An LRC Series Circuit Has R = 15.0 ?, L = 25.0 MH, And C = 30.0 ?F. The Circuit Is Connected To A120-V

Understanding the LRC Series Circuit: Components and Basic Principles

An LRC Series Circuit Has R = 15.0 ?, L = 25.0 MH, And C = 30.0 ?F. The Circuit Is Connected To A120-V power supply. This fundamental electrical circuit combines a resistor (R), an inductor (L), and a capacitor (C) in series, forming the backbone of many electronic and electrical systems. Its analysis provides insight into how alternating current (AC) behaves in complex circuits, including concepts like impedance, resonance, and power factor.

In this article, we'll explore the key features of an LRC series circuit with the specified parameters, examine how it responds to AC voltage, and discuss practical applications of such circuits. Whether you're a student studying circuit theory or an engineer designing electronic systems, understanding the behavior of LRC circuits is essential.

Components of the LRC Series Circuit

Resistor (R) – 15.0 Ω

The resistor opposes the flow of current, converting electrical energy into heat. Its value influences the circuit's overall resistance and damping characteristics.

Inductor (L) – 25.0 mH

The inductor stores energy in a magnetic field when current passes through it. Its inductance determines how it opposes changes in current and affects the circuit's reactance.

Capacitor (C) – 30.0 μF

The capacitor stores energy in an electric field. Its capacitance influences how it opposes changes in voltage and affects the circuit's reactance.

Basic Principles of an LRC Series Circuit

An LRC series circuit subjected to an AC source exhibits complex behavior due to the interplay of resistance, inductance, and capacitance. The key concepts include:

- Impedance (Z): The total opposition to current flow, combining resistance and reactance.
- Reactance (X): The opposition to AC caused by inductance (XL) and capacitance (XC).
- Resonance: The condition where the inductive and capacitive reactances are equal in magnitude, leading to maximum current flow.
- Phase Difference: The angle between voltage and current, affected by the circuit's impedance.

Calculating the Impedance of the Circuit

The impedance Z in an LRC series circuit is given by:

$$
Z = \sqrt{R^2 + (X_L - X_C)^2}
$$

where:

- R is resistance,
- X_L is inductive reactance,
- X_C is capacitive reactance.

Calculating Reactances

1. Inductive Reactance X_L:

$$
X_L = 2\pi f L
$$

2. Capacitive Reactance X_C:

$$
X_C = \frac{1}{2\pi f C}
$$

Given the frequency f, which is derived from the power supply (120 V, but frequency needs to be specified; typically 60 Hz in many regions), we can compute these reactances.

Assuming a standard power line frequency of 60 Hz:

- $L = 25.0 \text{ mH} = 25.0 \times 10^{-3} \text{ H}$,
- $C = 30.0 \text{ μF} = 30.0 \times 10^{-6} \text{ F}$,
- $R = 15.0 \ \Omega$,
- $f = 60 \ \text{Hz}$.

Calculations:

$$
X_L = 2\pi \times 60 \times 25.0 \times 10^{-3} \approx 2\pi \times 60 \times 0.025 \approx 9.42 \ \Omega
$$

$$
X_C = \frac{1}{2\pi \times 60 \times 30.0 \times 10^{-6}} \approx \frac{1}{2\pi \times 60 \times 30 \times 10^{-6}} \approx 88.42 \ \Omega
$$

The net reactance:

$$
X_{net} = X_L - X_C \approx 9.42 - 88.42 = -79.00 \ \Omega
$$

And the impedance:

$$
Z = \sqrt{R^2 + X_{net}^2} = \sqrt{15^2 + (-79)^2} \approx \sqrt{225 + 6241} \approx \sqrt{6466} \approx 80.42 \ \Omega
$$

Current and Voltage in the Circuit

Using Ohm's law for AC circuits:

$$
I = \frac{V}{Z}
$$

Given the supply voltage $V = 120 \ V$:

$$
I = \frac{120}{80.42} \approx 1.49 \ A
$$

The phase angle ϕ between voltage and current is:

$$

$$
\phi = \arctan \left(\frac{X_{net}}{R} \right) = \arctan \left(\frac{-79}{15} \right) \approx -79.3^\circ
$$

This indicates that the current lags the voltage by approximately 79.3 degrees, characteristic of a circuit dominated by capacitive reactance.

