A Solenoid Of Length 0.700 M Having A Circular Cross-section Of Radius 5.00 Cm Stores 6.00 J Of Energy

A Solenoid Of Length 0.700 M Having A Circular Cross-section Of Radius 5.00 Cm Stores 6.00 J Of Energy

Understanding the behavior of magnetic fields generated by solenoids is fundamental in electromagnetism and various practical applications, including electromagnets, inductors, and transformers. The detailed analysis of a specific solenoid with given dimensions and stored energy provides insights into the relationships between physical parameters like magnetic field strength, inductance, and energy storage. In this article, we explore the properties of a solenoid measuring 0.700 meters in length with a circular cross-section of radius 5.00 centimeters, which stores 6.00 joules of magnetic energy.

Introduction to Solenoids and Their Significance

A solenoid is a long coil of wire wound in the form of a helix, which produces a nearly uniform magnetic field when an electric current flows through it. Due to their ability to generate controlled magnetic fields, solenoids are widely used in electromagnetic devices such as relays, actuators, MRI machines, and inductors in electronic circuits.

The importance of understanding the parameters of a solenoid—such as its length, cross-sectional area, magnetic field, inductance, and stored energy—cannot be overstated. These parameters determine the efficiency and functionality of devices utilizing solenoids.

Given Data and Problem Overview

Let's restate the key data points for clarity:

- Length of the solenoid, \(L = 0.700\, \text{m}\)
- Radius of the circular cross-section, \(r = 5.00\, \text{cm} = 0.0500\, \text{m}\)

- Stored magnetic energy, $U = 6.00\, \text{J}$

Our goal is to analyze this solenoid's magnetic properties, calculate its inductance, magnetic field, and understand how these parameters relate to the stored energy.

Physical Principles Governing the Solenoid

To analyze the solenoid, we rely on fundamental electromagnetism principles:

Magnetic Field Inside a Solenoid

The magnetic field B inside a long, ideal solenoid is given by:

$$
B = \mu_0 n I
$$

where:
- $\mu_0 = 4\pi \times 10^{-7}\, \text{T}\cdot\text{m/A}$ (permeability of free space)
- $n = \frac{N}{L}$ = number of turns per unit length
- I = current flowing through the solenoid

Magnetic Energy Stored in the Solenoid

The energy stored in an inductor (the solenoid, in this case) is:

$$
U = \frac{1}{2} L I^2
$$

where:
- L = inductance of the solenoid
- I = current through the coil

Inductance of a Solenoid

The inductance of a solenoid with N turns is:

$$
L = \mu_0 n^2 A L
$$

where:
- $A = \pi r^2$ = cross-sectional area
- L = length of the solenoid

Calculating the Cross-Sectional Area

The cross-sectional area A is:

$$
A = \pi r^2 = \pi (0.0500\, \text{m})^2 = \pi \times 0.0025\, \text{m}^2 \approx 0.007854\, \text{m}^2
$$

This area influences the magnetic flux density and the inductance value.

Determining the Inductance and Current

Since the stored energy U is known, and $U = \frac{1}{2} L I^2$, we can relate the inductance L and current I.

Rearranged:

$$
L I^2 = 2 U = 12.00\, \text{J}
$$

To find L and I, we need an additional relation. However, there's an alternative approach: expressing L in terms of N and I, then solving for these parameters.

Expressing Inductance in Terms of Turns and Current

The inductance of a solenoid:

$$
L = \mu_0 n^2 A L
$$

where:
- $n = \frac{N}{L}$

Thus:

$$
L = \mu_0 \left(\frac{N}{L}\right)^2 A L = \mu_0 N^2 \frac{A}{L}
$$

Rearranged:

$$
L = \mu_0 N^2 \frac{A}{L}
$$

And the magnetic energy:

$$
U = \frac{1}{2} L I^2
$$

From the above, if we express I:

$$
I = \frac{B}{\mu_0 n}
$$

But since B also relates to N and I, a more straightforward approach is to express L directly in terms of N, then find the current.

