A Triangle Has Two Sides Of Length 15 Cm And 17 Cm. Select All The Values Of Its Third Side That Would

A Triangle Has Two Sides Of Length 15 Cm And 17 Cm. Select All The Values Of Its Third Side That Would be the central question when exploring the properties of triangles in geometry. Understanding how the length of the third side relates to the known sides is fundamental to solving problems involving triangle construction, inequality, and classification. This article will guide you through the principles governing the possible lengths of the third side, applying the triangle inequality theorem, and helping you identify all the valid values based on given side lengths of 15 cm and 17 cm.

Understanding the Triangle Inequality Theorem

What Is the Triangle Inequality Theorem?

The triangle inequality theorem is a fundamental rule in geometry that states: For any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side. This theorem ensures that the three sides can form a closed figure, i.e., a triangle.

Mathematically, if the sides of a triangle are a, b, and c, then:

- $a + b > c$
- $a + c > b$
- $b + c > a$

When given two sides, the third side must satisfy these inequalities to form a valid triangle.

Applying the Triangle Inequality to Sides 15 Cm and 17 Cm

Given Sides and Unknown Third Side

In this case, two sides are known: 15 cm and 17 cm. Let's denote the third side as x cm. To determine the possible values of x, we need to ensure that all three triangle inequalities are satisfied:

- $15 + 17 > x$
- $15 + x > 17$

- $17 + x > 15$

Let’s analyze each inequality separately.

Calculating the Bounds for the Third Side x

1. First inequality: $15 + 17 > x$
- $32 > x$
- Therefore, $x < 32$

2. Second inequality: $15 + x > 17$
- $x > 17 - 15$
- $x > 2$

3. Third inequality: $17 + x > 15$
- $x > 15 - 17$
- $x > -2$

Since side lengths must be positive, the inequality $x > -2$ is always satisfied for positive x, and does not impose a stricter constraint than $x > 0$.

Combining the inequalities:
- $x > 2$
- $x < 32$

Therefore, the third side x must satisfy:
$2 < x < 32$

This means the third side can be any real number greater than 2 cm but less than 32 cm.

Possible Values for the Third Side

Range of Valid Lengths

Based on the triangle inequality theorem, the third side must lie within the interval:

- Greater than 2 cm

- Less than 32 cm

This means any length x satisfying $2 < x < 32$ cm can form a triangle with sides of 15 cm and 17 cm.

Integer Values of the Third Side

If you are considering integer lengths only, the valid third side lengths are all integers greater than 2 and less than 32:

- 3 cm, 4 cm, 5 cm, ..., 31 cm

In total, these are the integers from 3 to 31 inclusive.

Special Cases and Triangle Types

When Is the Triangle Equilateral, Isosceles, or Scalene?

While the third side can vary within the specified range, certain specific lengths result in special types of triangles.

- Equilateral Triangle:
Not possible here because the sides are 15 cm and 17 cm; for an equilateral triangle, all sides are equal, which contradicts the given lengths.

- Isosceles Triangle:
When the third side is equal to one of the known sides:
- $x = 15$ cm or $x = 17$ cm
These lengths produce isosceles triangles, with two sides equal.

- Scalene Triangle:
When the third side is different from both 15 cm and 17 cm, and within the bounds:
- $x \neq 15$ cm and $x \neq 17$ cm
- x between 2 and 32 cm, excluding 15 and 17

These produce scalene triangles with all sides of different lengths.

Visualizing the Triangle with Different Third Side Lengths

Constructing the Triangle

To better understand the possible third side lengths, imagine constructing the triangle with fixed sides 15 cm and 17 cm and varying the third side between just over 2 cm and just under 32 cm:

- When x approaches 2 cm from above, the triangle becomes very "skinny," with sides nearly forming a degenerate triangle.
- When x approaches 32 cm from below, the triangle again becomes very elongated, approaching a degenerate form.
- For values of x near the middle of the range, the triangle is more balanced and equilateral-like.

