B.Tech First Year 18. A Body Of Mass M Falls From Rest Under Gravity In A Field Whose Resistance Is Mk

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Understanding the motion of a body falling under gravity is fundamental in physics, especially in classical mechanics. When a body of mass M falls from rest in a medium that offers resistance proportional to its velocity, the problem becomes a classic example of analyzing forces, accelerations, and terminal velocity. This article provides an in-depth explanation of this scenario, including derivations, key concepts, and practical implications, structured to enhance your understanding for B.Tech first-year studies and improve your SEO visibility for related physics queries.

Introduction to Motion Under Resistance

In real-world scenarios, objects falling through a medium such as air or water experience resistive forces that oppose their motion. These resistive forces depend on various factors including the velocity of the object, the properties of the medium, and the shape and size of the body.

In the case under consideration, the resistive force is proportional to the velocity, represented mathematically as:

$$F_{resistance} = Mk v$$

Where:
- M is the mass of the body
- k is a proportionality constant representing the resistance per unit velocity
- v is the velocity of the body at any instant

This is known as linear resistance or viscous resistance and is often applicable to small objects moving in viscous media.

Problem Statement and Physical Setup

The problem involves a body of mass M, initially at rest, falling freely under gravity g, through a medium that exerts a resistive force proportional to velocity. The key points are:

- Initial velocity $v_0 = 0$
- Gravitational force $F_{gravity} = Mg$
- Resistive force $F_{resistance} = Mk v$
- No other forces act on the body

The goal is to analyze:

- The velocity of the body as a function of time
- The time to reach terminal velocity
- The terminal velocity itself
- The displacement during the fall

Mathematical Formulation of the Problem

Equation of Motion

Applying Newton's second law:

$$
M \frac{dv}{dt} = Mg - Mk v
$$

Dividing through by M:

$$
\frac{dv}{dt} = g - k v
$$

This is a first-order linear differential equation describing the velocity of the falling body over time.

Initial Conditions

Since the body starts from rest:

$$
v(0) = 0
$$

Solution to the Differential Equation

Integrating the Equation

The differential equation:

$$
\frac{dv}{dt} + k v = g
$$

can be solved using integrating factors.

The integrating factor:

$$
\mu(t) = e^{k t}
$$

Multiplying through by $e^{k t}$:

$$
e^{k t} \frac{dv}{dt} + k e^{k t} v = g e^{k t}
$$

Recognizing the left side as the derivative of $v e^{k t}$:

$$
\frac{d}{dt} [v e^{k t}] = g e^{k t}
$$

Integrate both sides:

$$
v e^{k t} = \int g e^{k t} dt + C
$$

$$
v e^{k t} = \frac{g}{k} e^{k t} + C
$$

Solve for $v(t)$:

$$
v(t) = \frac{g}{k} + C e^{-k t}
$$

Using initial condition $v(0) = 0$:

$$
0 = \frac{g}{k} + C
$$

$$
C = - \frac{g}{k}
$$

Thus, the velocity as a function of time:

$$
v(t) = \frac{g}{k} (1 - e^{-k t})
$$

Understanding Terminal Velocity

Definition of Terminal Velocity

Terminal velocity v_{t} is achieved when the acceleration becomes zero, i.e., the net force is zero, and the body moves with constant velocity.

Set $\frac{dv}{dt} = 0$:

$$
0 = g - k v_{t}
$$

Solve for v_{t}:

$$
v_{t} = \frac{g}{k}
$$

This is the terminal velocity in the case of linear resistance.

Characteristics of Terminal Velocity

- The body approaches v_{t} asymptotically.
- The velocity reaches approximately 63% of v_{t} at $t = \frac{1}{k}$.
- The time to reach near terminal velocity depends on k and the initial conditions.

Time to Reach Terminal Velocity

From the velocity expression:

$$
v(t) = v_{t} (1 - e^{-k t})
$$

To find the time $t_{0.9}$ when the velocity reaches 90% of terminal velocity:

$$
0.9 v_{t} = v_{t} (1 - e^{-k t_{0.9}})
$$

Dividing both sides:

$$
0.9 = 1 - e^{-k t_{0.9}}
$$

\]

$$
e^{-k t_{0.9}} = 0.1
$$

Taking natural logarithm:

$$
- k t_{0.9} = \ln 0.1
$$

$$
t_{0.9} = - \frac{\ln 0.1}{k} = \frac{\ln 10}{k}
$$

Since $\ln 10 \approx 2.3026$:

$$
t_{0.9} \approx \frac{2.3026}{k}
$$

This result indicates that the higher the resistance constant k, the faster the body reaches near terminal velocity.

Displacement During the Fall

To find the displacement $s(t)$, integrate the velocity:

$$
s(t) = \int_{0}^{t} v(t) dt
$$

Substitute the expression for $v(t)$:

$$
s(t) = \int_{0}^{t} \frac{g}{k} (1 - e^{-k t'}) dt'
$$

Integrate term-by-term:

$$
s(t) = \frac{g}{k} \left[t - \frac{1}{k} (1 - e^{-k t}) \right]
$$

or equivalently:

$$
s(t) = \frac{g}{k} t - \frac{g}{k^{2}} (1 - e^{-k t})
$$

This formula provides the displacement traveled after time t.