Estimating the Number of Turns N

Suppose we assume the current I is such that the magnetic energy stored is 6 J. We can express L:

$$
L = \frac{2U}{I^2}
$$

But without I, this seems to be a circular problem. To resolve this, we can consider the magnetic flux and the relation between flux and inductance:

$$
\Phi = N \times \text{Flux per turn}
$$

and the magnetic flux density B relates to flux:

$$
\Phi = B A
$$

For a long solenoid, the magnetic field is approximately uniform and:

$$
B = \mu_0 n I
$$

Expressed as:

$$
\Phi = \mu_0 n I A
$$

Then, the flux linkage:

$$
N \Phi = N \times \mu_0 n I A = \mu_0 N n I A
$$

which simplifies to:

$$
\text{Inductance} \quad L = \frac{N \Phi}{I} = \mu_0 N n A
$$

But since $n = N / L$:

$$
L = \mu_0 N \times \frac{N}{L} \times A = \mu_0 \frac{N^2 A}{L}
$$

This confirms the earlier formula.

Calculating the Number of Turns (N)

Let's define N as unknown, and aim to find it.

From the energy relation:

$$
U = \frac{1}{2} L I^2
$$

Express \(L\):

\[
L = \mu_0 \frac{N^2 A}{L}
\]

Rearranged:

\[
L = \mu_0 \frac{N^2 A}{L}
\]

Note that this expression involves \(L\) (length of the solenoid), so:

\[
L_{ind} = \mu_0 \frac{N^2 A}{L}
\]

Now, express \(I\):

\[
I = \frac{B}{\mu_0 n} \quad \text{or} \quad B = \mu_0 n I
\]

Alternatively, considering the energy stored:

\[
U = \frac{1}{2} L I^2
\]

Express \(I\) in terms of \(U\) and \(L\):

\[
I = \sqrt{\frac{2U}{L}}
\]

Since \(L = \mu_0 \frac{N^2 A}{L}\), substitute back:

\[
L = \mu_0 \frac{N^2 A}{L}
\]

Rearranged:

\[
L^2 = \mu_0 N^2 A
\]

which gives:

\[

$$
N = \frac{L}{\sqrt{\mu_0 A}}
$$

Plugging in the known values:

$$
\sqrt{\mu_0 A} = \sqrt{4\pi \times 10^{-7} \times 0.007854} \approx \sqrt{9.8696 \times 10^{-10}} \approx 3.14 \times 10^{-5}
$$

Therefore:

$$
N = \frac{0.700}{3.14 \times 10^{-5}} \approx 22,289
$$

This is the approximate number of turns.

Now, compute the inductance:

$$
L = \mu_0 \frac{N^2 A}{L} = 4\pi \times 10^{-7} \times \frac{(22,289)^2 \times 0.007854}{0.700}
$$

Calculations:

$$
N^2 \approx (22,289)^2 \approx 4.97 \times 10^8
$$

Then:

Frequently Asked Questions

What is the magnetic energy stored in a solenoid with 6.00 J of energy?

The magnetic energy stored is 6.00 Joules, as given in the problem statement.

How can we calculate the magnetic field inside the solenoid from the stored energy?

The magnetic field B can be found using the formula $U = (1/2) L I^2$, where U is stored energy, L is inductance, and I is current. Alternatively, B can be related to the current and geometry via $B = \mu_0 n I$, where n is the number of turns per unit length.

What is the significance of the solenoid's radius in calculating its magnetic properties?

The radius determines the cross-sectional area, which influences the magnetic flux and inductance calculations, as the flux linkage depends on the area through which the magnetic field lines pass.

How do you determine the inductance of the solenoid given its dimensions and stored energy?

Using the energy formula $U = (1/2) L I^2$, if the current I is known or can be calculated, the inductance L can be found as $L = 2U / I^2$.

What is the approximate magnetic field strength inside the solenoid based on the given energy?

To estimate B, first calculate the inductance and current, then use $B = \mu_0 n I$. Without the number of turns, an exact value isn't possible, but the energy stored relates to the magnetic field strength.

Why is the energy stored in a solenoid proportional to the square of the current?

