

An Inductor Of $L = 250$ Is Subjected To A Voltage $V(t) = 8 \times 10^{-4}t$ V:A. Knowing That, Integrate Both Sides

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Introduction

When analyzing electrical circuits, especially those involving inductors, understanding the relationship between voltage and current over time is crucial. In this scenario, we are presented with an inductor characterized by its inductance ($L = 250$ H), subjected to a time-varying voltage ($V(t) = 8 \times 10^{-4}t$ V). The task involves integrating both sides of the governing differential equation to find the current ($i(t)$) flowing through the inductor over time.

This problem provides an excellent opportunity to explore the fundamental concepts of inductor behavior, differential equations in circuit analysis, and the application of integration techniques to solve for unknown quantities. Understanding these principles is essential for electrical engineers, students, and professionals working with transient circuit analysis, filter design, power systems, and various electronic applications.

Fundamental Concepts of Inductors and Voltage-Current Relationship

The Inductor in Electrical Circuits

An inductor is a passive electrical component that stores energy in a magnetic field when current flows through it. Its key characteristic is the opposition to changes in current, which is described mathematically by its inductance (L). The inductance measures how much magnetic flux is generated per unit current in the inductor.

Voltage-Current Relationship for an Inductor

The voltage across an inductor ($V(t)$) relates to the current ($i(t)$) flowing through it by the fundamental differential equation:

$$V(t) = L \frac{di(t)}{dt}$$

where:

- $V(t)$ is the instantaneous voltage across the inductor,
- L is the inductance (in Henrys, H),
- $i(t)$ is the current through the inductor (in Amperes, A),
- $\frac{di(t)}{dt}$ is the derivative of the current with respect to time.

This relationship indicates that the voltage across an inductor is proportional to the rate of change of current through it.

Problem Statement and Mathematical Formulation

Given:

- Inductance $L = 250$, H ,
- Voltage $V(t) = 8 \times 10^{-4} t$, V .

The goal is to find the current $i(t)$ as a function of time.

Starting from the fundamental relation:

$$V(t) = L \frac{di(t)}{dt}$$

Substituting the given values:

$$8 \times 10^{-4} t = 250 \frac{di(t)}{dt}$$

Rearranged, the differential equation becomes:

$$\frac{di(t)}{dt} = \frac{8 \times 10^{-4} t}{250}$$

which simplifies to:

$$\frac{di(t)}{dt} = 3.2 \times 10^{-6} t$$

Integrating Both Sides to Find the Current $i(t)$

The goal is to find $i(t)$. To do this, integrate both sides with respect to time:

$$\int i(t) dt = \int 3.2 \times 10^{-6} t dt$$

Performing the integration:

$$i(t) = 3.2 \times 10^{-6} \int t dt$$

$$i(t) = 3.2 \times 10^{-6} \times \frac{t^2}{2} + C$$

where C is the constant of integration, determined by initial conditions.

Simplifying:

$$i(t) = 1.6 \times 10^{-6} t^2 + C$$

Determining the Integration Constant C

The value of C depends on the initial current $i(0)$. If the initial current $i(0)$ at $t=0$ is known, then:

$$i(0) = 1.6 \times 10^{-6} \times 0^2 + C \Rightarrow C = i(0)$$

Common scenarios:

- Initially unenergized inductor: $i(0) = 0$, so $C = 0$,
- Known initial current: plug in the value to find C .

Assuming the inductor is initially unenergized:

$$i(0) = 0$$

then,

$$i(t) = 1.6 \times 10^{-6} t^2$$

This quadratic relation describes how the current increases over time as the voltage is applied.

Analysis and Interpretation of the Results

Current Behavior Over Time

The current $i(t) = 1.6 \times 10^{-6} t^2$ indicates a parabolic growth, starting from zero at $t=0$. The rate at which current increases depends directly on time, reflecting the linearly increasing voltage $V(t)$.

Key observations:

- At $t=0$, $i(0)=0$,
- For larger t , the current increases quadratically,
- The inductor opposes sudden changes; the current builds up gradually.

Physical Significance

This solution shows how an inductor responds to a linearly increasing voltage:

- The current increment is proportional to t^2 ,
- The energy stored in the magnetic field increases as $\frac{1}{2} L i^2(t)$,
- The inductor's behavior is predictable and smooth, avoiding abrupt current jumps.

Additional Considerations and Practical Applications

Initial Conditions and Circuit Context

In real-world applications, initial conditions are essential:

- If the inductor had an initial current $i(0) \neq 0$, the constant C would reflect that,

- The presence of other circuit elements (resistors, capacitors) would modify the differential equation.