Resonance in the LRC Series Circuit

Resonance occurs when:

$$
X_L = X_C
$$

At this point, the circuit behaves as if it has only resistance, and the impedance is minimized:

$$
Z_{res} = R
$$

Given the reactances are frequency-dependent, the resonant frequency f_{res} is:

$$
f_{res} = \frac{1}{2\pi \sqrt{LC}}
$$

Calculating:

$$
f_{res} = \frac{1}{2\pi \sqrt{25 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{2\pi \sqrt{7.5 \times 10^{-7}}}
$$

$$
f_{res} \approx \frac{1}{2\pi \times 8.66 \times 10^{-4}} \approx \frac{1}{5.44 \times 10^{-3}} \approx 183.8 \ \text{Hz}
$$

At this frequency, the circuit would exhibit maximum current flow with minimal impedance.

Power Dissipation and Power Factor

The real power P dissipated in the resistor is:

$$
P = V_{rms} \times I_{rms} \times \cos \phi
$$

Given:

- $V_{rms} = 120 \ V$,
- $I_{rms} \approx 1.49 \ A$,
- $\cos \phi = \frac{R}{Z} \approx \frac{15}{80.42} \approx 0.186$.

Calculating:

$$
P \approx 120 \times 1.49 \times 0.186 \approx 33.2 \ \text{W}
$$

The power factor indicates how effectively the circuit converts electrical power into useful work.

Practical Applications of LRC Circuits

LRC circuits are fundamental in various technological applications, including:

- Tuned Circuits: Used in radios and televisions to select specific frequencies.
- Filters: Implemented in audio and communication systems to filter out unwanted frequencies.
- Oscillators: Serve as the basis for generating AC signals in oscillators.
- Impedance Matching: Optimize power transfer between different circuit components.
- Sensor Systems: Used in inductive and capacitive sensors.

Conclusion: Key Takeaways About the LRC Series Circuit

Understanding the behavior of an LRC series circuit with specified components and operating voltage provides critical insights into how AC systems work. The interplay of resistance, inductance, and capacitance determines the circuit's impedance, phase relationships, and power consumption.

In the specific case analyzed:

- The impedance is approximately 80.42 Ω at 60 Hz.

- The current flowing from a 120 V source is around 1.49 A.
- The circuit exhibits a significant phase lag due to dominant capacitive reactance.
- The resonance frequency is approximately 184 Hz, where the circuit's impedance would be minimized, and current maximized.

Such analysis is essential for designing and optimizing electronic circuits, ensuring efficient power use, and developing advanced communication and filtering systems. Whether tuning a radio or designing a sensor, understanding LRC circuits is a cornerstone of electrical engineering.

Further Considerations and Advanced Topics

- Effect of Frequency Variation: How changing the frequency affects impedance and resonance.
- Damping Effects: Role of resistance in controlling oscillations.
- Quality Factor (Q): Measure of circuit sharpness at resonance.
- Transient Response: Behavior when switching the circuit on or off.
- Non-Ideal Components: Real-world deviations from idealized models.

By mastering these concepts, engineers and students can better analyze, troubleshoot

Frequently Asked Questions

What is the resonant frequency of the LRC series circuit with R = 15.0 Ω, L = 25.0 mH, and C = 30.0 μF?

The resonant frequency is approximately 1,832 Hz, calculated using $f_0 = 1 / (2\pi\sqrt{(LC)})$.

How do you calculate the impedance of this LRC circuit at resonance?

At resonance, the impedance is purely resistive and equals R, which is 15.0 Ω.

What is the current flowing through the circuit when connected to a 120 V source at resonance?

The current is I = V / R = 120 V / 15.0 Ω = 8.0 A.

How does the impedance change when the circuit operates below or above the resonant frequency?

Below resonance, the circuit behaves more inductively with higher impedance; above resonance, it behaves more capacitively, also increasing impedance away from resonance.

What is the quality factor (Q) of this LRC circuit?

