An Atom Moving At Its Root-mean-square Speed At 20 Oc Has A Wavelength Of 3.28 X 10-11 M. Identify The

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Understanding atomic motion and wave behavior is fundamental in quantum mechanics and atomic physics. When an atom moves at its root-mean-square (RMS) speed at a given temperature, it exhibits wave-like properties described by de Broglie wavelength. The given wavelength of 3.28 x 10^-11 meters provides insight into the atom's characteristics, including its mass, velocity, and quantum behavior. This article delves into the physics behind this phenomenon, explaining how to identify the atom based on its wavelength and speed, and exploring the principles of de Broglie wavelength, thermal motion, and atomic properties.

Introduction to Atomic Wave-Particle Duality

The concept of wave-particle duality is central in quantum physics, asserting that particles such as atoms and electrons exhibit both particle-like and wave-like behavior. Louis de Broglie famously proposed that matter particles have associated wavelengths, now known as de Broglie wavelengths, which depend on their momentum.

Key points:

- Particles exhibit wave-like behavior at small scales.
- De Broglie wavelength (λ) relates to the particle's momentum (p): λ = h / p.
- This principle allows us to analyze atomic motion in terms of wave properties.

Understanding Root-Mean-Square Speed in Atomic Context

The root-mean-square speed (v_rms) is a statistical measure of the velocity of particles in a gas, representing the square root of the average of the squares of their speeds. It reflects the typical speed of atoms or molecules at a given temperature, and is derived from kinetic theory.

Formula for RMS speed:

$$
v_{rms} = \sqrt{\frac{3k_B T}{m}}
$$

Where:

- k_B is Boltzmann's constant ($1.38 \times 10^{-23} \mathrm{J/K}$)
- T is temperature in Kelvin
- m is the mass of the atom in kilograms

At 20°C (which is 293 K), the RMS speed provides an estimate of the typical atomic velocity.

De Broglie Wavelength and Atomic Mass

Given the de Broglie wavelength of an atom, we can identify the atom by calculating its momentum and, consequently, its mass or velocity.

De Broglie wavelength formula:

$$
\lambda = \frac{h}{p} = \frac{h}{m v}
$$

Where:

- h is Planck's constant ($6.626 \times 10^{-34} \mathrm{Js}$)
- p is momentum
- m is mass
- v is velocity

Given the wavelength, if we know the velocity (or RMS speed), we can find the mass of the atom.

Calculating the Atom's Speed and Mass

Given data:

- Wavelength, $\lambda = 3.28 \times 10^{-11}$ meters
- Temperature, $T = 20^\circ C = 293\,K$

Step 1: Express RMS speed in terms of mass

From the RMS speed formula:

$$
v_{rms} = \sqrt{\frac{3k_B T}{m}}
$$

Step 2: Use the de Broglie relation to find momentum

$$
p = \frac{h}{\lambda}
$$

Step 3: Determine the velocity

Assuming the atom's speed is approximately equal to RMS speed,

$$
v = \frac{p}{m} = \frac{h}{m \lambda}
$$

Step 4: Combine equations

From the RMS speed formula and the de Broglie relation, rearranged:

$$
v = \sqrt{\frac{3k_B T}{m}}
$$

Since v can also be expressed as:

$$
v = \frac{h}{m \lambda}
$$

Squaring both sides:

$$
v^2 = \frac{h^2}{m^2 \lambda^2}
$$

and from RMS:

$$
v^2 = \frac{3k_B T}{m}
$$

Set equal:

$$
\frac{h^2}{m^2 \lambda^2} = \frac{3k_B T}{m}
$$

Multiply both sides by $m^2 \lambda^2$:

$$
h^2 = 3k_B T m \lambda^2
$$

Solve for m:

$$
m = \frac{h^2}{3k_B T \lambda^2}
$$

Plugging in the values:

$$
m = \frac{(6.626 \times 10^{-34})^2}{3 \times 1.38 \times 10^{-23} \times 293 \times (3.28 \times 10^{-11})^2}
$$

Calculations:

- Numerator: $(6.626 \times 10^{-34})^2 = 4.39 \times 10^{-67}$
- Denominator:

$$
3 \times 1.38 \times 10^{-23} \times 293 \times (3.28 \times 10^{-11})^2
$$

Calculate step-by-step:

- $3 \times 1.38 \times 10^{-23} = 4.14 \times 10^{-23}$
- $4.14 \times 10^{-23} \times 293 = 1.213 \times 10^{-20}$
- $(3.28 \times 10^{-11})^2 = 10.75 \times 10^{-22}$

Now multiply:

$$
1.213 \times 10^{-20} \times 10.75 \times 10^{-22} = 1.213 \times 10.75 \times 10^{-42} \approx 13.05 \times 10^{-42} = 1.305 \times 10^{-41}
$$

Finally, compute:

$$
m = \frac{4.39 \times 10^{-67}}{1.305 \times 10^{-41}} \approx 3.36 \times
$$

$$
10^{-26} \, \mathrm{kg}
$$

Conclusion:

The atomic mass $m \approx 3.36 \times 10^{-26}$ kg.

Identifying the Atom Based on Mass

The approximate mass of the atom corresponds closely to the mass of a hydrogen isotope:

- Hydrogen atom: $1.0078\, \mathrm{amu}$
- Mass in kg: $1.0078\, \mathrm{amu} \times 1.6605 \times 10^{-27}\, \mathrm{kg/amu} \approx 1.673 \times 10^{-27}\, \mathrm{kg}$

Our calculated mass is about 3.36×10^{-26} kg, which is roughly 20 times the mass of a hydrogen atom. Therefore, the atom is likely a heavier element.

Estimate:

$$
\frac{3.36 \times 10^{-26}}{1.673 \times 10^{-27}} \approx 20.1
$$

This indicates the atom is approximately 20 times heavier than hydrogen, suggesting it could be:

- Neon (Ne): atomic mass ~20 amu
- Neon atom: $20 \times 1.6605 \times 10^{-27} \approx 3.32 \times 10^{-26}$ kg

Hence, the atom is most likely a neon atom.

Understanding the Significance of de Broglie Wavelength of 3.28 x 10^-11 M

The de Broglie wavelength of approximately 3.28×10^{-11} meters falls within the scale of atomic dimensions, consistent with the size of electron orbitals and interatomic distances. This wavelength indicates the wave-like nature of the neon atom at thermal velocities, relevant in fields such as quantum mechanics, spectroscopy, and nanotechnology.

Implications include:

- Validating quantum behavior of atoms at room temperature.
- Understanding atomic diffraction and interference experiments.
- Designing nanoscale devices where atomic wave properties are significant.

Summary and Key Takeaways

This exploration into the de Broglie wavelength and atomic motion reveals several key insights:

1. De Broglie wavelength relates the wave nature of matter to its momentum.
2. Root-mean-square speed provides an estimate of the atom's velocity at a given temperature.
3. Calculations suggest that an atom with a wavelength of \(3.28 \times 10^{-11}\) meters moving at RMS speed at 20°C is likely a neon atom.
4. Atomic mass determination relies on combining quantum formulas with thermodynamic data.
5. Wave-particle duality plays a crucial role in understanding atomic behavior in quantum physics.

Conclusion

The analysis of an atom moving at its RMS speed at 20°C with a wavelength of \(3.28 \times 10^{-11}\) meters

Frequently Asked Questions

What is the significance of the root-mean-square (RMS) speed in the context of atomic motion?

The RMS speed represents the average speed of atoms in a sample, indicating their thermal energy and kinetic activity at a given temperature, such as 20°C.

How is the de Broglie wavelength related to an atom's momentum?

The de Broglie wavelength is inversely proportional to an atom's momentum, given by $\lambda = h / p$, where h is Planck's constant.

Given the wavelength of 3.28 x 10^-11 meters, how can we determine the atom's RMS speed?

Using the de Broglie relation $\lambda = h / (m v)$, rearranged to $v = h / (m \lambda)$, where m is the atomic mass; this allows calculation of the RMS speed from the wavelength.

Which atomic species is likely to have a de Broglie wavelength of approximately 3.28 x 10^-11 meters at 20°C?

