Answer The Following Questions: (1) Determine If The Sequence 2n+1/n+1, N 1 Is Increasing, Decreasing,

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Introduction

Understanding the behavior of sequences is fundamental in calculus and mathematical analysis. One common task is to determine whether a given sequence is increasing, decreasing, or neither as the term index n progresses. In this article, we will analyze the sequence $a_n = \frac{2n + 1}{n + 1}$ for $n \geq 1$ to determine its monotonicity. Through detailed examination, including algebraic manipulation and limits, we will establish whether the sequence is increasing, decreasing, or exhibits other behaviors.

Understanding the Sequence $a_n = \frac{2n + 1}{n + 1}$

Before diving into the analysis, let's familiarize ourselves with the sequence's structure.

Sequence Definition

The sequence is given by:

$$
a_n = \frac{2n + 1}{n + 1}
$$

where n is a natural number starting from 1 (i.e., $n \geq 1$).

Initial Terms

Calculating the first few terms:

- For $n=1$:

$$
a_1 = \frac{2(1) + 1}{1 + 1} = \frac{3}{2} = 1.5
$$

- For $n=2$:

$$
a_2 = \frac{4 + 1}{2 + 1} = \frac{5}{3} \approx 1.6667
$$

- For $n=3$:

$$
a_3 = \frac{6 + 1}{3 + 1} = \frac{7}{4} = 1.75
$$

- For $n=4$:

$$
a_4 = \frac{8 + 1}{4 + 1} = \frac{9}{5} = 1.8
$$

From these initial terms, it appears the sequence is increasing. Our goal is to rigorously verify this behavior.

Methodology for Determining Monotonicity

To analyze whether a_n is increasing or decreasing, we examine the difference between successive terms:

$$
a_{n+1} - a_n
$$

If this difference is positive for all n, the sequence is increasing; if negative, decreasing.

Alternatively, for sequences defined on integers, analyzing the difference is a direct approach. For more general functions or sequences, derivatives can be employed, but since a_n is discrete, difference analysis is appropriate.

Calculating the Difference $a_{n+1} - a_n$

Let's derive an expression for the difference between consecutive terms:

$$
a_{n} = \frac{2n + 1}{n + 1}
$$

$$
a_{n+1} = \frac{2(n+1) + 1}{(n+1) + 1} = \frac{2n + 2 + 1}{n + 2} = \frac{2n + 3}{n + 2}
$$

Now, compute:

$$
a_{n+1} - a_n = \frac{2n + 3}{n + 2} - \frac{2n + 1}{n + 1}
$$

Find a common denominator:

$$
a_{n+1} - a_n = \frac{(2n + 3)(n + 1) - (2n + 1)(n + 2)}{(n + 2)(n + 1)}
$$

Expand the numerators:

$$
(2n + 3)(n + 1) = (2n)(n + 1) + 3(n + 1) = 2n^2 + 2n + 3n + 3 = 2n^2 + 5n + 3
$$

$$
(2n + 1)(n + 2) = 2n(n + 2) + 1(n + 2) = 2n^2 + 4n + n + 2 = 2n^2 + 5n + 2
$$

Subtract:

$$
(2n^2 + 5n + 3) - (2n^2 + 5n + 2) = 1
$$

Thus,

$$
a_{n+1} - a_n = \frac{1}{(n + 2)(n + 1)}
$$

Since $n \geq 1$, both $n + 1$ and $n + 2$ are positive, making the denominator positive. Therefore,

$$
a_{n+1} - a_n = \frac{1}{(n + 2)(n + 1)} > 0
$$

for all $n \geq 1$.

Conclusion: The difference between successive terms is always positive, indicating that the sequence a_n is strictly increasing for all $n \geq 1$.

Limit of the Sequence as $n \to \infty$

While the sequence is increasing, it's also insightful to examine its behavior as n approaches infinity.

Calculating the limit:

$$
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{2n + 1}{n + 1}
$$

Divide numerator and denominator by n:

$$
= \lim_{n \to \infty} \frac{2 + \frac{1}{n}}{1 + \frac{1}{n}} = \frac{2 + 0}{1 + 0} = 2
$$

Since the sequence increases and approaches 2, we can conclude:

$$
a_n \to 2 \quad \text{as} \quad n \to \infty
$$

This indicates the sequence is bounded above by 2 and increases monotonically towards this limit.

Summary of Results

Based on the calculations and analysis:

- The sequence $a_n = \frac{2n + 1}{n + 1}$ is strictly increasing for all $n \geq 1$.
- The difference between successive terms is positive for all $n \geq 1$, confirming the monotonic increase.
- The sequence converges to 2 as $n \to \infty$.

