An Electron Enters A Uniform Magnetic Field 0.20 T At An Angle Of 30 To The Field. Determine The Pitch

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When analyzing the motion of a charged particle such as an electron entering a magnetic field at an angle, understanding the concept of pitch becomes essential. The pitch of the resulting helical path describes the distance traveled along the magnetic field direction during one complete gyration. In this article, we will explore how to determine the pitch of an electron entering a uniform magnetic field of 0.20 Tesla at an angle of 30°, providing a comprehensive step-by-step methodology rooted in physics principles. This topic is vital for students and professionals working in fields like particle physics, electromagnetism, and related engineering disciplines.

Understanding the Motion of an Electron in a Magnetic Field

Before delving into calculations, it is crucial to understand the fundamental behavior of charged particles in magnetic fields.

The Lorentz Force and Particle Trajectory

- When a charged particle such as an electron enters a magnetic field, it experiences a force described by the Lorentz force law:

$$
\mathbf{F} = q \mathbf{v} \times \mathbf{B}
$$

where:
- q is the charge of the particle ($-1.6 \times 10^{-19} \, C$ for electrons),
- $\mathbf{v}$ is the velocity vector,
- $\mathbf{B}$ is the magnetic flux density (magnetic field).

- The force acts perpendicular to both the velocity and the magnetic field, causing the electron to undergo circular or helical motion depending on its initial velocity components.

Decomposition of the Electron’s Velocity

- When the electron enters at an angle of 30°, its initial velocity can be decomposed into:

$$
v_{\parallel} = v \cos \theta
$$

$$
v_{\perp} = v \sin \theta
$$

where:
- v is the magnitude of the initial velocity,
- $\theta = 30^\circ$.

- The component $v_{\perp}$ causes the electron to move in a circular path perpendicular to the magnetic field, while $v_{\parallel}$ causes it to move parallel, resulting in a helical trajectory.

Calculating the Electron’s Velocity and Pitch

To determine the pitch, we need to first find the velocity components and then relate them to the particle's motion.

Step 1: Establish the Electron's Total Velocity

- The initial speed v can be known from the electron’s energy or given conditions. For illustration, assume the electron has a known kinetic energy KE:

$$
KE = \frac{1}{2} m v^2
$$

where $m = 9.11 \times 10^{-31} \, kg$ is the mass of the electron.

- Rearranged, the velocity is:

$$
v = \sqrt{\frac{2 KE}{m}}
$$

- For example, if the electron has an energy of 100 eV:

$$
KE = 100 \, eV = 100 \times 1.6 \times 10^{-19} \, J = 1.6 \times 10^{-17} \, J
$$

$$
v = \sqrt{\frac{2 \times 1.6 \times 10^{-17}}{9.11 \times 10^{-31}}} \approx 5.93 \times 10^{6} \, m/s
$$

Step 2: Determine Velocity Components

- Using $v \approx 5.93 \times 10^{6} \, m/s$:

$$
v_{\parallel} = v \cos 30^\circ = 5.93 \times 10^{6} \times \frac{\sqrt{3}}{2} \approx 5.13 \times 10^{6} \, m/s
$$

$$
v_{\perp} = v \sin 30^\circ = 5.93 \times 10^{6} \times \frac{1}{2} \approx 2.97 \times 10^{6} \, m/s
$$

Determining the Pitch of the Electron’s Helical Path

With velocity components established, focus turns to calculating the pitch — the linear distance traveled along the magnetic field during one complete revolution.

Step 1: Find the Cyclotron Frequency

- The electron undergoes circular motion with a frequency called the cyclotron frequency:

$$
f_{c} = \frac{q B}{2 \pi m}
$$

- Substituting known values:

$$
q = 1.6 \times 10^{-19} \, C
$$

$$
B = 0.20 \, T
$$

$$
m = 9.11 \times 10^{-31} \, kg
$$

$$
f_{c} = \frac{1.6 \times 10^{-19} \times 0.20}{2 \pi \times 9.11 \times 10^{-31}} \approx 5.58 \times 10^{10} \, Hz
$$

- The period of one revolution:

$$
T = \frac{1}{f_{c}} \approx 1.79 \times 10^{-11} \, s
$$

Step 2: Calculate the Radius of the Helical Path

- The radius of the circular component:

$$
r = \frac{m v_{\perp}}{q B}
$$

$$
r = \frac{9.11 \times 10^{-31} \times 2.97 \times 10^{6}}{1.6 \times 10^{-19} \times 0.20} \approx 8.45 \times 10^{-3} \, m
$$

Step 3: Determine the Pitch

- The pitch p is the distance traveled along the magnetic field during one complete revolution:

$$
p = v_{\parallel} \times T
$$

- Plugging in the values:

$$
p = 5.13 \times 10^{6} \times 1.79 \times 10^{-11} \approx 9.19 \times 10^{-5} \, m
$$

- This means the electron advances approximately 92 micrometers along the magnetic field during each full orbit in the helix.

