An Annuity Pays $12,000 Every Year For 5 Years With The First Payment At The End Of Year 4. Given A 13

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Understanding financial products such as annuities is essential for effective retirement planning and wealth management. In this article, we will explore a specific scenario: an annuity that pays $12,000 annually over five years, with the first payment occurring at the end of Year 4, given a discount rate of 13%. This case involves complex time value of money calculations, and grasping it thoroughly will help you make informed investment decisions.

What Is an Annuity and How Does It Work?

An annuity is a series of equal payments made at regular intervals over a specified period. It is a common financial instrument used for retirement income, insurance, and investment purposes.

Types of Annuities

- **Immediate Annuities:** Payments start immediately after the initial investment.
- **Deferred Annuities:** Payments start after a certain accumulation period.
- **Fixed Annuities:** Payments are fixed and predictable.
- **Variable Annuities:** Payments vary based on investment performance.

In our scenario, the annuity involves fixed payments starting after a delay, making it a deferred annuity.

Understanding the Specific Scenario

The key details are:

1. Payment amount: $12,000 per year
2. Number of payments: 5
3. Payment timing: First payment at the end of Year 4
4. Discount rate (interest rate): 13%

This setup implies that payments are deferred and begin after a certain period, which impacts their present value significantly.

Calculating the Present Value of the Annuity

The core concept here is to determine the present value (PV) of the series of future payments, considering the time value of money at a 13% discount rate.

Step 1: Recognize the Timing of Payments

- Payments occur annually at the end of Years 4 through 8.
- The first payment occurs at the end of Year 4.
- The last payment occurs at the end of Year 8.

This means the series of payments is:

Payment Number	Payment Year	Timing
1	Year 4	End of Year 4
2	Year 5	End of Year 5
3	Year 6	End of Year 6
4	Year 7	End of Year 7

| 5 | Year 8 | End of Year 8 |

Step 2: Find the Present Value at the Time of the First Payment

Since payments start at Year 4, the present value of these payments as of Year 3 (just before the first payment at Year 4) can be calculated as the present value of an ordinary annuity with 5 payments of $12,000 each, discounted back five years (from Year 4 to Year 3).

The present value of an ordinary annuity formula:

$$
PV_{annuity} = P \times \frac{1 - (1 + r)^{-n}}{r}
$$

Where:
- $P = \$12,000$
- $r = 13\% = 0.13$
- $n = 5$

Calculating:

$$
PV_{Year\,3} = 12,000 \times \frac{1 - (1 + 0.13)^{-5}}{0.13}
$$

Step 3: Calculate the Discount Factor

First, compute $(1 + 0.13)^{-5}$:

$$
(1 + 0.13)^5 = 1.13^5
$$

Calculating 1.13^5:

- $1.13^1 = 1.13$
- $1.13^2 = 1.13 \times 1.13 = 1.2769$
- $1.13^3 = 1.2769 \times 1.13 \approx 1.4419$

- $1.13^4 = 1.4419 \times 1.13 \approx 1.6294$
- $1.13^5 = 1.6294 \times 1.13 \approx 1.8412$

Now, $(1 + 0.13)^{-5} = 1 / 1.8412 \approx 0.5429$.

Plug into the annuity formula:

$$
PV_{Year\,3} = 12,000 \times \frac{1 - 0.5429}{0.13} = 12,000 \times \frac{0.4571}{0.13}
$$

Calculate numerator:

$$
0.4571 / 0.13 \approx 3.5177
$$

Finally:

$$
PV_{Year\,3} = 12,000 \times 3.5177 \approx \$42,212.40
$$

This is the value at the end of Year 3.

Step 4: Discount the PV Back to Present (Year 0)

To find the present value today (at Year 0), discount this amount back three years (from Year 3 to Year 0):

$$
PV_{Year\,0} = PV_{Year\,3} \times (1 + r)^{-3}
$$

Calculate $(1 + 0.13)^3$:

- $1.13^3 = 1.4419$ (from earlier calculations)

So,

$$
\[

$$
PV_{Year\,0} = 42,212.40 \times \frac{1}{1.13^3} = 42,212.40 \times \frac{1}{1.4419} \approx 42,212.40 \times 0.6936 \approx \$29,278.66
$$

Summary of the Calculation

- The present value of the series of five $12,000 payments starting at the end of Year 4 is approximately \$29,279.

- This value represents the worth today of receiving these payments, considering a 13% discount rate.