Because magnetic energy stored in an inductor is given by $U = (1/2) L I^2$, which shows a quadratic dependence on current due to the magnetic field's energy density.

What additional information is needed to fully determine the solenoid's inductance and magnetic field?

The number of turns or the current passing through the solenoid is needed to calculate the inductance and magnetic field accurately.

Additional Resources

A Solenoid Of Length 0.700 M Having A Circular Cross-section Of Radius 5.00 Cm Stores 6.00 J Of Energy

---

Introduction

In the realm of electromagnetism, solenoids stand as fundamental components with widespread applications—from magnetic resonance imaging (MRI) machines to inductors in electrical circuits. Recently, a particular solenoid with specific dimensions and energy storage capacity has garnered scientific interest. This article explores the detailed physics behind a solenoid measuring 0.700 meters in length, with a circular cross-section of radius 5.00 centimeters, which stores an energy of 6.00 joules. Through a comprehensive analysis, we aim to understand its physical parameters, energy storage mechanisms, and implications for practical applications.

Understanding the Basics of Solenoids

What Is a Solenoid?

A solenoid is a long coil of wire wound in many turns, designed to produce a uniform magnetic field when an electric current flows through it. Its cylindrical shape and uniform winding make it ideal for creating controlled magnetic environments.

Fundamental Properties

- Length (L): The distance from one end of the solenoid to the other.
- Number of Turns (N): Total loops of wire wound along its length.
- Cross-Sectional Area (A): The area of the circle formed by the coil's cross-section.
- Magnetic Field (B): The magnetic flux density inside the solenoid.

Energy Storage in a Solenoid

A current-carrying solenoid stores magnetic energy, which can be expressed as:

$$ U = \frac{1}{2} L_{m} I^{2} $$

where:

- U is the stored magnetic energy,
- L_{m} is the inductance of the solenoid,
- I is the current passing through it.

Alternatively, the energy stored can be related directly to the magnetic field and volume:

$$ U = \frac{1}{2\mu_0} \int B^{2} dV $$

Determining the Physical Parameters

Given Data Recap

- Length of the solenoid, $L = 0.700 \mathrm{\,m}$

- Radius of cross-section, $r = 5.00 \mathrm{\,cm} = 0.0500 \mathrm{\,m}$
- Stored energy, $U = 6.00 \mathrm{\,J}$

Our goal is to determine the magnetic field inside the solenoid, the current flowing through it, and the inductance.

Calculating the Cross-Sectional Area

The cross-sectional area A of the solenoid, which is circular, is:

$$
A = \pi r^{2}
$$

Plugging in the radius:

$$
A = \pi \times (0.0500\, \mathrm{m})^{2} \approx 3.1416 \times 0.0025\, \mathrm{m^2} \approx 7.854 \times 10^{-3} \mathrm{\,m^2}
$$

This is the area through which the magnetic flux passes.

Estimating the Magnetic Field and Current

Step 1: Expressing Energy in Terms of Magnetic Field

Using the energy stored formula in terms of magnetic field:

$$
U = \frac{1}{2\mu_0} \times B^{2} \times V
$$

where:

- $\mu_0 = 4\pi \times 10^{-7} \mathrm{\,H/m}$,
- V is the volume of the solenoid.

The volume V of the solenoid:

$$
V = A \times L = 7.854 \times 10^{-3}\, \mathrm{m^2} \times 0.700\, \mathrm{m} \approx 5.498 \times 10^{-3} \mathrm{\,m^3}
$$

Rearranging the energy formula to solve for B:

$$
B = \sqrt{\frac{2 \mu_0 U}{V}}
$$

Plugging in the known values:

$$
B = \sqrt{\frac{2 \times 4\pi \times 10^{-7} \times 6.00}{5.498 \times 10^{-3}}}
$$

Calculating numerator:

$$
2 \times 4\pi \times 10^{-7} \times 6.00 = 8\pi \times 10^{-7} \times 6.00 \approx (8 \times 3.1416) \times 10^{-7} \times 6 \approx 25.1328 \times 10^{-7} \times 6 \approx 150.8 \times 10^{-7} = 1.508 \times 10^{-5}
$$

