An Object Of Mass M Is Launched From A Planet Of Mass M And Radius R. 50% Part (a) Derive And Enter An

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Understanding the physics behind objects launched from planetary surfaces is fundamental in astrophysics and aerospace engineering. When an object of mass M is launched from a planet with the same mass M and radius R, analyzing its motion involves gravitational principles, energy conservation, and kinematics. This article aims to provide a comprehensive derivation of the relevant equations, focusing on the key concepts involved in such a scenario. Whether you're a student preparing for exams, a researcher, or an enthusiast in gravitational physics, this detailed explanation will clarify the derivation process and help you understand how to calculate important parameters such as escape velocity and final velocity at infinity.

Fundamental Concepts in Gravitational Physics

Before delving into the derivation, it is essential to understand some foundational concepts that underpin the physics of objects launched from celestial bodies.

Gravitational Potential Energy

- The gravitational potential energy (U) of an object of mass m at a distance r from the center of a planet of mass M is given by:

- $U = -GMm / r$

- This negative sign indicates that the energy is lower (more negative) when the object is closer to the planet's center.

Escape Velocity

- The minimum velocity required for an object to escape the gravitational pull of a planet without further propulsion is called the escape velocity (v_e).
- It is derived by equating the initial kinetic energy to the magnitude of the gravitational potential energy at the surface:

- $v_e = \sqrt{2GM / R}$

- For the case where the mass of the planet is equal to the mass of the object (M = m), the formula simplifies accordingly.

Energy Conservation Principle

- The total mechanical energy (kinetic + potential) remains constant if only conservative forces are acting.
- This principle is crucial for deriving the velocity of the object as it moves away from the planet.

Derivation of the Velocity of the Object at Infinity

In this section, we will derive the velocity of the object as it moves infinitely far from the planet, assuming it is launched with an initial velocity v_0 from the surface.

Initial Conditions

- Mass of the planet: M
- Radius of the planet: R
- Mass of the object: M (as per the problem statement)
- Initial velocity at the surface: v_0
- Initial position: $r = R$

Applying Conservation of Energy

- At the initial point (surface), the total energy (E_0) is:

- $E_0 = (1/2) M v_0^2 - GM M / R$

- At a point infinitely far away ($r \to \infty$), the gravitational potential energy approaches zero, and the velocity is v_∞.
- The total energy at infinity (E_∞) is:

- $E_\infty = (1/2) M v_\infty^2$

Since energy is conserved:

$$E_0 = E_\infty$$

$$(1/2) M v_0^2 - GM M / R = (1/2) M v_\infty^2$$

Dividing through by M:

$$(1/2) v_0^2 - GM / R = (1/2) v_\infty^2$$

Rearranged:

$$v_\infty^2 = v_0^2 - 2GM / R$$

This fundamental equation relates the initial launch velocity, the gravitational parameters of the planet, and the velocity at infinity.

Special Cases

- If the initial velocity v_0 equals the escape velocity v_e:

- $v_0 = v_e = \sqrt{2GM / R}$

- Substituting into the equation:

- $v_\infty^2 = (\sqrt{2GM / R})^2 - 2GM / R = 0$

- Therefore, the object just escapes the gravitational field with zero residual velocity at infinity.

Calculating the Final Velocity at Infinity: Practical Applications

Knowing how to derive the velocity at infinity helps in various practical scenarios, such as designing spacecraft trajectories, analyzing meteorite escape conditions, or understanding planetary ejections.

Example Calculation

Suppose an object is launched from the surface of a planet with the same mass M and radius R, with an initial velocity v_0.

- If $v_0 < v_e$:

- The object will rise to a maximum height and fall back, eventually reaching zero velocity at some distance before falling back to the surface.

- If $v_0 = v_e$:

- Object escapes the planet's gravity but leaves with zero velocity at infinity.

- If $v_0 > v_e$:

- The object will escape with a residual velocity v_∞, computed as:

$$v_\infty = \sqrt{v_0^2 - v_e^2}$$

Conclusion: Significance of Deriving Velocity at Infinity

The derivation of the velocity of an object launched from a planet of mass M and radius R, particularly at infinity, is fundamental in understanding gravitational escape phenomena. It provides insights into how initial launch conditions influence the object's trajectory and ultimate fate. This understanding aids in the design of space missions, the study of natural celestial events, and the broader field of astrophysics.

By applying the principles of energy conservation and gravitational potential, scientists and engineers can predict whether an object will escape a planet's gravity or remain bound. Mastery of these derivations not only enhances theoretical knowledge but also informs practical applications in space exploration and planetary sciences.

