? Question The Point (x, Y) Is First Rotated 180 Clockwise About The Origin, Translated 6 Units To The

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Understanding transformations in coordinate geometry is fundamental to grasping how figures move within the plane. A common type of problem involves applying multiple transformations sequentially—such as rotations, translations, reflections, and dilations—to a point or a figure. This article explores a specific transformation sequence involving a 180-degree clockwise rotation about the origin, followed by a translation of 6 units. We will analyze each step thoroughly, derive the general formulas, and illustrate how to apply these transformations to any given point (x, y).

Understanding the Transformation Sequence

The Overall Transformation

The problem involves two key transformations applied to a point with coordinates (x, y):

1. Rotation 180° clockwise about the origin
2. Translation 6 units to the right (or along the x-axis)

The sequence of operations is significant because the order affects the final position of the point. First, the point is rotated, then translated.

Rotation of a Point About the Origin

Basics of Rotation

Rotation about the origin involves turning a point around the coordinate plane's origin (0, 0) by a specified angle. The rotation can be clockwise or counterclockwise, with the standard convention being:

- Counterclockwise rotation: positive angles
- Clockwise rotation: negative angles

In our case, the rotation is described as 180° clockwise. Since a 180° rotation is symmetric, rotating clockwise or counterclockwise by 180° yields the same final position.

Rotation Formula for 180°

The general rotation of a point (x, y) about the origin by an angle θ in the counterclockwise direction is given by:

$$
\begin{cases}
x' = x \cos \theta - y \sin \theta \\
y' = x \sin \theta + y \cos \theta
\end{cases}
$$

For $\theta = 180°$, the cosine and sine values are:

$$
\cos 180° = -1, \quad \sin 180° = 0
$$

Thus, the rotation formulas simplify to:

$$
x' = x \times (-1) - y \times 0 = -x
$$

$$
y' = x \times 0 + y \times (-1) = -y
$$

Therefore, rotating a point (x, y) by 180° clockwise about the origin results in:

$$
\boxed{
(x', y') = (-x, -y)
}
$$

Note: Since 180° rotation is symmetric about the origin, clockwise or counterclockwise rotation produces the same result.

Applying Rotation to the Point (x, y)

Step 1: Rotate 180° Clockwise

Given the point (x, y), after rotation:

$$
(x_1, y_1) = (-x, -y)
$$

This transformation effectively maps the point to its opposite across both axes.

Translation of the Rotated Point

Understanding Translation

Translation involves shifting a point by a certain distance along one or more axes. In this case, the translation is specified as 6 units, which typically implies moving the point along the x-axis unless otherwise specified.

Assuming the translation is 6 units to the right (positive x-direction), the translation can be expressed as:

$$
x_2 = x_1 + 6
$$

$$
y_2 = y_1
$$

Putting the rotation and translation together, the overall transformation is:

$$
x_2 = -x + 6
$$

$$
y_2 = -y
$$

Final Coordinates of the Transformed Point

Complete Transformation Formula

Starting from the original point (x, y), the final coordinates after both transformations are:

$$
\boxed{
\text{Transformed Point} = \left(-x + 6, \, -y \right)
}
$$

This formula allows us to find the position of any point after applying the sequence: 180° rotation clockwise about the origin, followed by a 6-unit translation to the right.

Visualizing the Transformation

Geometric Interpretation

- The initial rotation of 180° about the origin effectively flips the point to the opposite side of the origin, mirroring it across both axes.
- The subsequent translation shifts the entire point 6 units to the right, moving it horizontally without affecting its vertical position.

Graphical Steps

1. Original Point: (x, y)
2. After Rotation: (-x, -y)
3. After Translation: (-x + 6, -y)

Visualizing these steps helps in understanding how the point's position changes.

Examples of Transformation

Example 1: Point (2, 3)

- Rotation: (-2, -3)
- Translation: (-2 + 6, -3) = (4, -3)

Final position: (4, -3)

Example 2: Point (-4, 5)

- Rotation: (4, -5)
- Translation: (4 + 6, -5) = (10, -5)

Final position: (10, -5)

Example 3: Point (0, 0)

- Rotation: (0, 0)
- Translation: (0 + 6, 0) = (6, 0)

Final position: (6, 0)

Extensions and Variations

Different Translation Distances

If the translation is not 6 units but another value, say d, the formula adjusts to:

$$
x' = -x + d
$$
$$
y' = -y
$$

This generalizes the process for any horizontal translation.

