A Very Long Cylinder, Of Radius A, Carries A Uniform Polarization P Perpendicular To Its Axis.Find The

A Very Long Cylinder, Of Radius A, Carries A Uniform Polarization P Perpendicular To Its Axis.Find The problem involves analyzing the electric field and potential distribution generated by a uniformly polarized long cylinder. This scenario is a classic problem in electrostatics and materials science, often encountered in the study of dielectric materials and ferroelectric structures.

Understanding the behavior of such a polarized cylinder requires a comprehensive approach involving concepts like bound charges, electric fields, potential calculations, and boundary conditions. This article will explore these concepts in detail, providing step-by-step solutions, relevant equations, and physical insights to deepen your understanding of the problem.

Understanding the Physical Setup

Geometry and Polarization Direction

- The object in question is a very long cylinder, which implies that edge effects at the ends are negligible, allowing us to treat the problem as infinite in length.
- The radius of the cylinder is A.
- The polarization vector P is uniform throughout the volume of the cylinder.
- The polarization P is perpendicular to the axis of the cylinder, which we will assume to be aligned along the z-axis for simplicity.

For example:

- Cylinder axis along z-axis.
- Polarization P along x-axis (or any direction perpendicular to z).

Physical Implications of Uniform Polarization

- A uniform polarization P within a dielectric material results in bound charges that can be classified into:

1. Volume bound charge density:
$$
\rho_b = - \nabla \cdot \mathbf{P}
$$
2. Surface bound charge density:
$$
\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}
$$
where $\hat{\mathbf{n}}$ is the outward normal at the surface.

- Since P is uniform, the divergence inside the volume is zero:

$$
\nabla \cdot \mathbf{P} = 0
$$

Therefore, no volume bound charge exists within the bulk of the cylinder.

- However, at the surface, the bound charge appears due to the discontinuity of P at the boundary.

Bound Charges in the Polarized Cylinder

Surface Bound Charges

- Given the polarization is perpendicular to the axis, say along the x-axis, and the cylinder is aligned along the z-axis, the surface bound charge density is:

$$
\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}
$$

- The normal vector $\hat{\mathbf{n}}$ at the surface points radially outward, i.e., in the $\hat{\mathbf{r}}$ direction in cylindrical coordinates.

- For a cylinder of radius A:
- $\hat{\mathbf{n}}$ at the surface is $\hat{\mathbf{r}}$.
- The polarization $\mathbf{P}$ is perpendicular to the axis, say along the x-direction:

$$
\mathbf{P} = P \hat{\mathbf{x}}
$$

- Expressing $\hat{\mathbf{x}}$ in cylindrical coordinates:

$$
\hat{\mathbf{x}} = \cos\theta \, \hat{\mathbf{r}} - \sin\theta \, \hat{\boldsymbol{\theta}}
$$

- Therefore, the surface bound charge density becomes:

$$
\sigma_b(\theta) = \mathbf{P} \cdot \hat{\mathbf{r}} = P \cos \theta
$$

This indicates a non-uniform bound surface charge density around the circumference.

Implications of Surface Bound Charges

- The bound surface charge density varies sinusoidally around the cylinder:

$$
\sigma_b(\theta) = P \cos \theta
$$

- This creates a complex electric field distribution with positive and negative regions on the surface.

Electrostatic Potential and Field Calculation

Approach to the Solution

- Since the volume bound charge is zero, the primary contribution to the field arises from the surface bound charges.
- To find the electric field $\mathbf{E}$ and potential V, we solve Laplace's equation in the regions outside and inside the cylinder with appropriate boundary conditions.

Boundary Conditions

- The potential must be continuous across the boundary.
- The discontinuity in the electric displacement $\mathbf{D}$ across the surface relates to the bound surface charge:

$$
\hat{\mathbf{n}} \cdot (\mathbf{D}_{\text{outside}} - \mathbf{D}_{\text{inside}}) = \sigma_b
$$

- For a dielectric with dielectric constant ϵ, the relation between $\mathbf{D}$ and $\mathbf{E}$ is:

$$
\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}
$$

- Inside the dielectric, the bound charges produce a potential that can be obtained by solving Laplace's equation with boundary conditions incorporating σ_b.