Key Takeaways:
- The velocity at infinity is derived from energy conservation principles.
- Escape velocity for a planet of mass M and radius R is $v_e = \sqrt{2GM / R}$.
- The velocity at infinity after launch with initial velocity v_0 is $v_\infty = \sqrt{v_0^2 - 2GM / R}$.
- These formulas are crucial in understanding and predicting the motion of objects in gravitational fields.

Whether you're studying physics, preparing for exams, or working on space mission designs, grasping these fundamental derivations is essential for advancing in gravitational physics and aerospace engineering.

Frequently Asked Questions

What is the initial velocity needed for an object of mass M launched from a planet of mass M and radius R to escape the planet's gravitational pull?

The initial velocity required is the escape velocity, given by $v = \sqrt{2GM/R}$, where G is the gravitational constant. Since the planet and object have the same mass M, the formula remains the same: $v = \sqrt{2GM/R}$.

How does the mass of the planet and the launched object affect the object's trajectory after launch?

The mass of the planet influences the gravitational pull, affecting the trajectory, while the mass of the launched object does not affect the trajectory in a gravitational field (assuming no other forces), due to the equivalence principle. Thus, the path depends primarily on the planet's mass and the initial conditions.

Derive the expression for the gravitational potential energy of the object at the planet's surface.

The gravitational potential energy (U) at the surface is U = -GMm/R, where M is the planet's mass, m is the object's mass, and R is the radius of the planet. Since M = m in this problem, U = -G M^2 / R.

What is the significance of the 50% part mentioned in the problem statement?

The '50% part' likely refers to a specific portion of the problem, such as the object being given half of the escape velocity or analyzing the energy after traveling a certain fraction of the distance. Clarification is needed, but it typically relates to partial energy calculations or partial trajectories.

How does the launch angle influence the range and maximum height of the object?

The launch angle determines the projectile's trajectory; an angle of 45° maximizes the range in a uniform gravitational field, while the maximum height depends on the sine of twice the launch angle. In a planetary gravitational field, the angle influences the trajectory shape, with vertical launches reaching maximum height and horizontal launches covering maximum horizontal distance before falling back.

Can the derivation of the object’s trajectory be simplified assuming a uniform gravitational field?

While assuming a uniform gravitational field simplifies calculations, in reality, gravitational acceleration varies with distance from the planet's center. For precise results, the inverse-square law should be used. However, for small distances relative to the planet's radius, the uniform field approximation provides a reasonable estimate and simplifies the derivation.

Additional Resources

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Introduction

In the realm of astrophysics and classical mechanics, understanding the motion of objects under gravitational influence is fundamental. Consider a scenario where an object with mass \(M \) is launched from a planet that has the same mass \(M \) and radius \(R \). This scenario, though simplified, offers rich insights into gravitational potential energy, escape velocity, and the energy considerations involved in space travel. In this article, we will delve into the derivation of key expressions relevant to this situation, focusing particularly on part (a) of the problem: deriving the expression for the velocity of the object at a given point, the energy considerations involved, and the conditions necessary for escape.

The Physical Setup

Imagine a planet with a known mass M and radius R. An object of the same mass M is launched vertically from the surface of this planet with an initial velocity v_0. The primary questions we seek to answer include:

- What is the velocity of the object at a distance r from the planet's center?
- What are the energy requirements for the object to escape the planet's gravitational pull?
- How does the mass of the object factor into the overall energy considerations?

To analyze this, we will utilize Newtonian gravity and the conservation of mechanical energy, which states that the total energy (kinetic plus potential) remains constant in the absence of external forces.

Gravitational Potential Energy and Escape Velocity

Gravitational Potential Energy

The gravitational potential energy U of an object of mass m at a distance r from the center of a planet of mass M is given by:

$$
U(r) = - \frac{G M m}{r}
$$

Here, G is the universal gravitational constant ($6.674 \times 10^{-11} \, \mathrm{Nm^2/kg^2}$). The negative sign indicates that the potential energy is zero at infinity and becomes more negative as the object approaches the planet.

Initial Conditions at the Surface

At the planet's surface ($r = R$), the object has initial velocity v_0 and potential energy:

$$
U(R) = - \frac{G M m}{R}
$$

and kinetic energy:

$$
K_0 = \frac{1}{2} m v_0^2
$$

The total mechanical energy initially is:

$$
E_{initial} = K_0 + U(R) = \frac{1}{2} m v_0^2 - \frac{G M m}{R}
$$

Conservation of Mechanical Energy

Assuming no external forces or energy losses, the total energy remains constant as the object moves away from the planet:

$$
E_{initial} = E(r) = \frac{1}{2} m v(r)^2 - \frac{G M m}{r}
$$

where $v(r)$ is the velocity of the object at a distance r.