Additional Considerations

Impact of Discount Rate

- The higher the discount rate, the lower the present value.
- At 13%, future payments are heavily discounted, reflecting higher opportunity costs or risk.

Alternatives and Uses

- Investors can compare this present value to other investment opportunities.
- Financial planners may use this calculation to determine fair value or to structure similar annuities.

Extensions and Variations

- Adjusting the number of payments or the timing affects PV.
- Changing the discount rate impacts valuation significantly.
- One can also calculate the future value of such an annuity for planning purposes.

Conclusion

In summary, when analyzing an annuity that pays $12,000 annually over five years starting at the end of Year 4 with a 13% discount rate, the present value is approximately \$29,279. This calculation involves understanding the timing of payments, applying the present value of an ordinary annuity formula, and discounting back to the present. Such calculations are essential for evaluating investment opportunities, designing retirement plans, and understanding the true worth of deferred income streams. With careful consideration of the discount rate and payment schedule, investors and financial professionals can make more informed and strategic financial decisions.

Frequently Asked Questions

What is the total amount paid by the annuity over the 5-year period?

The total amount paid is $12,000 per year for 5 years, totaling $60,000.

Why does the first payment of the annuity occur at the end of Year 4 instead of Year 1?

This is an example of a deferred annuity, where payments begin after a specified deferral period, in this case starting at the end of Year 4.

Given a 13% discount rate, how do we calculate the present value of this annuity?

The present value is calculated by discounting each of the future payments back to the present using the 13% rate, considering the deferred start at Year 4.

What is the present value of the annuity payments starting at the end of Year 4 with a 13% discount rate?

The present value can be calculated as the sum of the discounted payments, which involves discounting the 5 payments of $12,000 each back to the present, starting from the end of Year 4.

How does the delay of payments until Year 4 impact the overall valuation of the annuity?

The delay reduces the present value because payments are received later, and the discounting effect diminishes their current worth compared to immediate payments.

What is the significance of the discount rate (13%) in valuing this annuity?

The discount rate reflects the time value of money and risk associated with the payments; a higher rate decreases the present value of future payments.

Can this annuity be considered a typical ordinary annuity? Why or why not?

No, because the payments start at the end of Year 4, making it a deferred or delayed annuity rather than a standard ordinary annuity, which begins payments immediately at the end of the first period.

Additional Resources

Annuity Planning and Valuation: A Deep Dive into a Unique 5-Year Annuity with Deferred Payments at a 13% Discount Rate

When evaluating investment opportunities, retirement income strategies, or insurance products, understanding the nuances of different annuity structures is essential. One intriguing case involves an annuity that pays $12,000 annually over five years, with the first payment deferred until the end of Year 4. This setup is further complicated by a high discount rate of 13%, which significantly impacts the present value and overall attractiveness of the product. In this comprehensive analysis, we will explore the mechanics, valuation, and implications of such an annuity, providing clarity for financial planners, investors, and consumers alike.

Understanding the Structure of the Annuity

Before delving into calculations, it's crucial to clarify the specific features of this annuity:

- Payment Amount: $12,000 annually
- Number of Payments: 5 payments in total
- Timing of Payments: Payments occur at the end of each period, with the first payment at the end of Year 4 and the last at the end of Year 8
- Payment Schedule:
- Payment 1: End of Year 4
- Payment 2: End of Year 5
- Payment 3: End of Year 6

- Payment 4: End of Year 7
- Payment 5: End of Year 8
- Discount Rate: 13% per annum (compounded annually)

This structure is somewhat unconventional because most annuities start payments immediately or after a short deferment. Here, the payments are heavily deferred, beginning only at the close of Year 4, which impacts the present value significantly.

Key Concepts in Valuing Deferred Annuities

To accurately value this annuity, several core financial concepts must be understood:

1. Present Value (PV):

The current worth of future cash flows, discounted at an appropriate rate. The higher the discount rate, the lower the present value of future payments.

2. Discount Rate:

A critical factor in valuation. Here, it is 13%, which reflects a relatively high risk environment or a market with significant opportunity costs.

3. Deferred Annuities:

An annuity where payments begin after a delay. These are valued by discounting the future payments back to the present, considering the deferral period.

4. Discounting Future Payments:

To determine the present value of each payment, we discount it by the number of periods until it occurs:
$$
PV = \frac{Payment}{(1 + r)^n}
$$
where:
- $ r $ = discount rate (13% or 0.13)
- $ n $ = number of periods until the payment

Step-by-Step Valuation of the Annuity

Let's proceed systematically to calculate the present value (PV) of this specific annuity.