Vertical Translations

Suppose the translation is along the y-axis by k units instead of x-axis. The formulas become:

$$
x' = -x
$$

$$
y' = -y + k
$$

Understanding these variations broadens the applicability of the transformation process.

Summary and Key Takeaways

- The rotation of a point (x, y) by 180° clockwise about the origin results in (-x, -y).
- A translation of 6 units to the right modifies the x-coordinate by adding 6, leaving the y-coordinate unchanged.
- The combined transformation yields the final coordinates: $\left(-x + 6, -y \right)$.
- The order of transformations is crucial; performing rotation first, then translation, leads to the derived formula.
- This process can be extended to different translation distances and directions, highlighting the versatility of coordinate transformations.

Conclusion

Transformations in coordinate geometry serve as powerful tools for understanding how figures move within the plane. The specific sequence of rotating a point 180° clockwise about the origin, then translating it 6 units to the right, demonstrates the importance of understanding both the individual transformations and their order. With the derived formulas, one can easily compute the final position of any point subjected to these operations, facilitating problem-solving in geometry, computer graphics, physics, and engineering applications.

By mastering these fundamental transformations, students and professionals can analyze complex geometric manipulations with confidence and precision.

Frequently Asked Questions

What are the steps to rotate a point 180 degrees clockwise about the origin?

To rotate a point 180 degrees clockwise about the origin, you can multiply both the x and y coordinates by -1, effectively reflecting the point across the origin.

How do you translate a point after rotating it 180 degrees clockwise around the origin?

After rotating the point, add 6 units to the x-coordinate (or y-coordinate, depending on the translation direction), to translate it accordingly.

If a point (x, y) is rotated 180 degrees clockwise about the origin, what is its new coordinate?

The new coordinate will be (-x, -y).

What is the combined effect of rotating a point 180 degrees and then translating it?

First, rotate the point by multiplying both coordinates by -1, then add the translation units to the desired coordinate axis (e.g., add 6 to x), resulting in the final position.

How do you perform a translation of 6 units to the right on a point's coordinates?

Add 6 to the x-coordinate of the point, so the new point becomes (x + 6, y).

Can you provide an example of rotating (x, y) 180 degrees clockwise and then translating 6 units to the right?

Yes. For example, starting with (2, 3), rotate 180 degrees to get (-2, -3), then translate 6 units to the right to get (4, -3).

What is the formula for the final coordinates after rotation and translation?

If the original point is (x, y), after rotation 180 degrees the point is (-x, -y). After translating 6 units to the right, the final point is (-x + 6, -y).

What are common mistakes to avoid when performing rotation and translation on a point?

Common mistakes include forgetting to invert both coordinates during rotation or incorrectly applying the translation after rotation. Always perform operations in the correct order and double-check calculations.

How does rotation 180 degrees clockwise about the origin affect the point's quadrant location?

Rotating 180 degrees about the origin moves a point to the opposite quadrant; for example, a point in Quadrant I moves to Quadrant III, and vice versa.

Is the order of operations important when rotating and translating a point?

Yes, the order is important. Typically, you should rotate first, then translate, to achieve the correct final position.

Additional Resources

Question: The Point (x, y) Is First Rotated 180° Clockwise About the Origin, Then Translated 6 Units to the…

Understanding the transformations of points in the coordinate plane is fundamental in fields such as geometry, computer graphics, robotics, and engineering. When a point undergoes multiple transformations—such as rotation and translation—it’s important to analyze each step carefully to determine the final position. In this guide, we will explore the process of transforming a point (x, y) through a 180° clockwise rotation about the origin, followed by a translation of 6 units in a specified direction. This comprehensive analysis will help you grasp the mechanics of these transformations and enable you to apply the concepts to various problems.

Introduction to Coordinate Transformations

Transformations are operations that alter the position or orientation of points in the coordinate plane. The most common transformations include:

- Rotation: Turning a point around a fixed point (typically the origin) by a certain angle.
- Translation: Moving a point by a specified distance in a particular direction.
- Reflection: Flipping a point across a line.
- Scaling: Resizing a point’s distance from a fixed point by a scale factor.

In our case, the focus is on rotation and translation, with the rotation specified as 180° clockwise about the origin. It’s crucial to understand how these transformations are performed sequentially and how they combine to produce the final position of the point.