Transient Response and Steady-State Behavior

- For this specific scenario, as $t \rightarrow \infty$, the current continues to grow quadratically,
- In practical circuits, resistive elements limit current growth, leading to a steady-state value.

Applications in Power Systems and Circuit Design

- Understanding inductor responses to voltage variations is critical in designing filters, transformers, and energy storage systems,
- The ability to calculate $i(t)$ helps in predicting circuit behavior during transient events,
- These calculations are fundamental for designing circuits with specific dynamic responses.

Conclusion

The process of integrating both sides of the voltage-current differential equation for an inductor provides a clear pathway to understanding the current evolution over time. Starting from the fundamental relationship $V(t) = L \frac{di(t)}{dt}$, substituting the given voltage function, and performing the integration yields a quadratic current function:

$$i(t) = 1.6 \times 10^{-6} t^2 + C$$

where the constant C depends on initial conditions. Under the assumption of zero initial current, the solution simplifies to:

$$i(t) = 1.6 \times 10^{-6} t^2$$

This analysis demonstrates the importance of integration in solving differential equations in electrical engineering and provides insight into the transient behavior of inductors subjected to time-varying voltages. Mastery of these concepts is essential for designing and analyzing complex electrical systems, ensuring their safe and efficient operation.

Keywords: inductor, voltage, current, differential equation, integration, transient analysis, inductance, circuit analysis, energy storage, power systems

Frequently Asked Questions

What is the differential equation governing the current in an inductor with inductance $L = 250 \text{ H}$ subjected to a voltage $V(t) = 8 \times 10^{-4} t \text{ V}$?

Using $V(t) = L (di/dt)$, the differential equation is $8 \times 10^{-4} t = 250 (di/dt)$.

How do you integrate both sides of the differential equation to find the current $i(t)$ in the inductor?

Rearranged as $di/dt = (8 \times 10^{-4} t)/250$, then integrate both sides with respect to t to find $i(t)$: $i(t) = \int [(8 \times 10^{-4} t)/250] dt + C$.

What is the integral of the voltage function $V(t) = 8 \times 10^{-4} t \text{ V}$ with respect to time?

The integral of $V(t) = 8 \times 10^{-4} t$ with respect to t is $(8 \times 10^{-4}) (t^2/2) + \text{constant}$, which simplifies to $4 \times 10^{-4} t^2 + C$.

How is the current $i(t)$ expressed after integrating the voltage over time for the inductor?

$i(t) = (1/L) \int V(t) dt + i(0) = (1/250) (4 \times 10^{-4} t^2) + i(0) = (4 \times 10^{-4}/250) t^2 + i(0)$.

What is the significance of the constant of integration when solving for current in this problem?

The constant of integration, $i(0)$, represents the initial current through the inductor at $t=0$, which must be specified or determined from initial conditions.

If the initial current $i(0)$ is zero, what is the expression for current $i(t)$?

With $i(0) = 0$, $i(t) = (4 \times 10^{-4}/250) t^2 = 1.6 \times 10^{-6} t^2 \text{ A}$.

How does knowing the integral of $V(t)$ help in analyzing the behavior of the inductor's current?

Integrating $V(t)$ over time gives the magnetic flux linkage change, which directly relates to the inductor's current, allowing us to predict current response to the applied voltage.

What are the units of the integral of $V(t)$ with respect to time in this context?

The integral of voltage over time has units of volt-seconds (V·s), which relates to magnetic flux linkage in the inductor.

Additional Resources

An Inductor of $L = 250$ H Subjected to a Voltage $V(t) = 8 \times 10^{-4} t$ V: An In-Depth Analysis

Introduction

In the realm of electrical engineering and circuit analysis, inductors play a pivotal role due to their unique property of opposing changes in current. The behavior of an inductor under various voltage conditions is fundamental to understanding transient phenomena, filter design, and energy storage systems. This article delves into a specific scenario involving a high-inductance component—an inductor with inductance $(L = 250, \text{H})$ —being subjected to a time-dependent voltage $(V(t) = 8 \times 10^{-4} t, \text{V})$. The primary focus is to analyze the resulting circuit behavior by examining the governing differential equations and their integrations, elucidating the underlying physics, and discussing implications for circuit design.

Fundamental Principles: Inductor Voltage-Current Relationship

Before dissecting the specific case, it is essential to revisit the fundamental relationship governing inductors:

$$V(t) = L \frac{di(t)}{dt}$$

where:

- $(V(t))$ is the voltage across the inductor as a function of time.
- (L) is the inductance, measured in henrys (H).
- $(i(t))$ is the current flowing through the inductor.

This first-order differential equation encapsulates the inductor's innate opposition to rapid changes in

current. When subjected to a time-varying voltage, the current's evolution can be obtained by integrating this equation with respect to time.