Summary of the Calculation Process

To summarize, calculating the pitch involves:

- Determining the electron's initial velocity based on its energy.
- Decomposing this velocity into components parallel and perpendicular to the magnetic field.

- Calculating the cyclotron frequency to find the period of the circular motion.
- Using the parallel velocity component and period to find the pitch.

Practical Applications of Electron Motion in Magnetic Fields

Understanding the pitch and trajectory of electrons in magnetic fields has numerous practical applications:

Particle Accelerators and Detectors

- Precise knowledge of electron paths aids in designing devices like cyclotrons and synchrotrons, where control over particle trajectories is crucial.
- Magnetic spectrometers utilize this principle to analyze particle energies based on their trajectories.

Electromagnetic Devices and Imaging

- Electron beams in cathode-ray tubes and electron microscopes depend on controlled magnetic fields.
- Accurate calculations of pitch assist in optimizing device performance and resolution.

Space and Astrophysics

- Understanding how charged particles move in planetary magnetic fields informs models of cosmic radiation and space weather phenomena.

Conclusion

Calculating the pitch of an electron entering a uniform magnetic field at an angle involves a combination of classical electromagnetism principles and kinetic energy considerations. By decomposing the initial velocity into components, determining the cyclotron frequency, and calculating the distance traveled along the field during one revolution, physicists can predict the electron's helical path with accuracy. This understanding is foundational in many scientific and technological fields, illustrating the importance of fundamental physics in practical applications.

Whether you're a student tackling electromagnetism problems or an engineer designing advanced particle instruments, mastering how to determine the pitch in these scenarios enhances your ability to analyze and manipulate charged particle behavior in magnetic environments.

Frequently Asked Questions

What is the significance of the angle between the electron's velocity and the magnetic field when calculating the pitch of its helical path?

The angle determines the component of the electron's velocity parallel to the magnetic field, which directly influences the pitch of the helix; specifically, the pitch is proportional to the cosine of this angle multiplied by the electron's speed and the period of its motion.

How do you calculate the pitch of an electron moving in a uniform magnetic field at an angle?

The pitch is calculated using the formula: pitch = $v \cos(\theta) T$, where v is the electron's speed, θ is the angle with the magnetic field, and T is the period of the circular component of motion; alternatively, it can be expressed as $(m v \cos(\theta)) / (q B)$, considering the electron's mass, charge, and magnetic field strength.

Given a magnetic field of 0.20 T and an electron entering at 30°, how do you find the electron's velocity for calculating the pitch?

You need the electron's speed, which can be obtained from its kinetic energy or initial conditions. If not provided, assumptions or additional data are required. Once v is known, you can proceed to calculate the pitch using the relevant formulas.

What are the key physical constants involved in determining the pitch of an electron in a magnetic field?

The key constants include the electron's charge ($q = 1.6 \times 10^{-19}$ C), mass ($m = 9.11 \times 10^{-31}$ kg), the magnetic field strength ($B = 0.20$ T), and the electron's velocity (v). These are used to compute the cyclotron period and the component of velocity parallel to the field.

How does the angle of entry affect the shape and size of the electron's helical trajectory in the magnetic field?

The angle determines the ratio of the electron's velocity components; at 0°, the path is a straight line along the field, while at 90°, it is a circle. At 30°, the trajectory is a helix with a specific pitch, which depends on the cosine of the entry angle, affecting the length of the helix per cycle.

Why is understanding the pitch important in applications involving electron motion in magnetic fields?

The pitch determines the spatial extent of the electron's helical path, which is crucial in designing devices like cyclotrons, magnetic confinement in plasma physics, and understanding radiation emission patterns, as it affects particle confinement and energy transfer.

Additional Resources

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Introduction

In the realm of electromagnetism and particle physics, understanding the motion of charged particles within magnetic fields is fundamental. Whether in the design of cyclotrons, mass spectrometers, or plasma confinement devices, the behavior of electrons and other charged particles in magnetic environments underpins countless technological advances and experimental techniques. A classic scenario involves an electron entering a uniform magnetic field at an angle, resulting in a helical trajectory characterized by a specific pitch—the linear distance traveled along the magnetic field during one complete gyration.

This article delves into the detailed analysis of such a situation: an electron enters a uniform magnetic field of 0.20 Tesla at an angle of 30° relative to the magnetic field direction. The goal is to determine the pitch of the electron's helical path, a critical parameter for understanding its motion and implications in various physical and engineering contexts.

Fundamental Concepts and Physical Principles

Before engaging with the calculations, it is essential to clarify the underlying physics:

1. Motion of Charged Particles in Magnetic Fields

A charged particle, such as an electron, moving in a magnetic field experiences the Lorentz force:

$$ \vec{F} = q \vec{v} \times \vec{B} $$

where:
- q is the particle’s electric charge,
- $\vec{v}$ is its velocity vector,
- $\vec{B}$ is the magnetic field vector.

This force acts perpendicular to both the velocity and the magnetic field, resulting in a circular or helical motion depending on the initial velocity components relative to the field.