Implications and Applications

Understanding the monotonicity of sequences like a_n has several practical applications:

- Series convergence analysis: Knowing whether a sequence converges and its monotonicity helps in evaluating the convergence of related series.

- Limit approximation: Recognizing the limit aids in estimating large terms of the sequence or sums involving such sequences.

- Mathematical modeling: Sequences that increase towards a finite limit are common in models where gradual stabilization or equilibrium is observed.

Additional Examples of Sequence Analysis

To deepen understanding, consider analyzing other sequences using similar methods:

- Sequence decreasing toward a limit:

$$
b_n = \frac{n}{n + 1}
$$

Here, the difference $b_{n+1} - b_n$ is negative, indicating decreasing behavior.

- Sequence with oscillations:

$$
c_n = (-1)^n \frac{1}{n}
$$

This sequence oscillates but with diminishing amplitude, converging to 0.

Understanding these behaviors enhances the ability to analyze diverse sequences in calculus and real analysis.

Conclusion

In conclusion, the sequence $a_n = \frac{2n + 1}{n + 1}$ is strictly increasing for all $n \geq 1$ and converges to 2 as $n \to \infty$. This outcome is confirmed through direct difference calculation, demonstrating that each successive term is larger than the previous one. Recognizing such properties is crucial in advanced mathematical analysis, enabling precise understanding of sequence behaviors and their limits. Whether employed in theoretical contexts or practical applications, the ability to determine monotonicity and limits remains a fundamental skill in mathematics.

Frequently Asked Questions

Is the sequence defined by $a_n = (2n + 1)/(n + 1)$ increasing or decreasing for $n \geq 1$?

The sequence is decreasing for $n \geq 1$ because the ratio approaches 2 as n increases, and the terms decrease from 3 at n=1 towards 2.

How can we determine if the sequence (2n + 1)/(n + 1) is increasing or decreasing?

By analyzing the difference between successive terms or using the derivative of the function, we can determine whether the sequence is increasing or decreasing.

What is the limit of the sequence (2n + 1)/(n + 1) as n approaches infinity?

The limit is 2 because as n approaches infinity, the dominant terms give (2n)/(n) = 2.

Can the sequence (2n + 1)/(n + 1) be increasing for some values of n?

No, the sequence is decreasing for all $n \geq 1$ because the difference between successive terms is negative for all $n \geq 1$.

What is the first term of the sequence (2n + 1)/(n + 1) at n=1?

At n=1, the term is (21 + 1)/(1 + 1) = 3/2 = 1.5.

How does the behavior of the sequence (2n + 1)/(n + 1) change as n increases?

The sequence decreases and approaches the horizontal asymptote at 2 as n increases.

Is the sequence (2n + 1)/(n + 1) bounded?

Yes, the sequence is bounded between values just above 1.5 and approaching 2 as n grows large.

What mathematical tools can be used to analyze the monotonicity of the sequence?

Calculus tools such as derivatives of the corresponding function or difference tests between successive terms can be used to analyze whether the sequence is increasing or decreasing.

Additional Resources

Answer The Following Questions: (1) Determine If The Sequence 2n+1/n+1, N 1 Is Increasing, Decreasing

Understanding the behavior of sequences is fundamental in mathematical analysis, particularly in calculus and discrete mathematics. It allows us to grasp how a sequence evolves as its index grows large, which has implications across various scientific and engineering disciplines. In this article, we delve into a specific sequence defined by the formula $a_n = \frac{2n + 1}{n + 1}$ for $n \geq 1$, and analyze whether this sequence is increasing, decreasing, or exhibits some other pattern as n progresses.

This exploration not only enhances our comprehension of sequence behavior but also provides insight into the techniques used to determine monotonicity—an essential aspect of mathematical analysis. By applying algebraic manipulations, limits, and derivative-based tests, we aim to unravel the trend of this sequence, offering both a rigorous and intuitive understanding.

Understanding the Sequence $a_n = \frac{2n + 1}{n + 1}$

Before analyzing whether the sequence is increasing or decreasing, it is vital to understand its structure and what it signifies.

Sequence Definition and Basic Intuition

The sequence in question is:

$$a_n = \frac{2n + 1}{n + 1} \quad \text{for} \quad n \geq 1$$

At first glance, this sequence involves a rational function where both numerator and denominator are linear in n. As n increases, both numerator and denominator increase, but the key is to understand how their ratio behaves.

Initial Values and Pattern Recognition

Calculating the first few terms:

- For $n=1$:

$$a_1 = \frac{2(1) + 1}{1 + 1} = \frac{3}{2} = 1.5$$

- For $n=2$:

$$a_2 = \frac{4 + 1}{2 + 1} = \frac{5}{3} \approx 1.6667$$

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$$a_3 = \frac{6 + 1}{3 + 1} = \frac{7}{4} = 1.75$$

- For $n=4$:

$$a_4 = \frac{8 + 1}{4 + 1} = \frac{9}{5} = 1.8$$

- For $n=5$:

$$a_5 = \frac{10 + 1}{5 + 1} = \frac{11}{6} \approx 1.8333$$

From these initial values, it appears that a_n is increasing as n increases. To confirm this trend rigorously, we need to analyze the sequence's behavior mathematically.

