the same current.
- Distance from the loop: The magnetic field decreases with increasing x, following the derived formula.
- Loop geometry: The square shape influences the field distribution; circular loops have a different but related formula.

Applications and Implications:

- Wireless Power Transfer: Understanding how magnetic fields diminish with distance aids in designing efficient systems.
- Electromagnetic Compatibility (EMC): Knowing the field strengths helps in shielding and minimizing interference.
- Magnetic Field Sensing: The calculated values are essential for calibrating sensors and understanding environmental magnetic influences.

Extensions and Further Considerations

While the calculation provides a good estimate, real-world factors such as:

- Finite thickness of the wire
- Surrounding materials and their magnetic properties
- Non-idealities in current distribution

can influence the actual magnetic field. For more precise modeling, numerical methods or finite element analysis (FEA) can be employed.

Conclusion

In summary, a small square loop carrying a 41A current generates a magnetic field of approximately 33.9 μT at a point 48 cm along its axis, assuming a side length of 10 cm. This calculation demonstrates how fundamental electromagnetic principles enable us to predict and analyze magnetic field distributions, which are critical in designing and understanding various electrical and electronic systems.

Understanding these principles allows engineers and physicists to optimize devices, ensure safety standards, and innovate in fields such as wireless communication, medical imaging, and power systems. The ability to estimate magnetic field strengths accurately is a vital skill in the realm of applied electromagnetism.

Note: If the actual size of the loop differs, the

Frequently Asked Questions

What is the magnetic field strength at 48 cm on the axis of a small square loop carrying 41A current?

The magnetic field strength at 48 cm on the axis of a small square loop depends on the loop's size and current; it can be calculated using the magnetic field formula for a square loop, considering the distance and current. Precise value requires the loop's side length.

How does the distance from a small square loop affect the magnetic field strength along its axis?

The magnetic field strength decreases with increasing distance from the loop along its axis, approximately following an inverse cubic or similar relation depending on the loop's geometry.

What is the formula to calculate the magnetic field along the axis of a square current loop?

The magnetic field at a point on the axis of a square loop can be calculated using the Biot-Savart law, often simplified as $B = (\mu_0 I a^2) / (2\pi (z^2 + (a/\sqrt{2})^2)^{(3/2)})$, where a is the side length and z is the distance from the loop.

How does the current of 41A influence the magnetic field produced by the square loop?

A higher current, such as 41A, proportionally increases the magnetic field strength generated by the loop, as magnetic field is directly proportional to current.

If the magnetic field strength 48 cm from the loop is known, how can one determine the loop's side length?

By rearranging the magnetic field formula for a square loop and substituting the known field strength and distance, you can solve for the side length of the loop.

What practical applications rely on calculating the magnetic field of a small square loop?

Applications include electromagnetic sensors, inductors in circuits, magnetic field calibration, and wireless power transfer systems.

How does the shape of the loop (square vs. circular) affect the magnetic field calculation?

While the fundamental principles are similar, the exact magnetic field expressions differ; circular loops have simpler formulas, but square loops require summation over four sides, leading to more complex calculations.

Can the magnetic field at 48 cm from a small square loop be considered uniform?

No, the magnetic field varies with distance; at 48 cm, the field is weaker and non-uniform compared to points closer to the loop.

What factors should be considered when measuring the magnetic field of a small square loop in experiments?

Factors include the current magnitude, precise loop dimensions, distance measurement accuracy, environmental magnetic interference, and the orientation of the measurement device.

How does increasing the current from 41A to a higher value affect the magnetic field strength at the same point?

Increasing the current proportionally increases the magnetic field strength at the same point, as the relationship between current and magnetic field is linear.

Additional Resources

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Introduction

The magnetic field generated by current-carrying conductors has long been a fundamental subject in electromagnetism, underpinning everything from electric motors to medical imaging devices. Among the various configurations, the magnetic field produced by a small, square loop of wire is a classic problem with both theoretical and practical significance. In this analysis, we explore the magnetic field strength along the axis of such a loop, given a current of 41 amperes and a measurement point located 48 centimeters from the loop. Understanding this specific scenario provides insight into the behavior of magnetic fields in finite loops and offers foundational knowledge applicable to designing electromagnetic devices.

Theoretical Background of Magnetic Fields in Current Loops

Magnetic Fields from Currents: Basic Principles

At the core of electromagnetism lies Ampère's law and the Biot-Savart Law, which describe how steady currents produce magnetic fields:

- Ampère's Law (integral form):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

- Biot-Savart Law (differential form):

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

where:
- μ_0 is the permeability of free space ($4\pi \times 10^{-7}\ \text{H/m}$),
- I is the current,
- $d\mathbf{l}$ is an element of the current-carrying wire,
- $\hat{\mathbf{r}}$ is the unit vector pointing from the element to the observation point,
- r is the distance from the element to the observation point.

Magnetic Field of a Circular vs. Square Loop

While the magnetic field of a circular loop is well-documented and often used as a reference, real-world applications sometimes involve square or rectangular loops due to manufacturing constraints or design preferences. The magnetic field along the axis of a square loop is more complex but can be calculated by superposing the contributions of each side, considering the Biot-Savart Law.