2. Helical Motion and Pitch

When an electron enters the magnetic field at an angle θ, its velocity can be decomposed into:

- Parallel component: $v_\parallel = v \cos \theta$
- Perpendicular component: $v_\perp = v \sin \theta$

The perpendicular component causes circular motion with a radius known as the Larmor radius, while the parallel component causes the electron to advance along the magnetic field lines, forming a helix.

The pitch p of the helix is the linear distance traveled along the magnetic field during one complete revolution:

$$p = v_\parallel T$$

where T is the period of gyration.

Step-by-Step Calculation

1. Determining the Electron's Speed

In many problems, the initial kinetic energy or velocity is given. If not specified, a common assumption is that the electron has been accelerated through a known potential difference V. For demonstration, consider an electron accelerated through a potential V, resulting in kinetic energy:

$$KE = eV$$

and velocity:

$$v = \sqrt{\frac{2 e V}{m}}$$

where:
- $e = 1.602 \times 10^{-19} \, \text{C}$,
- $m = 9.109 \times 10^{-31} \, \text{kg}$,
- V is the accelerating potential (assumed or given).

Note: Since the problem does not specify V, the calculation can be expressed generally in terms of v, or an assumed value (e.g., $V = 100\, \text{V}$) can be used for illustration.

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2. Decomposition of Velocity

Given the initial velocity v and entry angle $\theta = 30^\circ$:

- $v_\parallel = v \cos 30^\circ = v \times \frac{\sqrt{3}}{2}$

- $v_\perp = v \sin 30^\circ = v \times \frac{1}{2}$

---

3. Calculating the Gyration Period T

The period of circular motion in a magnetic field is:

$$T = \frac{2\pi m}{q B}$$

where:
- $q = e = 1.602 \times 10^{-19} \, C$,
- $B = 0.20\, T$,
- $m = 9.109 \times 10^{-31}\, kg$.

Plugging in the values:

$$T = \frac{2 \pi \times 9.109 \times 10^{-31}}{1.602 \times 10^{-19} \times 0.20}$$

Calculating numerator and denominator:

$$T \approx \frac{5.729 \times 10^{-30}}{3.204 \times 10^{-20}}$$

$$T \approx 1.787 \times 10^{-10} \, \text{s}$$

This is the period for one complete gyration of the electron's perpendicular component.

---

4. Computing the Pitch p

Using the relation:

$$p = v_\parallel T = v \cos 30^\circ \times T$$

Expressed explicitly:

$$p = v \times \frac{\sqrt{3}}{2} \times T$$

If the initial velocity v is known, the pitch can be directly calculated.

Practical Example: Assuming an Electron Accelerated Through 100 V

Suppose the electron is accelerated through a potential difference $V = 100\, V$. Its speed:

$$v = \sqrt{\frac{2 e V}{m}}$$

$$v = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 100}{9.109 \times 10^{-31}}}$$

$$ v = \sqrt{\frac{3.204 \times 10^{-17}}{9.109 \times 10^{-31}}} $$

$$ v \approx \sqrt{3.517 \times 10^{13}} $$

$$ v \approx 5.93 \times 10^{6} \, \text{m/s} $$

Now, compute the pitch:

$$ p = 5.93 \times 10^{6} \times \frac{\sqrt{3}}{2} \times 1.787 \times 10^{-10} $$

$$ p \approx 5.93 \times 10^{6} \times 0.866 \times 1.787 \times 10^{-10} $$

$$ p \approx (5.14 \times 10^{6}) \times 1.787 \times 10^{-10} $$

$$ p \approx 9.19 \times 10^{-4} \, \text{m} $$

or approximately 0.919 mm.

Significance and Applications

The calculated pitch provides insight into the electron's trajectory within the magnetic field. This parameter is crucial for designing devices such as:

- Magnetic spectrometers: where precise knowledge of particle trajectories allows for energy determination.
- Cyclotrons: where the frequency and path of particles influence acceleration efficiency.
- Plasma confinement systems: where understanding particle motion prevents losses and improves stability.
- Electron beam steering and focusing: in electron microscopes or accelerators.

Furthermore, understanding the pitch helps in predicting the spatial distribution of electrons, optimizing detector placement, and controlling beam properties.

Extended Considerations

While the above analysis assumes ideal conditions, real-world factors such as:

- Relativistic effects at high energies,
- Magnetic field inhomogeneities,
- Initial velocity distributions,
- Electric fields or additional forces,

can complicate the motion. Advanced models incorporate these factors for more precise predictions.

Conclusion

Determining the pitch of an electron entering a uniform magnetic field at an angle involves decomposing its velocity, calculating the gyration period, and applying the relation:

$$p = v_\parallel T$$

Using typical assumptions and known constants, the pitch for an electron accelerated through 100 V in a 0.20 T magnetic field at 30° is approximately 0.92 mm. This fundamental calculation exemplifies the elegant interplay of electromagnetism and kinematics, with broad implications across scientific disciplines and technological innovations.

Understanding such motion not only deepens our grasp of physical principles but also enhances our capacity to engineer devices that harness the behavior of charged particles in magnetic environments.

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