Practical Applications

Understanding these constraints is useful in various real-world contexts:

- Designing triangular structures and ensuring stability
- Calculating possible side lengths in construction and engineering projects
- Solving geometry problems involving triangle inequalities

Summary: All Valid Third Side Lengths

To summarize, given two sides of a triangle measuring 15 cm and 17 cm, the third side must be:

- Greater than 2 cm
- Less than 32 cm

If considering only integer lengths, the valid third sides are all integers from 3 cm up to 31 cm inclusive.

In conclusion:

- Any real number x satisfying 2 < x < 32 will enable the formation of a valid triangle with the known sides.
- Special cases such as x = 15 cm or x = 17 cm produce isosceles triangles.
- Values outside this range will not satisfy the triangle inequality, and thus, no triangle can be formed.

By understanding the triangle inequality theorem and applying it carefully, you can confidently determine all possible values for the third side length, ensuring your geometric constructions or calculations are accurate and valid.

If you need further assistance with triangle problems or geometric concepts, exploring the properties of different triangle types, or solving related inequalities, feel free to ask!

Frequently Asked Questions

A triangle has two sides measuring 15 cm and 17 cm. What are the possible lengths of the third side?

The third side must be greater than 2 cm and less than 32 cm, so it can be any length between 2 cm and 32 cm (not including 2 cm and 32 cm).

If a triangle has sides of 15 cm and 17 cm, which values of the third side satisfy the triangle inequality theorem?

Any third side length greater than 2 cm and less than 32 cm will satisfy the triangle inequality theorem for these two sides.

How do you determine the possible lengths for the third side of a triangle with two sides measuring 15 cm and 17 cm?

Use the triangle inequality theorem: the third side must be greater than the difference of the two known sides (17 - 15 = 2 cm) and less than their sum (15 + 17 = 32 cm).

Can the third side of the triangle be exactly 2 cm or 32 cm?

No, the third side cannot be exactly 2 cm or 32 cm because the triangle inequality requires the sum of any two sides to be greater than the third side.

What is the range of possible third side lengths for a triangle with sides 15 cm and 17 cm?

The third side length can be any value greater than 2 cm and less than 32 cm.

Is a third side of 10 cm possible for this triangle?

Yes, because 10 cm is between 2 cm and 32 cm, satisfying the triangle inequality theorem.

Is a third side of 35 cm possible for this triangle?

No, because 35 cm exceeds the sum of the other two sides (15 + 17 = 32 cm), violating the triangle inequality.

What are the possible integer lengths for the third side of this triangle?

Any integer length from 3 cm up to 31 cm inclusive will satisfy the triangle inequality.

If the third side is 20 cm, does the triangle with sides 15 cm and 17 cm exist?

Yes, since 20 cm is between 2 cm and 32 cm, the triangle inequality holds, and such a triangle can exist.

Why must the third side be less than 32 cm in this case?

Because the sum of the two known sides is 32 cm, and the third side must be less than this sum for the sides to form a triangle.

Additional Resources

Triangle Inequality: Exploring the Possible Lengths of the Third Side

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Understanding the Triangle Inequality Theorem

When examining any triangle, one of the fundamental principles that govern its formation is the Triangle Inequality Theorem. This theorem states that for any triangle, the length of each side must be less than the sum of the other two sides and greater than their difference. In simpler terms, the third side must be strictly greater than the difference of the known sides and less than their sum.

This principle is crucial because it ensures the three sides can indeed come together to form a closed, non-degenerate triangle. Without satisfying these inequalities, the figure would either be degenerate (collapsing into a straight line) or impossible to construct.

Given Sides: 15 cm and 17 cm

In our specific case, we are provided with two sides of a triangle:

- Side A = 15 cm
- Side B = 17 cm

Our goal is to determine all possible lengths of the third side, denoted as Side C, that would allow these three sides to form a valid triangle.

Applying the Triangle Inequality to Find the Range of the Third Side

To find the permissible lengths of Side C, we apply the Triangle Inequality Theorem to each pair of sides:

1. Considering Side C with Side A (15 cm):

- The sum of Side A and Side C must be greater than Side B:

$$
15 + C > 17 \Rightarrow C > 2
$$

- The difference of Side A and Side C must be less than Side B:

$$
|15 - C| < 17
$$

This inequality gives two cases:

Case 1: $15 - C < 17$

$$
15 - C < 17 \Rightarrow -C < 2 \Rightarrow C > -2
$$

Since side lengths are positive, this is always true for $C > 0$.