Dividing through by m (since it appears in every term), we get:

$$
\frac{1}{2} v_0^2 - \frac{G M}{R} = \frac{1}{2} v(r)^2 - \frac{G M}{r}
$$

Rearranged to solve for $v(r)$:

$$
v(r) = \sqrt{v_0^2 + 2 G M \left(\frac{1}{r} - \frac{1}{R} \right)}
$$

This expression describes how the velocity of the object changes as it moves away from the planet.

Part (a): Derivation of Velocity at a Distance r

Step-by-step Derivation

1. Start with conservation of energy:

$$
\frac{1}{2} v_0^2 - \frac{G M}{R} = \frac{1}{2} v(r)^2 - \frac{G M}{r}
$$

2. Isolate $v(r)$:

$$
\frac{1}{2} v(r)^2 = \frac{1}{2} v_0^2 + G M \left(\frac{1}{r} - \frac{1}{R} \right)
$$

3. Multiply both sides by 2:

$$
v(r)^2 = v_0^2 + 2 G M \left(\frac{1}{r} - \frac{1}{R} \right)
$$

4. Take the square root:

$$
v(r) = \sqrt{v_0^2 + 2 G M \left(\frac{1}{r} - \frac{1}{R} \right)}
$$

This is the velocity of the object at any distance r from the planet's center, given initial launch velocity v_0.

Critical Conditions: Escape Velocity

One key concept in this context is the escape velocity v_{esc}, which is the minimum initial velocity needed for the object to reach infinity with zero residual velocity, overcoming the gravitational pull of the planet.

Setting $v(r) = 0$ at $r \to \infty$, the energy conservation condition becomes:

$$
0 = v_0^2 + 2 G M \left(0 - \frac{1}{R} \right)
$$

Simplifying:

$$
v_0^2 = \frac{2 G M}{R}
$$

Thus, the escape velocity is:

$$
v_{esc} = \sqrt{\frac{2 G M}{R}}
$$

This value is fundamental for space missions and understanding the energy thresholds needed for objects to leave planetary gravitational fields.

Special Case: Launching from a Planet of Mass M and Radius R

In our scenario, the object is launched from a planet with the same mass M. The initial velocity v_0 can be less than, equal to, or greater than v_{esc}, depending on whether the goal is to escape or to reach specific altitudes.

- If $v_0 < v_{esc}$: The object will reach some maximum height and fall back.
- If $v_0 = v_{esc}$: The object reaches infinity with zero residual speed.
- If $v_0 > v_{esc}$: The object escapes the planet’s gravitational influence, continuing to infinity with some residual velocity.

Energy Considerations for a Given Initial Velocity

The total initial energy per unit mass is:

$$
E_{initial} = \frac{1}{2} v_0^2 - \frac{G M}{R}
$$

To determine whether an object can escape, compare $E_{initial}$ to zero:

- $E_{initial} \geq 0$: Escape is possible.
- $E_{initial} < 0$: The object is gravitationally bound.

This energy perspective is crucial when planning launches, as it directly informs the required initial velocity or energy input.

The Role of the Object’s Mass M

An interesting aspect of the problem is that both the planet and the object have the same mass M. This symmetry simplifies the analysis but also introduces considerations about the mutual gravitational interaction:

- Mutual gravitational force: Both masses attract each other with equal magnitude but opposite directions.
- Center of mass: The combined system's center of mass remains stationary if initially at rest, affecting the motion of each body.
- Reduced mass concept: For two bodies of equal mass M, the relative motion can be analyzed using the reduced mass $\mu = M/2$.

In classical mechanics, when considering the motion of two bodies under mutual attraction, the problem reduces to a single body of mass μ moving in the gravitational potential of the other. Since both are equal in this scenario, the dynamics involve their relative motion, but as the focus here is on the launched object, we treat the planet as a fixed mass for simplicity, assuming its mass is much larger or that the planet is stationary initially.

Practical Implications and Applications

Understanding the derivation of velocity and energy requirements has practical implications:

- Designing space missions: Knowing the necessary launch velocity to reach specific orbits or escape velocity.
- Assessing energy costs: Estimating fuel and energy needs for space travel.
- Planetary defense and exploration: Planning missions to other planets or moons with similar mass and radius parameters.

Furthermore, these principles underpin technologies like rockets, satellites, and interplanetary probes, making the theoretical derivations directly relevant to real-world engineering.

Conclusion

In analyzing an object of mass M launched from a planet of identical mass M and radius R, the core physics revolves around gravitational potential energy, kinetic energy, and conservation principles. The key takeaway from part (a) is the derivation of the velocity at any distance r, encapsulated in the expression:

\[
v(r) = \sqrt{v_0^2 + 2 G M \left(\frac{1}{r} - \frac{1}{R} \right

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