Using Potential Theory and Fourier Expansion

- The problem exhibits angular dependence, suggesting that the potential can be expanded as a Fourier series:

$$
V(r, \theta) = \sum_{n=0}^\infty \left(A_n r^n + B_n r^{-n} \right) \cos n \theta
$$

- The coefficients A_n and B_n are determined from boundary conditions involving the surface charge distribution.

Analytical Solution for the Electric Potential

Potential Inside the Cylinder (r < a)

- Inside the cylinder, the potential must be finite at $r=0$, implying $B_n=0$ for all n.
- The general form becomes:

$$
V_{\text{inside}}(r, \theta) = \sum_{n=0}^\infty A_n r^n \cos n \theta
$$

\]

Potential Outside the Cylinder (r > A)

- Outside the cylinder, the potential must vanish at infinity, implying:

$$
V_{\text{outside}}(r, \theta) = \sum_{n=0}^\infty B_n r^{-n} \cos n \theta
$$

Matching Boundary Conditions at r = A

- The boundary conditions involve continuity of potential:

$$
V_{\text{inside}}(A, \theta) = V_{\text{outside}}(A, \theta)
$$

- And the discontinuity in the normal component of $\mathbf{D}$:

$$
\epsilon_0 \left. \frac{\partial V_{\text{outside}}}{\partial r} \right|_{r=A} - \epsilon \left. \frac{\partial V_{\text{inside}}}{\partial r} \right|_{r=A} = - \sigma_b(\theta)
$$

- Substituting the Fourier expansions and solving for the coefficients yields explicit expressions for the potential.

Physical Interpretation and Results

Electric Field Distribution

- The resulting electric field inside and outside the cylinder can be derived from the potential:

$$
\mathbf{E} = - \nabla V
$$

- Inside the cylinder, the field is primarily determined by the surface bound charges, leading to a non-uniform field with angular dependence.

- Outside the cylinder, the field resembles that of a dipole distribution, decreasing with distance as $1/r^2$ or faster.

Induced Dipole Moments and Polarization Effects

- The polarization induces a bound surface charge distribution that acts like a continuous dipole sheet.
- The net dipole moment per unit length of the cylinder can be calculated by integrating the surface bound charges:

$$
\mathbf{p} = \int \sigma_b(\theta) \mathbf{r} \, d\theta
$$

- For the given $\sigma_b(\theta) = P \cos \theta$, the dipole moment aligns along the x-axis:

$$
p_x = \pi P a^2
$$

Applications and Relevance

Dielectric Materials and Ferroelectric Devices

- Understanding the electric fields generated by polarized cylinders aids in designing ferroelectric devices, sensors, and capacitors.
- The analysis guides engineers in predicting how polarized dielectric structures influence electric fields and potential distributions.

Electrostatics in Composite Materials

- Cylindrical inclusions with uniform polarization are common in composite materials, influencing their dielectric properties.
- Accurate modeling of such structures helps optimize material performance.

Electromagnetic Compatibility and Shielding

- The distribution of bound charges affects electromagnetic field interactions, relevant in shielding applications.

Summary of Key

Frequently Asked Questions

What is the electric field inside a very long cylinder of radius a with uniform polarization P perpendicular to its axis?

The electric field inside the cylinder can be derived using bound charge distributions and is given by $E = (P / (2 \pi \varepsilon_0 a))\, r$, where r is the

RADIAL DISTANCE FROM THE AXIS, ASSUMING A UNIFORM POLARIZATION PERPENDICULAR TO THE AXIS.

HOW DOES THE POLARIZATION P PERPENDICULAR TO THE AXIS AFFECT THE BOUND CHARGE DISTRIBUTION IN THE CYLINDER?

A UNIFORM POLARIZATION PERPENDICULAR TO THE AXIS RESULTS IN BOUND SURFACE CHARGES ON THE CURVED SURFACE OF THE CYLINDER, WITH NO BOUND VOLUME CHARGES, CREATING AN EXTERNAL BOUND CHARGE DISTRIBUTION THAT INFLUENCES THE INTERNAL ELECTRIC FIELD.

WHAT IS THE SIGNIFICANCE OF THE CYLINDER BEING 'VERY LONG' IN CALCULATING ITS ELECTRIC FIELD?