Concept of On-Axis Magnetic Field

The "on-axis" magnetic field refers to the magnetic field along the central axis perpendicular to the plane of the loop, passing through its center. This is a common measurement point because it simplifies calculations and provides a clear picture of the field's strength and distribution.

Geometry and Parameters of the Square Loop

Dimensions of the Loop

For calculations, the key parameters include:

- Side length of the square loop: a (unknown, but can be determined or assumed)
- Current: $I = 41\, \text{A}$
- Distance from the loop along its axis: $z = 48\, \text{cm} = 0.48\, \text{m}$

Assumptions and Considerations

Given the problem statement, we need to determine the magnetic field strength at a point 48 cm along the axis of a small, square loop carrying a 41A current. Since the side length isn't explicitly provided, we can analyze the problem in two ways:

1. Assuming a specific size (e.g., a loop with a side length of 10 cm)
2. Deriving a general expression that can be applied to any side length

In this article, we will explore the general case, then illustrate with an example assuming a typical small loop size.

Calculating the Magnetic Field Along the Axis of a Square Loop

Step 1: Break Down the Square Loop into Four Line Segments

Each side of the square contributes to the magnetic field at the observation point. Due to symmetry, the fields from opposite sides can be combined vectorially.

Step 2: Express the Magnetic Field of a Single Side

The magnetic field at a point along the axis of a finite straight conductor of length L carrying current I is given by:

$$
B_{\text{side}} = \frac{\mu_0 I}{4\pi R} (\sin \theta_2 + \sin \theta_1)
$$

where:
- R is the perpendicular distance from the side to the observation point,
- θ_1 and θ_2 are the angles between the line from the observation point to each end of the segment and the perpendicular from the segment to the point.

However, for a square loop, the calculation simplifies by considering the entire contributions of each side along the axis.

Step 3: Derive the Total Magnetic Field

The total magnetic field B_z at distance z along the axis of the square loop with side length a is:

$$
B_z = \frac{2 \sqrt{2} \mu_0 I}{\pi a} \times \frac{a^2/2}{(a^2/2 + z^2)^{3/2}}
$$

This expression results from summing the contributions of each side, considering symmetry and the geometry involved.

Numerical Example: Calculating the Magnetic Field

Assumed Parameters

Let’s assume the side length of the square loop is 10 cm (0.1 m), a typical small loop size.

Given:
- $I = 41\, \text{A}$

- $a = 0.1\, \text{m}$
- $z = 0.48\, \text{m}$

Step 4: Plugging Values into the Formula

Using the derived formula:

$$
B_z = \frac{2 \sqrt{2} \times 4\pi \times 10^{-7}\ \text{H/m} \times 41}{\pi \times 0.1} \times \frac{(0.1)^2 / 2}{\left((0.1)^2 / 2 + (0.48)^2 \right)^{3/2}}
$$

Calculations:

1. Compute constants:

$$
\mu_0 = 4\pi \times 10^{-7} \approx 1.2566 \times 10^{-6}\ \text{H/m}
$$

2. Numerator:

$$
2 \sqrt{2} \times \mu_0 \times I \approx 2 \times 1.4142 \times 1.2566 \times 10^{-6} \times 41 \approx 2.8284 \times 1.2566 \times 10^{-6} \times 41
$$

$$
\approx 3.551 \times 10^{-6} \times 41 \approx 1.455 \times 10^{-4}
$$

3. Denominator:

$$
\pi \times 0.1 \approx 0.31416
$$

4. The geometric factor numerator:

$$
(0.1)^2 / 2 = 0.005\, \text{m}^2
$$

5. The denominator inside the root:

$$
(0.005) / \left((0.005) + (0.48)^2 \right)^{3/2}
$$

Calculating:

$$
(0.48)^2 = 0.2304
$$

$$
0.005 + 0.2304 = 0.2354
$$

$$
(0.2354)^{3/2} = (0.2354) \times \sqrt{0.2354} \approx 0.2354 \times 0.485 \approx 0.1144
$$

Now, the entire expression:

$$
B_z \approx \frac{1.455 \times 10^{-4}}{0.31416} \times \frac{0.005}{0.1144}
$$

Calculate each part:

$$
\frac{1.455 \times 10^{-4}}{0.31416} \approx 4.63 \times 10^{-4}
$$

$$
\frac{0.005}{0.1144} \approx 0.0437
$$

Finally:

$$
B_z \approx 4.63 \times 10^{-4} \times 0.0437 \approx 2.02 \times 10^{-5}\ \text{Tesla}
$$

or approximately 20.2 microteslas.

Analysis and Interpretation

Significance of the Magnetic Field Strength

The computed magnetic field strength of about 20 microteslas at 48 cm from the loop demonstrates the rapid decay of magnetic fields with distance, especially for small loops. For comparison, Earth's magnetic field averages about 25 to 65 microteslas, indicating that the field from this small loop is of similar magnitude at that distance, though notably weaker than fields generated by larger or more current-heavy configurations.

Effect of Loop Size and Current

- Loop Size: Increasing the side length a enhances the magnetic field strength at a given distance

due to a larger magnetic dipole moment.
- Current: The magnetic field scales linearly with the current I; doubling the current doubles the magnetic field

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