A hydrogen atom or a light element's atom, since lighter atoms have longer de Broglie wavelengths at given speeds.

Why is understanding the wavelength of atoms important in quantum mechanics?

Because it illustrates wave-particle duality, showing that atoms exhibit wave-like properties that influence behaviors at microscopic scales.

How does temperature (20°C) affect the RMS speed of an atom?

Higher temperatures increase the average kinetic energy and RMS speed of atoms, following the relation between temperature and thermal motion.

What formula can be used to identify the atom based on its de Broglie wavelength and RMS speed?

Using $\lambda = h / (m v)$, where λ is the wavelength, h is Planck's constant, m is atomic mass, and v is RMS speed, to solve for the atom's identity or properties.

Additional Resources

An Atom Moving At Its Root-mean-square Speed At 20 °C Has A Wavelength Of 3.28×10^{-11} m. Identify The

Understanding the behavior of atoms on a microscopic scale often involves exploring their wave-like properties. When an atom moves at its root-mean-square (rms) speed at 20 °C, and this motion corresponds to a specific wavelength—here, 3.28×10^{-11} meters—it invites a detailed analysis of quantum mechanics, thermodynamics, and atomic physics. This article delves into the significance of this wavelength, how it relates to the atom's physical parameters, and what this information reveals about the atom's identity, state, and behavior.

Introduction

The statement "An atom moving at its root-mean-square speed at 20 °C has a wavelength of 3.28×10^{-11} m" encapsulates a fascinating intersection of classical and quantum physics. It prompts us to ask:

- What does the wavelength tell us about the atom?
- How does the atom's velocity relate to its mass?
- Can we identify the atom based on this wavelength and temperature?
- What implications does this have for understanding atomic motion and wave-particle duality?

In this guide, we will systematically explore these questions, starting from fundamental principles and progressing to more complex interpretations.

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1. Fundamental Concepts

1.1 Wave-Particle Duality

At the core of quantum mechanics lies the wave-particle duality, which states that particles such as atoms and electrons exhibit both particle-like and wave-like properties. The de Broglie wavelength (λ) describes this wave aspect and is given by:

$$ \lambda = \frac{h}{p} $$

where:

- h is Planck's constant ($6.626 \times 10^{-34}\, \mathrm{Js}$)
- p is the momentum of the particle

This relationship ties the microscopic motion of particles directly to their wave characteristics.

1.2 Root-Mean-Square Speed

The root-mean-square speed v_{rms} is a measure of the average speed of particles in a system at thermal equilibrium, expressed by:

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

where:

- k_B is Boltzmann's constant ($1.381 \times 10^{-23}\, \mathrm{J/K}$)
- T is the absolute temperature (Kelvin)
- m is the mass of the particle (kg)

This parameter is crucial in thermodynamics and kinetic theory, providing a link between temperature and molecular speed.

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2. Data and Known Values

To analyze the problem, let's compile the necessary known data:

Parameter	Value	Description
Wavelength (λ)	$3.28 \times 10^{-11}\, \mathrm{m}$	Given
Temperature (T)	20 °C	Convert to Kelvin: $20 + 273.15 = 293.15\, \mathrm{K}$
Planck's constant h	$6.626 \times 10^{-34}\, \mathrm{Js}$	Fundamental constant
Boltzmann's constant k_B	$1.381 \times 10^{-23}\, \mathrm{J/K}$	Fundamental constant

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3. Step-by-Step Analysis

3.1 Calculating the Momentum

Using the de Broglie relation:

$$p = \frac{h}{\lambda}$$

Substituting the known values:

$$p = \frac{6.626 \times 10^{-34}\, \mathrm{Js}}{3.28 \times 10^{-11}\, \mathrm{m}}$$

Calculating:

$$p \approx 2.02 \times 10^{-23}\, \mathrm{kg \cdot m/s}$$

This is the momentum of the atom moving at its rms speed corresponding to the given wavelength.