THE ASSUMPTION OF THE CYLINDER BEING VERY LONG ALLOWS THE USE OF THE INFINITE-LENGTH APPROXIMATION, NEGLECTING EDGE EFFECTS AND SIMPLIFYING THE CALCULATION OF THE ELECTRIC FIELD AS UNIFORM AND DEPENDENT ONLY ON THE RADIAL DISTANCE.

HOW IS THE POTENTIAL INSIDE THE POLARIZED CYLINDER RELATED TO THE POLARIZATION P?

THE ELECTRIC POTENTIAL INSIDE THE CYLINDER DEPENDS ON THE POLARIZATION P AND CAN BE DETERMINED BY INTEGRATING THE ELECTRIC FIELD, RESULTING IN A POTENTIAL THAT VARIES LINEARLY WITH THE RADIAL POSITION, INFLUENCED BY THE BOUND SURFACE CHARGES.

WHAT BOUNDARY CONDITIONS ARE APPLIED WHEN CALCULATING THE ELECTRIC FIELD IN A POLARIZED CYLINDER OF RADIUS A?

BOUNDARY CONDITIONS INCLUDE THE CONTINUITY OF THE TANGENTIAL component of the electric field and the discontinuity of the normal component according to the bound surface charge density, which is related to the polarization P at the surface of the cylinder.

Additional Resources

A Very Long Cylinder, Of Radius A, Carries A Uniform Polarization P Perpendicular To Its Axis Find The

The problem involving a very long cylinder with a uniform polarization perpendicular to its axis is a classic electrostatics scenario that provides deep insights into the behavior of polarized materials and their associated electric fields and potentials. This particular configuration offers a rich ground for exploring concepts such as bound charges, electric fields due to polarization, and the application of boundary conditions in cylindrical geometries. In this article, we will delve into the theoretical foundations, detailed calculations, and physical interpretations associated with this problem.

Introduction to Polarization and Bound Charges

Before tackling the specific problem, it's essential to understand the basics of polarization and bound charges.

What Is Polarization?

- Polarization ($\mathbf{P}$) is a vector field representing the electric dipole moment per unit volume within a material.
- When a material is uniformly polarized, each small volume element possesses a dipole moment aligned in a particular direction.
- In this case, the polarization $\mathbf{P}$ is uniform and oriented perpendicular to the cylinder's axis, implying a consistent dipole density throughout the cylinder.

Bound Charges and Their Types

- Bound charge arises due to the polarization within the material:
- Volume bound charge density (ρ_b): $\rho_b = -\nabla \cdot \mathbf{P}$
- Surface bound charge density (σ_b): $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the outward normal to the surface.
- For uniform polarization ($\mathbf{P}$ constant), $\nabla \cdot \mathbf{P} = 0$, so no volume bound charge exists inside the cylinder.
- Surface bound charges appear at the ends or the lateral surface depending on the orientation of $\mathbf{P}$.

Geometry and Orientation of the Cylinder

The problem involves a very long cylinder with:

- Radius: a
- Length: approaching infinity (to neglect edge effects)
- Axis: along the z-direction (say)

- Polarization: uniform and perpendicular to the axis (say along the x-direction), i.e., $\mathbf{P} = P \hat{\mathbf{x}}$

This orientation significantly simplifies the analysis because of the symmetry and the uniformity of the polarization.

Bound Charges on the Cylinder

Given $\mathbf{P} = P \hat{\mathbf{x}}$:

- Since $\mathbf{P}$ is uniform, $\nabla \cdot \mathbf{P} = 0$, so no volume bound charges.
- The bound surface charge density on the lateral surface:

$$
\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}
$$

where $\hat{\mathbf{n}}$ is the outward normal to the surface.

Surface Bound Charges on the Lateral Surface

- The lateral surface of the cylinder at radius a has an outward normal:

$$
\hat{\mathbf{n}} = \hat{\mathbf{r}}
$$

- The polarization is along $\hat{\mathbf{x}}$, which in cylindrical coordinates can be expressed as:

$$
\hat{\mathbf{x}} = \cos \theta \, \hat{\mathbf{r}} - \sin \theta \, \hat{\boldsymbol{\theta}}
$$

- Therefore, the bound surface charge density becomes:

$$
\sigma_b = P \hat{\mathbf{x}} \cdot \hat{\mathbf{r}} = P \cos \theta
$$

- This indicates that the surface bound charge density varies around the circumference, with maximum values at $\theta = 0$ and π:

$$
\sigma_b(\theta) = P \cos \theta
$$

- The total bound charge on the lateral surface integrates to zero over the entire circumference, since:

$$
\int_0^{2\pi} \sigma_b(\theta) \, A \, d\theta = P A \int_0^{2\pi} \cos \theta \, d\theta = 0
$$

Key Point: The bound charge distribution is dipolar, with positive and negative charges arranged around the circumference, but no net charge.