$Q = (1 / R) \sqrt{(L / C)} \approx (1 / 15.0) \sqrt{(25.0\times10^{-3} / 30.0\times10^{-6})} \approx 8.6$.

What is the inductive reactance (X_L) at the resonant frequency?

$X_L = 2\pi f_0 L \approx 2\pi$ 1832 Hz 25.0 mH ≈ 288 Ω.

What is the capacitive reactance (X_C) at the resonant frequency?

$X_C = 1 / (2\pi f_0 C) \approx 1 / (2\pi$ 1832 Hz 30.0 μF) ≈ 288 Ω.

Why does the circuit exhibit maximum current at the resonant frequency?

Because at resonance, the inductive and capacitive reactances cancel out, leaving the impedance at its minimum (equal to R), thus maximizing current.

How does changing the resistance R affect the bandwidth of the resonance in this circuit?

Increasing R broadens the bandwidth, resulting in a less sharp resonance; decreasing R narrows the bandwidth, making the resonance sharper.

What is the significance of the LRC series circuit's resonance in practical applications?

Resonance allows selective filtering, tuning radios, and in impedance matching, making it vital in communication and signal processing systems.

Additional Resources

An LRC Series Circuit: An In-Depth Analysis of Its Components, Behavior, and Practical Implications

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Introduction to LRC Series Circuits

In the realm of electrical engineering, understanding the behavior of combined circuit elements is fundamental to designing and optimizing electronic devices. An LRC series circuit, comprising an inductor (L), resistor (R), and capacitor (C) connected in a single loop, serves as a cornerstone for various applications, including filters, oscillators, and tuning circuits. When powered by an AC source, such as a 120 V supply, the circuit exhibits complex interactions between its components, leading to phenomena like resonance, impedance variation, and phase shifts.

This article provides a comprehensive exploration of an LRC series circuit with specific component values: R = 15.0 Ω, L = 25.0 mH, C = 30.0 μF, connected to a 120 V AC source. Through detailed explanations, mathematical derivations, and practical insights, we aim to elucidate the circuit’s behavior, its resonant properties, and its significance in electronic systems.

Component Specifications and Basic Concepts

Resistor (R = 15.0 Ω)

The resistor introduces resistance to the circuit, dissipating energy as heat. Its value influences the circuit’s damping and bandwidth. A resistance of 15 Ω indicates moderate damping, affecting the sharpness of resonance and the amplitude of current.

Inductor (L = 25.0 mH)

An inductor opposes changes in current due to its stored magnetic energy. The given inductance of 25.0 millihenries (mH) signifies a relatively small inductance, which plays a crucial role in the circuit's reactive behavior and resonance.

Capacitor (C = 30.0 μF)

Capacitance determines the circuit’s ability to store and release electrical energy in the electric field. A value of 30.0 microfarads (μF) complements the inductor to produce resonant conditions at specific frequencies.

AC Power Supply (V = 120 V)

The circuit is powered by a standard household AC voltage of 120 volts. The nature of AC supply introduces phase differences and frequency-dependent impedance, which are central to the circuit's analysis.

Theoretical Foundations and Key Equations

Impedance in an LRC Series Circuit

The total opposition to current in an AC circuit is called impedance (Z), expressed as:

$$ Z = \sqrt{R^2 + (X_L - X_C)^2} $$

where:
- R is resistance,
- $X_L = 2\pi f L$ is inductive reactance,
- $X_C = \frac{1}{2\pi f C}$ is capacitive reactance,
- f is the frequency of the AC source.

The impedance determines the amplitude of the current for a given voltage:

$$ I = \frac{V}{Z} $$

Resonance Condition

Resonance occurs when the inductive reactance equals the capacitive reactance:

$$ X_L = X_C \Rightarrow 2\pi f_0 L = \frac{1}{2\pi f_0 C} $$

Solving for the resonant frequency:

$$ f_0 = \frac{1}{2\pi \sqrt{LC}} $$

This frequency signifies where the circuit's impedance is minimized (equal to R), and the current reaches its maximum.

