# An Isotope Of An Element X Has A Half-life Of 3 Minutes. What Is The Fifth Life (T1/5) Of This Isotope

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## Introduction to Radioactive Decay and Half-life

Radioactive isotopes are unstable variants of elements that undergo spontaneous decay, transforming into different elements or isotopes over time. The rate at which this decay occurs is characterized by a property known as the half-life. Understanding the half-life is crucial in fields like nuclear physics, medicine, and environmental science, as it helps determine the stability and longevity of radioactive materials.

The half-life ($t_{1/2}$) of an isotope is the time it takes for half of a sample of that isotope to decay. For the isotope in question, the half-life is 3 minutes, which means that after 3 minutes, only half of the initial amount remains; after another 3 minutes (6 minutes total), only a quarter remains, and so forth.

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## Understanding the Concept of "Life" in Radioactive Decay

In the context of radioactive decay, the term "life" often refers to the mean life ($\tau$), which is related to the half-life but differs mathematically. The mean life is the average lifetime of a nucleus before it decays. It is connected to the half-life through the relation:

$$
\tau = \frac{t_{1/2}}{\ln 2}
$$

where $\ln 2 \approx 0.693$.

This concept is important because it provides a continuous measure of the decay process, contrasting with the discrete nature of the half-life.

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## Calculating the Mean Life ($T_1/5$) From the Half-life

Given the half-life $t_{1/2} = 3$ minutes, we can determine the mean life $\tau$ as follows:

$$
\tau = \frac{3}{\ln 2} \approx \frac{3}{0.693} \approx 4.33 \text{ minutes}
$$

This value — approximately 4.33 minutes — is often called the average lifetime or mean life of the isotope.

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## Defining the "Fifth Life" ($T_1/5$)

The term "fifth life" ($T_1/5$) in radioactive decay refers to the expected or average time it takes for the isotope to decay to one-fifth of its original quantity, or more generally, it can be understood as the time associated with a certain fraction remaining.

However, in many contexts, "life" can be synonymous with the mean life, or sometimes with the decay constant or mean lifetime.

In this case, since the question asks for $T_1/5$, it most likely refers to the time at which only 1/5 (20%) of the original isotope remains.

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## Calculating the Time for the Isotope to Reduce to 20% (One-Fifth) of Its Initial Quantity

Radioactive decay follows an exponential decay law:

$$
N(t) = N_0 e^{-\lambda t}
$$

where:

- $N(t)$ is the quantity remaining after time $t$,
- $N_0$ is the initial quantity,
- $\lambda$ is the decay constant.

The decay constant $\lambda$ relates to the half-life:

$$
\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{3} \approx 0.231 \text{ min}^{-1}
$$

To find the time $T_{1/5}$ when only 20% remains:

$$
\frac{N(T_{1/5})}{N_0} = \frac{1}{5} = e^{-\lambda T_{1/5}}
$$

Taking natural logarithms on both sides:

$$
\ln \left(\frac{1}{5}\right) = -\lambda T_{1/5}
$$

$$
T_{1/5} = - \frac{\ln (1/5)}{\lambda}
$$

Since $\ln (1/5) = - \ln 5 \approx -1.609$:

$$
T_{1/5} = \frac{1.609}{0.231} \approx 6.97 \text{ minutes}
$$

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# Interpreting the Result: The Fifth Life ($T_1/5$)

The calculation indicates that it takes approximately 6.97 minutes for only 20% of the original isotope to remain. This duration can be considered the fifth life or the time to decay to one-fifth of the initial quantity.

This period is roughly 2.3 times the half-life (which is 3 minutes), aligning with the exponential decay behavior since:

$$
\text{Number of half-lives to reach } \frac{1}{5} \text{ of the initial amount} \approx \frac{\ln 5}{\ln 2} \approx 2.32
$$

and

$$
2.32 \times 3 \text{ minutes} \approx 6.96 \text{ minutes}
$$

which matches our previous calculation.

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# Summary of Key Points

- The half-life ($t_{1/2}$) of the isotope is 3 minutes.
- The decay constant ($\lambda$) is approximately 0.231 min$^{-1}$.
- The time for the isotope to decay to 20% (one-fifth) of its original amount — the "fifth life" — is approximately 6.97 minutes.
- This duration is about 2.32 half-lives, reflecting exponential decay behavior.

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# Practical Implications and Applications

Understanding the "fifth life" of an isotope is important in various fields:

## Medical Applications

- Radioisotopes used in diagnostic imaging or radiotherapy often have specific decay times to optimize imaging quality and minimize patient exposure.