Electric Field Due to Bound Charges

Understanding the electric field generated by the polarization involves calculating the potential and field resulting from the bound charges.

Methodology Overview

- Since volume bound charges are zero, the problem reduces to calculating the field due to surface bound charges $\sigma_b(\theta)$.
- The symmetry suggests that the resulting field will have specific characteristics, especially considering the long cylinder's extent.

Using the Potential Approach

- The scalar potential ϕ at a point $\mathbf{r}$ can be expressed as:

$$
\phi(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{\sigma_b(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dS'
$$

- Since the bound charges are distributed on the lateral surface, and the cylinder is infinitely long, the problem simplifies by considering the superposition of dipoles.

Alternative: Effective Dipole Layer

- The bound surface charge acts like a dipole layer, and for a uniformly polarized infinite cylinder, the external field resembles that of a dipole array.
- The resulting field outside the cylinder at points far from it is similar to that of a line of dipoles aligned along the cylinder, producing a field analogous to that of an infinite sheet of dipoles.

Electric Field Inside and Outside the Cylinder

Field Inside the Cylinder

- Due to the symmetry and the fact that $\nabla \cdot \mathbf{P} = 0$, the internal electric field $\mathbf{E}_{\text{inside}}$ can be derived from the relation:

$$
\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}
$$

- For a uniformly polarized dielectric, the field within a long, uniformly polarized cylinder is generally uniform and can be obtained from boundary conditions and known electrostatics results.

- It is known that a uniformly polarized infinite dielectric medium has an internal field:

$$
\mathbf{E}_{\text{inside}} = - \frac{\mathbf{P}}{\varepsilon_0}
$$

- However, this applies in the absence of boundary effects; in our case, the surface bound charges modify the field near the surface.

Field Outside the Cylinder

- The external field resembles that of a dipole sheet with a dipole moment per unit area:

$$
\mathbf{m} = \mathbf{P} \times \text{thickness (or length in this case)}
$$

- Since the cylinder is infinitely long, the field at points outside is primarily due to the surface dipole layer.

- The external electric field at a point far away (say, in the plane perpendicular to $\mathbf{P}$) behaves like that of an infinite sheet of dipoles, with the field:

$$
\mathbf{E}_{\text{outside}} = \frac{\mathbf{P}}{\varepsilon_0}
$$

- The direction of the field outside depends on the orientation of $\mathbf{P}$, and for the polarization perpendicular to the axis, the field is transverse to the cylinder.

Potential and Field Calculations

The detailed calculations involve advanced methods, but key results are:

- Inside the cylinder:

$$
\mathbf{E}_{\text{inside}} \approx -\frac{\mathbf{P}}{\varepsilon_0}
$$

- Outside the cylinder:

$$
\mathbf{E}_{\text{outside}} \approx \frac{\mathbf{P}}{\varepsilon_0}
$$

- These approximate results are valid for an infinitely long cylinder, neglecting edge effects.

Physical Interpretation and Applications

Understanding the behavior of a polarized cylinder has several practical and theoretical implications:

- It exemplifies how boundary conditions influence the internal and external fields in dielectric materials.

- It demonstrates the role of bound charges in shaping the electrostatic environment.
- Applications include designing polarized dielectric components, understanding electrostatic shielding, and analyzing the behavior of materials in external fields.

Summary of Key Results

- The polarization $\mathbf{P}$ perpendicular to the cylinder's axis induces surface bound charges with a $\cos \theta$ distribution.
- No volume bound charges exist inside the cylinder due to uniform polarization.
- The electric field inside the cylinder is approximately uniform and directed opposite to $\mathbf{P}$, given by $-\mathbf{P}/\varepsilon_0$.
- The external field resembles that of a dipole sheet, with magnitude $\mathbf{P}/\varepsilon_0$.

Pros and Cons / Features Summary

Features:

- Analytical simplicity due to symmetry and uniform polarization.
- Clear boundary conditions facilitate

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