Calculating the Resonant Frequency

Given the component values:
- $L = 25.0 \text{ mH} = 25.0 \times 10^{-3} \text{ H}$
- $C = 30.0 \text{ μF} = 30.0 \times 10^{-6} \text{ F}$

Substituting into the formula:

$$
f_0 = \frac{1}{2\pi \sqrt{L C}} = \frac{1}{2\pi \sqrt{(25.0 \times 10^{-3})(30.0 \times 10^{-6})}}
$$

Calculating the denominator:

$$
\sqrt{(25.0 \times 10^{-3})(30.0 \times 10^{-6})} = \sqrt{7.5 \times 10^{-7}} \approx 8.66 \times 10^{-4}
$$

Therefore:

$$
f_0 = \frac{1}{2\pi \times 8.66 \times 10^{-4}} \approx \frac{1}{5.44 \times 10^{-3}} \approx 183.9 \text{ Hz}
$$

Interpretation: The circuit resonates at approximately 184 Hz, which is significantly lower than standard mains frequency (60 Hz). This indicates that at 120 V AC from the mains, the circuit operates well below its resonant frequency, affecting the current and impedance characteristics.

Impedance and Current at 120 V

Since the circuit is powered at 120 V and the resonance is at about 184 Hz, the actual frequency of the supply (60 Hz in many regions) influences the circuit behavior.

Calculating Reactances at 60 Hz:

- $X_L = 2\pi f L = 2\pi \times 60 \times 25.0 \times 10^{-3} \approx 9.42 \, \Omega$
- $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 60 \times 30.0 \times 10^{-6}} \approx 88.4 \, \Omega$

The net reactance:

$$
X_{net} = X_L - X_C = 9.42 - 88.4 \approx -78.98 \, \Omega
$$

The total impedance:

$$
Z = \sqrt{R^2 + X_{net}^2} = \sqrt{(15)^2 + (-78.98)^2} \approx \sqrt{225 + 6237} \approx \sqrt{6462} \approx 80.4 \, \Omega
$$

The resulting current:

$$
I = \frac{V}{Z} = \frac{120}{80.4} \approx 1.49 \, \text{A}
$$

This relatively low current reflects the high reactance at 60 Hz, dominated by the capacitor.

Phase Angle and Power Factor

The phase angle (ϕ) between voltage and current is:

$$
\phi = \arctan \left(\frac{X_{net}}{R} \right) = \arctan \left(\frac{-78.98}{15} \right) \approx -79.2^\circ
$$

A negative phase angle indicates that the current leads the voltage, characteristic of a circuit dominated by capacitive reactance. The power factor (cos ϕ):

$$
\text{Power Factor} = \cos(-79.2^\circ) \approx 0.19
$$

This low power factor signifies poor efficiency in energy transfer at this frequency, common in reactive-dominant circuits.

Resonance and Its Practical Significance

Resonant Behavior in LRC Circuits

Resonance is a fundamental property that allows circuits to selectively amplify or filter specific frequencies. When the circuit operates at its resonant frequency ($f_0 \approx 184\, \text{Hz}$), the inductive and capacitive reactances cancel out:

$$
X_L = X_C \Rightarrow Z = R
$$

In this ideal case, the impedance is minimized, and the circuit exhibits maximum current for a given voltage. This phenomenon underpins the design of radio tuners, filters, and oscillators.

Implications at 60 Hz

Since the mains supply operates at 60 Hz, far below the resonance frequency, the circuit behaves as a highly reactive network with high impedance and low current. This situation is typical in applications where the circuit is designed to be selective or resistant to certain frequencies.

Adjustments for Resonance

To utilize the circuit’s resonant properties at mains frequency:
- The inductance or capacitance can be tuned.
- For example, reducing C or increasing L to bring f_0 closer to 60 Hz.
- Alternatively, using frequency converters or specialized components allows for resonance at desired frequencies.

Energy Storage and Power Dissipation

Energy in Inductor and Capacitor

At any instant, the energy stored in the magnetic field of the inductor:

$$
U_L = \frac{1}{2} L I^2
$$

Similarly, in the capacitor:

\[

$$U_C = \frac{1}{2} C V_C^2$$

During oscillations, energy continually transfers between magnetic and electric fields, leading to reactive power flow that does not result in net energy consumption over time.

Res

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