Mathematical Analysis of Monotonicity

To determine whether the sequence is increasing or decreasing, we can approach the problem using algebraic manipulation, limits, or calculus-based derivative tests.

Method 1: Direct Comparison of Consecutive Terms

A straightforward way to analyze the monotonicity of a sequence is to compare consecutive terms:

$$
a_{n+1} \quad \text{and} \quad a_n
$$

The sequence is increasing if:

$$
a_{n+1} > a_{n}
$$

and decreasing if:

$$
a_{n+1} < a_{n}
$$

Calculating a_{n+1}:

$$
a_{n+1} = \frac{2(n+1) + 1}{(n+1) + 1} = \frac{2n + 2 + 1}{n + 2} = \frac{2n + 3}{n + 2}
$$

Now, compare a_{n+1} and a_n:

$$
a_{n+1} - a_n = \frac{2n + 3}{n + 2} - \frac{2n + 1}{n + 1}
$$

Find a common denominator:

$$
a_{n+1} - a_n = \frac{(2n + 3)(n + 1) - (2n + 1)(n + 2)}{(n + 2)(n + 1)}
$$

Expand numerator:

$$
(2n + 3)(n + 1) = 2n(n + 1) + 3(n + 1) = 2n^2 + 2n + 3n + 3 = 2n^2 + 5n + 3
$$

$$
(2n + 1)(n + 2) = 2n(n + 2) + 1(n + 2) = 2n^2 + 4n + n + 2 = 2n^2 + 5n + 2
$$

Subtract numerator:

$$
(2n^2 + 5n + 3) - (2n^2 + 5n + 2) = (2n^2 - 2n^2) + (5n - 5n) + (3 - 2) = 1
$$

Therefore:

$$
a_{n+1} - a_n = \frac{1}{(n + 2)(n + 1)} > 0
$$

Since the denominator is always positive for $n \geq 1$, the difference is positive for all $n \geq 1$.

Conclusion: The sequence a_n is strictly increasing for all $n \geq 1$.

Method 2: Limit Analysis for Large n

Another approach involves examining the limit of a_n as $n \to \infty$:

$$
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{2n + 1}{n + 1}
$$

Divide numerator and denominator by n:

$$
\lim_{n \to \infty} \frac{2 + \frac{1}{n}}{1 + \frac{1}{n}} = \frac{2 + 0}{1 + 0} = 2
$$

The sequence tends towards 2 as n increases, and since the sequence is strictly increasing (from the previous method), it converges to 2 from below.

Alternative Approach: Using Calculus

While the previous methods are algebraic and limit-based, calculus provides a more general and elegant way to analyze the monotonicity of sequences, especially when sequences are derived from functions.

Expressing the Sequence as a Function

Define the function:

$$
f(n) = \frac{2n + 1}{n + 1}
$$

To analyze the behavior for real $n \geq 1$, consider the derivative $f'(n)$:

$$
f(n) = \frac{2n + 1}{n + 1}
$$

Apply the quotient rule:

$$
f'(n) = \frac{(2)(n + 1) - (2n + 1)(1)}{(n + 1)^2}
$$

Simplify numerator:

$$
2(n + 1) - (2n + 1) = 2n + 2 - 2n - 1 = 1
$$

Therefore,

$$
f'(n) = \frac{1}{(n + 1)^2}
$$

Since $(n + 1)^2 > 0$ for all $n \geq 1$, we have:

$$
f'(n) > 0
$$

for all $n \geq 1$. This indicates that $f(n)$ is strictly increasing over this domain.

Implication: Because $f(n)$ is increasing for real $n \geq 1$, the sequence $a_n = f(n)$ is also increasing for all integers $n \geq 1$.

Summary of Findings

- Comparison of consecutive terms shows that $a_{n+1} - a_{n} = \frac{1}{(n+1)(n+2)} > 0$, which confirms the sequence is strictly increasing for all $n \geq 1$.

- Limit analysis demonstrates that $a_n \to 2$ as $n \to \infty$, and the sequence approaches this limit from below.

- Calculus-based derivative confirms that the continuous extension of the sequence's defining function is increasing since $f'(n) > 0$ for all $n \geq 1$.

Final conclusion:

The sequence $a_n = \frac{2n + 1}{n + 1}$ is strictly increasing for all integers $n \geq 1$, approaching the limit value of 2 as n tends

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