Step 1: Understanding Rotation About the Origin

Rotation Fundamentals

Rotation of a point (x, y) about the origin by an angle θ (counterclockwise) can be described using the following formulas:

- Counterclockwise rotation by θ:

```
x' = x cos(θ) - y sin(θ)
y' = x sin(θ) + y cos(θ)
```

Rotation in the Clockwise Direction

Since the problem specifies a clockwise rotation, note that rotating clockwise by θ is equivalent to rotating counterclockwise by -θ. Therefore, the formulas become:

- Clockwise rotation by θ:

```
x' = x cos(-θ) - y sin(-θ)
y' = x sin(-θ) + y cos(-θ)
```

Using the identities:

- cos(-θ) = cos(θ)
- sin(-θ) = -sin(θ)

We simplify to:

```
x' = x cos(θ) + y sin(θ)
y' = -x sin(θ) + y cos(θ)
```

Applying 180° Rotation

For θ = 180°, the trigonometric values are:

- cos(180°) = -1
- sin(180°) = 0

Plugging these into the formulas:

```
x' = x (-1) + y 0 = -x
y' = -x 0 + y (-1) = -y
```

Result:

After a 180° clockwise rotation about the origin, the point (x, y) becomes (-x, -y).

Step 2: Understanding Translation

What is Translation?

Translation involves shifting all points in the plane by a certain distance in a specified direction. It is expressed as:

- (x, y) → (x + dx, y + dy)

where (dx, dy) are the horizontal and vertical displacements.

Translating by 6 Units: Clarification Needed

The problem states: "Translated 6 Units To The...". Typically, such phrasing indicates a translation along a particular direction, such as:

- 6 units to the right (positive x-direction)
- 6 units upward (positive y-direction)
- 6 units in an arbitrary direction

Since the direction isn't specified in the prompt, common assumptions include:

- Translation 6 units to the right (most straightforward)
- Translation 6 units upwards
- Translation along a specific vector

For this guide, we will analyze the most common case: translation 6 units to the right. If you have a different direction in mind, you can adapt the steps accordingly.

Step 3: Combining the Transformations

Sequential Transformation Process

The order in which transformations are applied is crucial. The problem indicates:

1. Rotate 180° clockwise about the origin
2. Translate 6 units to the right

This sequence implies:

- First, perform the rotation on (x, y)
- Then, translate the resulting point

Applying the Rotation

From the previous section, after rotation:

```
(x, y) → (-x, -y)
```

Applying the Translation

Assuming translation 6 units to the right, the translation vector is (6, 0).

Thus, the final coordinates after translation:

```
Final x-coordinate: -x + 6
Final y-coordinate: -y + 0 = -y
```

Final position of the point:

$(-x + 6, -y)$

Summary of the Transformation

Step	Description	Formula	Resulting Coordinates
1	Rotate 180° clockwise about the origin	(x, y) → (-x, -y)	(-x, -y)
2	Translate 6 units to the right	(-x + 6, -y)	(-x + 6, -y)

Practical Example: Applying the Transformation

Suppose the original point is (4, 3).

Step 1: Rotation

```
(4, 3) → (-4, -3)
```

Step 2: Translation 6 units to the right

```
(-4 + 6, -3) → (2, -3)
```

Final position: (2, -3)

This example illustrates how the transformations work step-by-step.

Additional Considerations and Variations

Translating in Different Directions

If the translation is in a different direction, adjust the translation vector accordingly:

- Upward: (0, +6)
- Leftward: (-6, 0)
- Downward: (0, -6)
- Along a vector: (a, b)

Generalized Formula

For a point (x, y) undergoing:

1. Rotation θ clockwise about the origin:
```
(x, y) → (x', y') = (x cos(θ) + y sin(θ), -x sin(θ) + y cos(θ))
```
2. Translation by vector (dx, dy):
```
(x', y') → (x' + dx, y' + dy)
```

Applying to Other Angles or Distances

Use the same formulas with different θ or translation vectors to analyze various transformations.

Final Thoughts and Tips

- Always confirm the direction of rotation and the angle. Remember that clockwise rotations can be converted to negative angles in formulas.
- Keep track of the order of operations. Applying rotation before translation is different from applying translation before rotation.
- Use unit vectors and basic trigonometry to adapt these steps for rotations around points other than the origin.
- Practice with different points and transformations to build intuition and confidence.

Conclusion

Transforming points through rotations and translations is a fundamental skill in coordinate geometry. In the case of the point (x, y) being rotated 180° clockwise about the origin and then translated 6 units to the right, the process simplifies to:

- Rotation: (x, y) → (-x, -y)
- Translation: (-x + 6, -y)

Understanding each step thoroughly allows for accurate problem-solving and lays the groundwork for tackling more complex transformations in geometry and related fields. Whether you're working on geometric proofs, computer graphics, or robotics, mastering these transformations is essential for precise spatial manipulation.

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