Step 1: Identify Payment Timing and Discount Factors

Payment Number	Payment at End of Year	Year of Payment	Discount Factor $\frac{1}{(1 + r)^n}$
1	$12,000	Year 4	$\frac{1}{(1 + 0.13)^4}$
2	$12,000	Year 5	$\frac{1}{(1 + 0.13)^5}$
3	$12,000	Year 6	$\frac{1}{(1 + 0.13)^6}$
4	$12,000	Year 7	$\frac{1}{(1 + 0.13)^7}$
5	$12,000	Year 8	$\frac{1}{(1 + 0.13)^8}$

Step 2: Calculate Discount Factors

Using $r = 0.13$:

- Year 4: $(1 + 0.13)^4 = 1.13^4$

Let's compute each:

$$
1.13^4 \approx 1.13 \times 1.13 \times 1.13 \times 1.13 \approx 1.13 \times 1.13 = 1.2769
$$

Next:

$$
1.2769 \times 1.13 \approx 1.4419
$$

Finally:

$$
1.4419 \times 1.13 \approx 1.6293
$$

Thus,

$$
PV \text{ factor at Year 4} \approx \frac{1}{1.6293} \approx 0.6138
$$

Similarly,

- Year 5: $1.13^5 \approx 1.6293 \times 1.13 \approx 1.842$

$$
PV \text{ factor at Year 5} \approx \frac{1}{1.842} \approx 0.5429
$$

- Year 6: $1.13^6 \approx 1.842 \times 1.13 \approx 2.082$

$$
PV \text{ factor at Year 6} \approx \frac{1}{2.082} \approx 0.4802
$$

- Year 7: $1.13^7 \approx 2.082 \times 1.13 \approx 2.351$

$$
PV \text{ factor at Year 7} \approx \frac{1}{2.351} \approx 0.4252
$$

- Year 8: $1.13^8 \approx 2.351 \times 1.13 \approx 2.656$

$$
PV \text{ factor at Year 8} \approx \frac{1}{2.656} \approx 0.3768
$$

Step 3: Calculate Present Values of Each Payment

Multiply each payment by its respective discount factor:

Payment Number	Year	Discount Factor	PV of Payment (\$12,000)
1	4	0.6138	\$12,000 $\times 0.6138 \approx$ \$7,365.60
2	5	0.5429	\$12,000 $\times 0.5429 \approx$ \$6,514.80
3	6	0.4802	\$12,000 $\times 0.4802 \approx$ \$5,762.40
4	7	0.4252	\$12,000 $\times 0.4252 \approx$ \$5,102.40
5	8	0.3768	\$12,000 $\times 0.3768 \approx$ \$4,521.60

Calculating the Total Present Value

Summing the PVs of all five payments:

$$
PV_{total} = 7,365.60 + 6,514.80 + 5,762.40 + 5,102.40 + 4,521.60
$$

$$
PV_{total} \approx \$29,266.20
$$

Final Note: Since the first payment occurs at the end of Year 4, the entire annuity's present value as of Year 0 (today) is approximately \$29,266.

Implications of the High Discount Rate

A 13% discount rate is notably high, often indicative of:

- Elevated investment risk
- Inflation expectations
- Market conditions with higher opportunity costs
- Strategic valuation assumptions for risk adjustment

This high rate significantly reduces the present value of future cash flows, making the annuity less attractive from a valuation perspective compared to a lower rate scenario.

Impact on Investment Decisions

- Lower Present Value: The high discount rate diminishes the attractiveness of the deferred payments, which might influence pricing, underwriting, or consumer decisions.
- Risk Premiums: Sellers or insurers might require higher premiums or interest to compensate for the risk reflected in the 13% rate.
- Comparative Analysis: When comparing this annuity to alternatives, adjustments for risk and time preference are essential.

Additional Considerations and Variations

Understanding the core valuation model opens avenues to explore other aspects:

- Sensitivity Analysis: How does changing the discount rate affect PV? For example, at 10%, the PV would be higher.
- Alternative Payment Structures: What if payments began immediately or at different intervals?
- Inflation Adjustments: Incorporating inflation can modify the real value of payments and discount rates.
- Tax Implications: Taxation can influence the net benefit of such an annuity.
- Inflows and Outflows: Evaluating the initial investment required to purchase such an annuity versus the present value of

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