Now, compute B:

$$
B = \sqrt{\frac{1.508 \times 10^{-5}}{5.498 \times 10^{-3}}} = \sqrt{2.744 \times 10^{-3}} \approx 0.0524\, \mathrm{T}
$$

Thus, the magnetic flux density inside the solenoid:

$$
\boxed{
B \approx 0.0524\, \mathrm{T}
}
$$

Determining the Current in the Solenoid

The magnetic field inside an ideal solenoid is given by:

$$
B = \mu_0 n I
$$

where:

- $n = \frac{N}{L}$ is the number of turns per unit length,
- I is the current.

Rearranged to find I:

$$
I = \frac{B}{\mu_0 n}
$$

But first, to determine I, we need n. To find n, we need the inductance L_m, which relates to the energy stored:

$$
U = \frac{1}{2} L_m I^{2}
$$

Rearranged:

$$
L_m = \frac{2U}{I^{2}}
$$

Alternatively, the inductance of a solenoid with N turns:

$$
L_m = \mu_0 n^{2} A L
$$

Expressed directly in terms of N:

$$
L_m = \mu_0 N^{2} \frac{A}{L}
$$

But since $N = n L$, it simplifies to:

$$
L_m = \mu_0 n^{2} A L
$$

Given the energy U, and the relation between L_m and I, we can determine I once N or n is known.

Calculating the Number of Turns and Turn Density

Step 1: Express L_m in terms of N:

$$
L_m = \mu_0 \frac{N^{2} A}{L}
$$

Using the energy relation:

$$

$$
U = \frac{1}{2} L_m I^{2}
$$

We can write:

$$
I^{2} = \frac{2U}{L_m}
$$

But to proceed, we need N or n. Alternatively, the magnetic field in a solenoid is:

$$
B = \mu_0 n I
$$

Expressed as:

$$
I = \frac{B}{\mu_0 n}
$$

Now, the magnetic field in an ideal solenoid also relates to the number of turns:

$$
B = \mu_0 \frac{N}{L} I
$$

From which:

$$
N = \frac{B L}{\mu_0 I}
$$

Alternatively, if we express N in terms of n:

$$
N = n L
$$

Let's first estimate the current I using the energy and inductance relationship.

Calculating the Inductance and Current

Using the energy stored:

$$
U = \frac{1}{2} L_m I^{2}
$$

Expressed for L_m:

$$
L_m = \frac{2 U}{I^{2}}
$$

But we don't know I yet. Instead, we can estimate the inductance directly using the formula for a long solenoid:

$$
L_m = \mu_0 N^{2} \frac{A}{L}
$$

Rearranged:

$$
N^{2} = \frac{L_m L}{\mu_0 A}
$$

Alternatively, considering the magnetic energy stored per unit volume:

$$
u = \frac{B^{2}}{2 \mu_0}
$$

Total energy:

$$
U = u \times V = \frac{B^{2}}{2 \mu_0} \times V
$$

which we've already used to find B.

Now, to find I, recall the relation:

$$
B = \mu_0 n I \Rightarrow I = \frac{B}{\mu_0 n}
$$

But to find n, we need the number of turns:

$$
N = n L
$$

Given the typical design of solenoids, the number of turns is usually large, but for this calculation,

A Solenoid Of Length 0 700 M Having A Circular Cross Section Of Radius 5 00 Cm Stores 6 00 J Of Energy

A Solenoid Of Length 0 700 M Having A Circular Cross Section Of Radius 5 00 Cm Stores 6 00 J Of Energy

Related Articles

- A Sled Is Moving Down A Steep Hill. The Mass Of The Sled Is 50 Kg And The Net Force Acting On It Is 20
- ? Question The Point (x, Y) Is First Rotated 180 Clockwise About The Origin, Translated 6 Units To The
- A Client Is Diagnosed With Bell's Palsy. The Nurse Assessing The Client Expects To Note Which Symptom?

Back to Home