## Environmental and Geological Dating

- Knowing how long it takes for a certain fraction of a radioactive isotope to decay helps in dating archaeological finds or geological formations.

## Radioactive Waste Management

- The decay timeline informs storage and disposal strategies for radioactive materials, ensuring safety over their decay period.

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# Conclusion

The concept of "life" in radioactive decay, especially the "fifth life" or the time to reach a specific fraction of the initial isotope, is critical for understanding the behavior of radioactive materials. For an isotope with a half-life of 3 minutes, the time for it to decay to one-fifth of its original amount — often referred to as the "fifth life" ($T_1/5$) — is approximately 6.97 minutes. This duration is derived from exponential decay laws and highlights the exponential nature of radioactive decay processes.

Understanding these principles enables scientists and engineers to accurately predict the behavior of radioactive isotopes in various applications, ensuring safety, effectiveness, and precision in their use.

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References:

- Glasstone, S., & Sesonske, A. (1994). Nuclear Reactor Physics. Springer.
- Krane, K. S. (1988). Introductory Nuclear Physics. Wiley.
- Nuclear Chemistry Resources. (2023). Understanding Half-life and Decay.

# Frequently Asked Questions

## What is the half-life of an isotope that decays to 1/32 of its original amount after 15 minutes?

The half-life is 3 minutes since 15 minutes divided by 3 minutes per half-life equals 5 half-lives, and 1/32 is $(1/2)^5$.

## How do you calculate the fifth life (T1/5) of an isotope with a half-life of 3 minutes?

The fifth life T1/5 is calculated as $T1/5 = T1/2 \times 5$, so it equals 3 minutes × 5 = 15 minutes.

## What does the fifth life (T1/5) represent in radioactive decay?

The fifth life is the total time it takes for the isotope to decay to 1/32 of its original quantity, which is five times its half-life.

## If an isotope has a half-life of 3 minutes, how long does it take to decay to 1/16 of its original amount?

It takes 4 half-lives, so 4 × 3 minutes = 12 minutes.

## Why is the concept of the fifth life important in understanding radioactive decay?

Because it helps determine the total time required for a substance to reduce to a specific fraction of its original amount, useful in various applications like medicine and archaeology.

## Can the fifth life (T1/5) be used to estimate the remaining quantity of an isotope after a certain time?

Yes, knowing T1/5 allows calculation of decay over multiple periods, especially since the decay follows exponential patterns based on the number of half-lives.

## What is the mathematical relationship between half-life and fifth life for an isotope?

Since the fifth life corresponds to five half-lives, $T1/5 = 5 \times T1/2$.

## If an isotope's fifth life is 15 minutes, what is its half-life?

The half-life is 15 minutes divided by 5, which equals 3 minutes.

# Additional Resources

An isotope of an element X has a half-life of 3 minutes. What is the fifth life ($T_1/_5$) of this isotope? This question leads us into a fascinating exploration of radioactive decay, half-lives, and the mathematical relationships that underpin nuclear physics. Understanding how isotopes decay over time not only helps in scientific research but also finds practical applications in medicine, archaeology, and energy production. In this guide, we will dissect what the fifth life ($T_1/_5$) means, how to calculate it, and why it's an important concept in understanding radioactive decay processes.

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Introduction to Radioactive Decay and Half-Lives

Radioactive isotopes, or radioisotopes, are unstable atoms that spontaneously emit radiation to reach a more stable state. The half-life ($T_1/_2$) of a radioactive isotope is the time it takes for half of a sample to decay. This measure is fundamental because it provides a characteristic timescale for the isotope's decay process.

Key points:
- The half-life remains constant regardless of the sample size.
- It is independent of the initial quantity of the isotope.
- The decay process follows an exponential pattern, meaning the amount of isotope decreases by half every $T_1/_2$ minutes.

For our isotope with a half-life of 3 minutes, this means:
- After 3 minutes, only 50% of the original isotope remains.
- After 6 minutes, only 25% remains.
- After 9 minutes, 12.5% remains, and so on.

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What Is the "Fifth Life" ($T_1/_5$)?

The term fifth life ($T_1/_5$) might seem unfamiliar at first glance. It typically refers to the time required for a substance to decay to 1/5 (or 20%) of its original amount. Unlike the half-life, which halves the quantity, $T_1/_5$ indicates a different decay fraction—specifically, reducing the original amount to 20%.

