As Shown In The Figure Above, Six Particles, Each With Charge Q, Are Held Fixed And Are Equally Spaced

As Shown In The Figure Above, Six Particles, Each With Charge Q, Are Held Fixed And Are Equally Spaced in a straight line, they create an intriguing scenario in electrostatics that highlights the fundamental principles governing electric forces and potentials. This configuration serves as a classic problem for understanding how multiple point charges interact, how the net electric field is determined at various points, and how the potential energy of such arrangements can be calculated. In this article, we will explore the physical principles underlying this setup, analyze the forces acting on each particle, and discuss the implications of the arrangement for electric potential and field distributions.

Understanding the Setup: Equally Spaced Charges in a Line

The Basic Configuration

Imagine six particles aligned along a straight line, each separated by an equal distance, say d. All particles possess the same charge, denoted by Q. Because the particles are fixed in position, their relative distances do not change, and they exert electrostatic forces on each other according to Coulomb's Law. This regular arrangement simplifies the analysis of the electric fields and potentials at various points along or near the line.

Significance of Equal Spacing and Identical Charges

The uniform spacing and identical charges create symmetry in the system, which is instrumental in simplifying calculations:

- Symmetry allows for the use of superposition principles effectively.
- It helps in determining points where the net electric field is zero.
- It facilitates the calculation of potential energy stored within the configuration.

This setup is a foundational model for understanding more complex arrangements and is often used in teaching electrostatics concepts.

Electrostatic Forces Between Particles

Applying Coulomb's Law

The force between any two point charges is given by Coulomb's Law:

$$ F = \frac{k |Q_1 Q_2|}{r^2} $$

where:

- k is Coulomb's constant ($8.99 \times 10^9\, \mathrm{Nm^2/C^2}$)
- Q_1 and Q_2 are the magnitudes of the charges (here both are Q)
- r is the distance between the charges

Since all charges are identical and equally spaced, the distance between any two charges can be expressed as multiples of d. For two charges separated by n spacings, the distance is n d.

Net Force on a Particle

To find the net force acting on a specific particle (say, the third particle from the left), we consider the forces due to all other particles. Because the charges repel each other if they are of the same sign, the forces tend to push particles apart.

For the third particle:
- It experiences repulsive forces from particles to its left (particles 1 and 2).
- It experiences repulsive forces from particles to its right (particles 4, 5, and 6).

The net force is the vector sum of all these individual forces, considering their directions along the line.

Example Calculation:
Suppose the third particle is at position x_3. The force exerted by the first particle (at x_1) on the third particle is:

$$ F_{13} = \frac{k Q^2}{(2d)^2} $$

Similarly, the force from the second particle (at x_2) is:

$$ F_{23} = \frac{k Q^2}{(d)^2} $$

The total force from particles on the left is the sum of these two, directed away from those charges:

$$ F_{left} = F_{13} + F_{23} $$

Similarly, forces from particles on the right can be calculated and summed.

Note:
- Since the system is linear and symmetric, analyzing the forces simplifies, especially if evaluating equilibrium conditions or net forces at specific points.

Electric Field and Potential Due to Multiple Charges

Superposition Principle

The electric field at any point due to multiple point charges is the vector sum of the fields due to each individual charge. Similarly, the electric potential is the algebraic sum of the potentials from each charge.

Electric Field at a Point:
$$
\vec{E} = \sum_{i=1}^{6} \vec{E}_i
$$
where each $\vec{E}_i$ is calculated as:

$$
\vec{E}_i = \frac{k Q}{r_i^2} \hat{r}_i
$$
with r_i being the distance from the ith charge to the point of interest.

Electric Potential at a Point:
$$
V = \sum_{i=1}^{6} \frac{k Q}{r_i}
$$
since potential is a scalar quantity.

Potential and Field at Special Points

- Midpoint between charges: Due to symmetry, the electric field may cancel out at the midpoint between two charges, leading to a point of zero electric field.
- Far away from the arrangement: The entire system behaves as a single charge of magnitude $6Q$, and the net electric field diminishes with distance.

Energy Considerations in the System

Potential Energy of the Configuration

The total electrostatic potential energy stored in the arrangement is the sum over all unique pairs:

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U = \sum_{i