Formulating the Differential Equation for the Given Scenario

Given:

$$V(t) = 8 \times 10^{-4} t, \text{ V}$$

$$L = 250, \text{ H}$$

Applying the fundamental inductor relation:

$$8 \times 10^{-4} t = 250 \frac{d i(t)}{dt}$$

which simplifies to:

$$\frac{d i(t)}{dt} = \frac{8 \times 10^{-4}}{250} t$$

Calculating the coefficient:

$$\frac{8 \times 10^{-4}}{250} = 3.2 \times 10^{-6}$$

Thus, the differential equation becomes:

$$\frac{d i(t)}{dt} = 3.2 \times 10^{-6} t$$

Integrating the Differential Equation: Deriving the

Inductor Current

The goal is to find $i(t)$. Since $\frac{d i(t)}{dt}$ is expressed as a function of t , integration is straightforward:

$$i(t) = \int \frac{d i(t)}{dt} dt + i(0)$$

Assuming initial conditions are known or zero (for simplicity), $i(0) = 0$. The integral becomes:

$$i(t) = \int 3.2 \times 10^{-6} t, dt$$

Calculating the integral:

$$i(t) = 3.2 \times 10^{-6} \int t, dt = 3.2 \times 10^{-6} \times \frac{t^2}{2} + C$$

where C is the constant of integration, which equals the initial current $i(0)$. With $i(0) = 0$, we find:

$$i(t) = \frac{3.2 \times 10^{-6}}{2} t^2 = 1.6 \times 10^{-6} t^2$$

This expression describes the current through the inductor as a quadratic function of time:

$$i(t) = 1.6 \times 10^{-6} t^2, \text{A}$$

Physical Interpretation and Analysis

The derived current indicates that, in response to the linearly increasing voltage $V(t) = 8 \times 10^{-4} t$, the inductor current exhibits a quadratic growth over time. This behavior reflects the inductor's opposition to sudden current changes, resulting in a smooth, predictable increase.

Key observations:

- Initial Conditions: Assuming zero initial current simplifies calculations; however, real-world scenarios may involve non-zero initial currents, which would modify the integration constant C .

- Time Dependence: The current increases quadratically with time, meaning the rate of change of

current accelerates as time progresses.

- Energy Storage: The inductor stores magnetic energy proportional to $(i^2(t))$, which can be expressed as:

$$W(t) = \frac{1}{2} L i^2(t) = \frac{1}{2} \times 250 \times (1.6 \times 10^{-6} t^2)^2$$

This quadratic increase of stored energy signifies a growing magnetic field within the inductor.

Implications for Circuit Design and Analysis

Understanding the current response to a linearly increasing voltage has several practical implications:

Transient Response Analysis

- The quadratic current growth illustrates how energy accumulates in the inductor during transient phases.
- Designers can predict the magnitude of current after a certain period, ensuring components are rated appropriately.

Energy Storage and Safety Considerations

- As energy stored increases, circuit components must withstand corresponding magnetic fields and currents.
- Rapid or unanticipated voltage changes can lead to excessive energy buildup, risking damage.

Applications in Filter and Oscillator Design

- Knowledge of inductor response helps in designing filters that rely on inductive reactance.
- The predictable current growth allows for precise timing in oscillator circuits.

Limitations and Assumptions

- The analysis assumes ideal components with no parasitic resistances or core losses.

- Real-world inductor behavior may involve resistive effects, saturation, and non-linearities.

Further Considerations: Integration in Complex Circuits

While the current analysis focuses on a single inductor subjected to a specific voltage, real circuits often involve multiple reactive elements. Extending this analysis involves:

- Incorporating resistive elements to evaluate damping effects.
- Considering initial conditions reflective of circuit states.
- Applying Laplace transforms for more complex, piecewise, or sinusoidal inputs.

Conclusion

The investigation into an inductor with $(L = 250\text{ H})$ subjected to a linearly increasing voltage $(V(t) = 8 \times 10^{-4} t, \text{ V})$ reveals a straightforward yet profound relationship: the inductor's current evolves quadratically with time, given by

$$i(t) = 1.6 \times 10^{-6} t^2, \text{ A}$$

This analysis underscores the importance of differential equations and their integration in predicting circuit responses, guiding design choices, and ensuring component resilience. Such foundational understanding is essential for engineers designing transient circuits, energy storage systems, and complex signal processing devices. Future work may involve incorporating non-ideal effects, initial conditions, and complex input waveforms to develop comprehensive models aligned with real-world applications.

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Note: The calculations assume ideal conditions and initial zero current. For precise circuit analysis, consider including resistive elements and initial state conditions accordingly.

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