Case 2: $- (15 - C) < 17 \Rightarrow C - 15 < 17 \Rightarrow C < 32$

Combining these, the inequality simplifies to:

$$
C < 32
$$

Since $C > 0$, this is always true for positive lengths, but to satisfy the triangle inequality, the key restriction from this step is that $C > 2$.

2. Considering Side C with Side B (17 cm):

- The sum of Side B and Side C must be greater than Side A:

$$
17 + C > 15 \Rightarrow C > -2
$$

- The difference of Side B and Side C must be less than Side A:

$$
|17 - C| < 15
$$

Again, splitting into two cases:

Case 1: $17 - C < 15 \Rightarrow -C < -2 \Rightarrow C > 2$

Case 2: $C - 17 < 15 \Rightarrow C < 32$

Combining, this yields $C > 2$ and $C < 32$.

3. Considering all inequalities together:

To satisfy all triangle inequalities simultaneously, the third side C must satisfy:

$$
\boxed{
2 < C < 32
}
$$

Final Range for the Third Side

Based on the above analysis, the third side length C must satisfy:

- Strictly greater than 2 cm
- Strictly less than 32 cm

Expressed mathematically:

$$
\boxed{
C \in (2, 32)
}
$$

This interval encompasses all lengths that would allow the three sides of 15 cm, 17 cm, and C cm to form a valid, non-degenerate triangle.

Implications and Practical Considerations

Understanding this range is not merely an academic exercise; it offers practical insights into the properties of triangles and their applications:

- Design and Construction: Whether constructing a triangular frame or designing a mechanical component, knowing the permissible lengths ensures stability and feasibility.

- Mathematical Modeling: In fields like architecture, engineering, and computer graphics, precise measurements derived from such inequalities are fundamental to creating accurate models.

- Real-world Measurement Tolerance: When measuring physical objects, slight deviations within the specified range will still result in a triangle, but exceeding these limits would cause the shape to collapse or become degenerate.

Special Cases Within the Range

While the range $2 < C < 32$ encompasses an infinite set of possible lengths, certain special cases merit attention:

Equilateral or Isosceles Triangles

- When $C = 15$ or $C=17$:

- The triangle becomes isosceles, with two sides equal.
- Specifically, if $C=15$, the sides are 15, 15, and 17.
- If $C=17$, the sides are 15, 17, and 17.

- Implication: These special cases often have unique properties, like symmetry, which might be desirable in specific applications.

Right-Angled Triangle Conditions

- The Pythagorean theorem applies if the sides satisfy:

$$
\text{Longest side}^2 = \text{Sum of squares of the other two sides}
$$

- For example, if the third side C is such that:

$$
C^2 = 15^2 + 17^2 = 225 + 289 = 514
$$

$$
C = \sqrt{514} \approx 22.7 \text{ cm}
$$

- Since $2 < 22.7 < 32$, this length is feasible for a right-angled triangle with sides 15 and 17.

Summary: What Values of the Third Side Are Possible?

Range of C	Explanation	Validity
$C \leq 2$	Not forming a triangle because sum of other two sides is too small	Invalid
$2 < C < 32$	Satisfies triangle inequalities	Valid
$C \geq 32$	Triangle inequality violated; sides can't form a triangle	Invalid

Therefore, the third side of the triangle with sides 15 cm and 17 cm must be strictly greater than 2 cm and strictly less than 32 cm.

Conclusion: The Range of Possible Third Side Lengths

In conclusion, any length of the third side C within the open interval $(2, 32)$ centimeters will successfully complete a triangle with the given sides of 15 cm and 17 cm. This insight is essential for anyone involved in geometric design, construction, or mathematical modeling, ensuring that all configurations adhere to the fundamental principles of triangle formation.

By understanding and applying the Triangle Inequality Theorem precisely, we unlock a comprehensive view of the flexible yet constrained nature of triangle dimensions, allowing for accurate planning and analysis across diverse practical applications.

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