Practical Applications and Implications

Understanding this model has numerous practical applications:

- Design of Parachutes: To determine how fast a parachute slows descent considering air resistance.
- Projectile Motion Analysis: For small objects moving through viscous media.
- Engineering of Fluid Dynamics: Modeling objects moving in fluids with viscous resistance.
- Sports Science: Analyzing the fall of objects like balls or athletes' bodies in mediums like water or air.

Summary of Key Results

- The velocity of a body falling under gravity with linear resistance:

$$
v(t) = \frac{g}{k} (1 - e^{-k t})
$$

- Terminal velocity:

$$
v_{t} = \frac{g}{k}
$$

\]

- Time to reach near terminal velocity:

$$
t_{0.9} \approx \frac{2.3026}{k}
$$

- Displacement traveled:

$$
s(t) = \frac{g}{k} t - \frac{g}{k^{2}} (1 - e^{-k t})
$$

Conclusion

Analyzing the motion of a body falling under gravity with resistive force proportional to velocity reveals how resistance influences acceleration, terminal velocity, and displacement. The exponential approach to terminal velocity underscores the importance of the resistance coefficient k in determining the rate of velocity change. This problem exemplifies core concepts in classical mechanics, especially in the context of real-world physics where resistive forces cannot be ignored. Mastery of these derivations and formulas is essential for B.Tech students and provides foundational knowledge applicable in various engineering and scientific domains.

Further Reading and Resources

- Classical Mechanics by Herbert Goldstein
- Engineering Mechanics: Dynamics by J.L. Meriam and L.G. Kraige
- Khan Academy Physics Modules on Motion with Resistance
- Online simulations demonstrating falling bodies with air resistance

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By understanding these principles, students can better analyze real-world problems involving motion under resistive forces, enhancing both their theoretical and practical physics skills.

Frequently Asked Questions

What is the main focus of the problem involving a body of mass M falling under gravity with resistance Mk?

The problem analyzes the motion of a body subjected to gravitational force and a resistive force proportional to its velocity, specifically for a body starting from rest and falling under these combined forces.

How is the resistive force modeled in this problem?

The resistive force is modeled as being proportional to the velocity, expressed as Mk, where M is the mass and k is a constant of proportionality.

What differential equation describes the motion of the body in this scenario?

The equation of motion is: M dv/dt = Mg - Mk v, where v is the velocity at time t, M is the mass, g is acceleration due to gravity, and k is the resistance coefficient.

What is the expression for the velocity of the body after time t?

The velocity v(t) = (g/k) (1 - e^(-k t)), showing that the velocity approaches a terminal velocity of g/k as time increases.

How does the resistance affect the terminal velocity of the falling body?

The terminal velocity is reduced to g/k, meaning higher resistance (larger k) results in a lower terminal velocity, limiting the body's speed during fall.

What is the significance of the body starting from rest in solving this problem?

Starting from rest simplifies the integration process and allows for straightforward calculation of velocity and displacement over time, given initial conditions v(0) = 0.

Additional Resources

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Introduction

In the realm of classical mechanics, understanding the motion of bodies under various forces is fundamental. The problem of a body of mass M falling under gravity in a resistive medium introduces intriguing complexities that deepen our grasp of dynamics. This particular scenario—where a body falls from rest in a field with resistance proportional to velocity—serves as a classic example to explore concepts such as terminal velocity, differential equations of motion, and steady-state behavior.

Conceptual Overview

Key Elements of the Problem

- Mass of the body, M: The inertial property that resists changes in motion.
- Initial condition: The body starts from rest, meaning initial velocity $u_0 = 0$.
- Gravitational field: The body experiences a constant acceleration g downward.
- Resistive force: Proportional to velocity, expressed as $R = Mk \times v$, where k is a positive constant, and v is the velocity of the body at time t.

This problem models a typical resistive medium where the opposition to motion increases linearly with velocity, such as viscous drag in certain fluids or air resistance at moderate speeds.

Mathematical Formulation of the Problem

Differential Equation of Motion

Applying Newton’s second law:

$$
\text{Net force} = M \frac{dv}{dt}
$$

The forces acting are:

- Downward gravitational force: Mg
- Upward resistive force: $Mk v$

Hence, the equation of motion becomes:

$$
M \frac{dv}{dt} = Mg - Mk v
$$

Dividing throughout by M:

$$
\frac{dv}{dt} = g - k v
$$

This is a first-order linear differential equation describing the velocity $v(t)$.

Solution of the Differential Equation

Step-by-step Solution

1. Standard Form:

$$
\frac{dv}{dt} + k v = g
$$

2. Integrating Factor:

$$
\mu(t) = e^{k t}
$$

3. Multiplying through by integrating factor:

$$
e^{k t} \frac{dv}{dt} + k e^{k t} v = g e^{k t}
$$

which simplifies to:

$$
\frac{d}{dt} \left(v e^{k t} \right) = g e^{k t}
$$

4. Integrate both sides with respect to t:

$$
v e^{k t} = \frac{g}{k} e^{k t} + C
$$

where C is the integration constant.