3.2 Determining the Atom’s Mass

The momentum relates to the mass m and velocity v_{rms}:

$$ p = m v_{rms} $$

But first, we need to find v_{rms}, which can be related to the wavelength:

$$ v_{rms} = \frac{p}{m} $$

Alternatively, since the atom's temperature is known, and the root-mean-square speed is:

$$ v_{rms} = \sqrt{\frac{3k_B T}{m}} $$

We can combine these two to solve for m.

---

4. Connecting Wavelength and Thermal Speed

Given that the atom's wavelength is tied to its momentum, and that the atom's speed at 20 °C is v_{rms}, we can set:

$$ v_{rms} = \frac{p}{m} $$

and

$$ v_{rms} = \sqrt{\frac{3k_B T}{m}} $$

Equating the two:

$$ \frac{p}{m} = \sqrt{\frac{3k_B T}{m}} $$

Squaring both sides:

$$ \frac{p^2}{m^2} = \frac{3k_B T}{m} $$

Rearranged:

$$ p^2 = 3k_B T \, m $$

Solving for m:

$$ m = \frac{p^2}{3k_B T} $$

Plugging in the known values:

$$ m = \frac{(2.02 \times 10^{-23})^2}{3 \times 1.381 \times 10^{-23} \times 293.15} $$

Calculating numerator:

$$ (2.02 \times 10^{-23})^2 = 4.08 \times 10^{-46} $$

Calculating denominator:

$$ 3 \times 1.381 \times 10^{-23} \times 293.15 \approx 1.214 \times 10^{-20} $$

Finally:

$$ m = \frac{4.08 \times 10^{-46}}{1.214 \times 10^{-20}} \approx 3.36 \times 10^{-26}\, \mathrm{kg} $$

This mass is characteristic of a light atom, suggesting it could be hydrogen or a similar element.

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5. Identifying the Atom

5.1 Estimating the Atomic Mass Number

Given $m \approx 3.36 \times 10^{-26}\, \mathrm{kg}$, and knowing that:

- The atomic mass of hydrogen nucleus (proton) is approximately $1.673 \times 10^{-27}\, \mathrm{kg}$

Dividing:

$$ \frac{m}{m_p} \approx \frac{3.36 \times 10^{-26}}{1.673 \times 10^{-27}} \approx 20.1 $$

This indicates the atom's mass is roughly 20 times the mass of a proton, corresponding to an element with atomic mass around 20 amu, such as neon ($^{20}\mathrm{Ne}$), or magnesium ($^{24}\mathrm{Mg}$).

However, the approximate mass suggests that it could be a neon isotope or possibly magnesium, depending on isotopic abundance.

5.2 Confirming with the Velocity

Calculate v_{rms}:

$$ v_{rms} = \frac{p}{m} = \frac{2.02 \times 10^{-23}}{3.36 \times 10^{-26}} \approx 601\, \mathrm{m/s} $$

This speed aligns reasonably with thermal velocities of light noble gases at room temperature.

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6. Final Conclusions

Based on the calculations and analysis:

- The atom is likely a noble gas, such as Neon or Magnesium, given its mass.
- Its root-mean-square speed at 20 °C is approximately 600 m/s.
- The wavelength of $3.28 \times 10^{-11} \, \mathrm{m}$ is consistent with de Broglie wavelengths of atoms moving at thermal speeds at room temperature.
- The mass estimate suggests a neon isotope, probably $^{20}\mathrm{Ne}$, which has an atomic mass close to 20 amu.

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7. Broader Implications and Applications

Understanding the quantum behavior of atoms in this context has broad implications:

- Quantum Gas Behavior: Atoms with such wavelengths exhibit wave-like behavior, which becomes significant in Bose-Einstein condensates and quantum gases.
- Spectroscopy and Atomic Identification: Precise knowledge of atomic mass and motion aids in spectroscopic analysis, crucial for astrophysics, plasma physics, and material science.
- Temperature and Motion Analysis: Comparing theoretical velocities with experimental data helps verify models of thermal motion and atomic interactions.

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8. Summary

In this detailed analysis, we explored how an atom moving at its root-mean-square speed at 20 °C with a wavelength of 3.28×10^{-11} m can be identified through fundamental physics principles. By calculating the atom's mass

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