In essence:
- $T_1/_5$ measures how long it takes for the isotope to decay from 100% to 20% of its initial quantity.

- It is useful in contexts where a specific residual amount indicates the end of its practical usability or when assessing decay over non-half-life intervals.

In our problem, we're asked to find $T_{1/5}$ given the half-life ($T_{1/2}$ = 3 minutes).

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Mathematical Foundations of Radioactive Decay

Radioactive decay is modeled by an exponential decay law:

$$ N(t) = N_0 e^{-\lambda t} $$

Where:
- $N(t)$ is the quantity remaining after time $t$,
- $N_0$ is the initial quantity,
- $\lambda$ is the decay constant (per unit time),
- $e$ is Euler's number (~2.71828).

Relationship between decay constant and half-life:

$$ T_{1/2} = \frac{\ln 2}{\lambda} $$

Given $T_{1/2} = 3$ minutes, we can find $\lambda$:

$$ \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{3} $$

Calculating:

$$ \lambda \approx \frac{0.6931}{3} \approx 0.231 \text{ min}^{-1} $$

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Calculating $T_{1/5}$: Time for 20% Remaining

Our goal is to find $T_{1/5}$—the time when only 20% of the original isotope remains:

$$ \frac{N(t)}{N_0} = 0.2 $$

Using the decay law:

$$ 0.2 = e^{-\lambda T_{1/5}} $$

Taking the natural logarithm of both sides:

$$ \ln 0.2 = -\lambda T_{1/5} $$

$$ T_{1/5} = - \frac{\ln 0.2}{\lambda} $$

Calculating $\ln 0.2$:

$$ \ln 0.2 \approx -1.6094 $$

Substituting $\lambda$:

$$ T_{1/5} = - \frac{-1.6094}{0.231} \approx \frac{1.6094}{0.231} $$

$$ T_{1/5} \approx 6.97 \text{ minutes} $$

Result: The fifth life, or the time for the isotope to decay to 20% of its initial amount, is approximately 6.97 minutes.

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Understanding the Decay Timeline

To visualize this, consider the following decay process:

| Time (minutes) | Remaining Fraction | Description |
|---|---|---|
| 0 | 100% | Initial amount |
| 3 | 50% | After first half-life |
| 6 | 25% | After second half-life |
| 9 | 12.5% | After third half-life |
| 12 | 6.25% | After fourth half-life |

In contrast, the $T_{1/5}$ occurs at approximately 6.97 minutes, which is just over two half-lives, consistent with the decay to 20%.

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Practical Implications and Applications

Understanding $T_{1/5}$ is valuable in many contexts:
- Medical applications: Determining how long a radioactive tracer remains effective.
- Nuclear waste management: Estimating decay times to assess safety.

- Archaeological dating: Calculating the decay of isotopes like Carbon-14 to determine age.
- Industrial applications: Managing isotopic materials in research and manufacturing.

Knowing how to compute $T_{1/5}$ allows scientists and engineers to predict the behavior of isotopes over time, especially when the decay to a specific residual amount is critical.

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Summary of Key Steps to Calculate $T_{1/5}$

1. Identify the half-life ($T_{1/2}$): Given as 3 minutes.
2. Calculate the decay constant ($\lambda$):
$$\lambda = \frac{\ln 2}{T_{1/2}}$$
3. Set up the decay equation for the desired fraction:
$$\frac{N(t)}{N_0} = \text{desired fraction}$$
4. Solve for $T_{1/5}$:
$$T_{1/5} = - \frac{\ln (\text{desired fraction})}{\lambda}$$

Applying this to our problem, the decay to 20% yields a $T_{1/5}$ of approximately 6.97 minutes.

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Final Thoughts

The calculation of the fifth life ($T_{1/5}$) of an isotope with a known half-life illustrates the power of mathematical modeling in understanding radioactive decay. It emphasizes that while the half-life provides a fundamental timescale for decay, other fractions—like one-fifth—are also crucial for specific applications. Mastery of these calculations enables scientists to make precise predictions about isotope behavior, ensuring safety, accuracy, and efficiency in a range of scientific and industrial domains.

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In conclusion, for an isotope with a half-life of 3 minutes, the fifth life ($T_{1/5}$)—the time for the isotope to decay to 20% of its original amount—is approximately 6.97 minutes. This insight underscores the exponential nature of radioactive decay and the importance of understanding decay constants and their relationships to different fractional decay times.

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