5. Expressing $v(t)$:

$$
v(t) = \frac{g}{k} + C e^{-k t}
$$

6. Applying initial condition $v(0) = 0$:

$$
0 = \frac{g}{k} + C \Rightarrow C = - \frac{g}{k}
$$

7. Final velocity expression:

$$
v(t) = \frac{g}{k} \left(1 - e^{-k t} \right)
$$

Physical Interpretation of the Solution

- Velocity Behavior:

As $t \to \infty$:

$$
v(t) \to \frac{g}{k}
$$

which is the terminal velocity—the maximum velocity attained when the net acceleration becomes zero because the resistive force balances the gravitational pull.

- Time-dependent behavior:

- Initially, at $t=0$, $v(0)=0$.
- The exponential term $e^{-k t}$ diminishes over time, causing $v(t)$ to asymptotically approach \(

g/k).

Displacement and Velocity as Functions of Time

To understand the body's position, derive the displacement $s(t)$:

$$
s(t) = \int_0^t v(t) dt
$$

which yields:

$$
s(t) = \int_0^t \frac{g}{k} (1 - e^{-k t'}) dt'
$$

Calculating:

$$
s(t) = \frac{g}{k} \left[t + \frac{1}{k} e^{-k t} - \frac{1}{k} \right]
$$

Simplifies to:

$$
s(t) = \frac{g}{k} t - \frac{g}{k^2} (1 - e^{-k t})
$$

Key Concepts and Implications

1. Terminal Velocity

- The velocity approaches $v_{term} = \frac{g}{k}$.
- This balance point signifies that the body continues to fall at a constant speed, where the gravitational force is exactly opposed by resistive force.

2. Time to Reach Near Terminal Velocity

- Typically, after about $3/k$ seconds, the velocity is within 95% of the terminal velocity.

- This helps in practical situations where the transient phase is brief relative to the steady state.

3. Displacement Analysis

- The displacement function indicates the body accelerates initially but the rate of increase diminishes as it approaches terminal velocity.
- The maximum displacement over long times is linear with a correction term for initial acceleration.

4. Energy Considerations

- The kinetic energy increases over time, but the resistive force dissipates energy as heat or other forms.
- Steady state entails a constant kinetic energy level, with energy input from gravity balanced by losses due to resistance.

Applications and Broader Context

This model applies to numerous physical scenarios:

- Objects falling in viscous fluids: Such as a sphere in honey or syrup.
- Air resistance at moderate speeds: For example, parachutists reaching terminal velocity.
- Engineering design: To estimate the speed of vehicles or projectiles in resistive media.
- Biomechanics: Analyzing how organisms or particles settle in fluids.

Limitations and Assumptions

- Linear Resistance: Assumes resistance is proportional to velocity; in reality, resistance often varies with the square of velocity at higher speeds.
- Constant g: Assumes uniform gravitational field.
- Neglects other forces: Such as buoyancy, lift, or additional external forces.
- Rigid body assumptions: Assumes the mass remains constant, no deformation.

Advanced Topics and Extensions

1. Nonlinear Resistance:

- For high velocities, resistance is proportional to v^2.
- Differential equations become nonlinear, requiring advanced solution techniques.

2. Variable Gravity:

- In astrophysical contexts, g may vary with altitude.
- The equations adapt to include $g(s)$, leading to more complex integrals.

3. Multiple Resistance Mechanisms:

- Combining viscous and quadratic resistances introduces complexities in the differential equations.

4. Numerical Methods:

- For complex resistance models, numerical integration approaches like Euler's method or Runge-Kutta are employed.

Summary and Concluding Remarks

The problem of a body of mass M falling from rest under gravity in a resistive medium characterized by resistance proportional to velocity provides a rich analytical framework to understand real-world phenomena like terminal velocity, transient behavior, and energy dissipation.

Key takeaways:

- The velocity approaches a limiting value $v_{term} = \frac{g}{k}$.
- The differential equation governing the system is linear and solvable with standard methods.
- The displacement and velocity functions reveal the transient and steady-state behavior of the falling body.
- Practical applications span various fields, from fluid dynamics to aerospace engineering.

This problem not only reinforces foundational concepts in differential equations and physics but also illustrates how idealized models can effectively approximate complex real-world systems. Mastery of such problems is essential for students embarking on advanced studies in mechanics, fluid dynamics, and applied physics.

References for Further Reading

- H. Goldstein, Classical Mechanics, Addison Wesley.
- L.D. Landau & E.M. Lifshitz, Mechanics, Course of Theoretical Physics.
- R.K. Singh, Fundamentals of Mechanics, Dhanpat Rai Publishing Co.
- N. L. Carver Mead, Introduction to Fluid Mechanics, McGraw-Hill.

In conclusion, analyzing the motion of a body under gravity with resistance proportional to velocity reveals fundamental principles of dynamics, equilibrium, and energy transfer. Such problems lay the groundwork for understanding complex systems where forces resist motion, emphasizing the importance of differential equations and their solutions in physics.

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