Calculate The Ten-year Duration Of A 6% + $1,000 Macaulay Bond With An Annual Coupon And Redemption Of

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Understanding bond duration is a fundamental aspect of fixed-income investing, as it provides insights into a bond's sensitivity to interest rate fluctuations. In this article, we will delve into the detailed process of calculating the ten-year duration of a specific bond: a 6% + $1,000 Macaulay bond with annual coupon payments and redemption. This comprehensive guide aims to clarify the methodology, assumptions, and significance of bond duration for investors and financial analysts alike.

Introduction to Bond Duration and Its Relevance

Bond duration measures the weighted average time until an investor receives the bond's cash flows. It is a vital metric because it offers a more precise indication of a bond's price volatility relative to interest rate changes than just its maturity.

Why is bond duration important?

- Interest Rate Risk Assessment: A higher duration indicates greater sensitivity to interest rate movements.
- Portfolio Management: Helps in immunizing portfolios against interest rate risk.
- Pricing and Valuation: Used in bond pricing models to estimate price changes with interest rate shifts.

Understanding the Bond's Features

Before we proceed with the calculation, let's clearly define the characteristics of the bond:

- Coupon Rate: 6% annually
- Face Value (Redemption Value): $1,000
- Annual Coupon Payment: 6% of $1,000 = $60
- Term: 10 years
- Redemption (Face Value Repayment): $1,000 at the end of 10 years
- Payment Frequency: Annual
- Yield to Maturity (YTM): Assumed to be the same as the coupon rate (6%) for simplicity, unless

otherwise specified.

Note: The assumption that the bond’s YTM equals its coupon rate simplifies calculations. If the YTM differs, the calculation would need to incorporate that rate.

Step-by-Step Calculation of the Macaulay Duration

Calculating the Macaulay duration involves determining the present value of each cash flow, weighing each by the time until receipt, and then summing these weighted times.

Key Formula:

$$
\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} t \times \frac{C_t}{(1 + y)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1 + y)^t}}
$$

Where:
- t = year of cash flow
- C_t = cash flow at time t (coupon or redemption)
- y = yield to maturity per period (annual in this case)
- n = total number of periods (10 years)

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1. List the Cash Flows

Year	Cash Flow (C_t)	Description
1	\$60	Coupon payment
2	\$60	Coupon payment
3	\$60	Coupon payment
4	\$60	Coupon payment
5	\$60	Coupon payment
6	\$60	Coupon payment
7	\$60	Coupon payment
8	\$60	Coupon payment
9	\$60	Coupon payment
10	\$1,060	Coupon + Redemption at Year 10

Note: The final year includes both the coupon and the redemption of the face value.

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2. Calculate Present Values of Each Cash Flow

Given the assumption that YTM equals the coupon rate (6%), the present value (PV) of each cash flow can be calculated using:

$$
PV = \frac{C_t}{(1 + y)^t}
$$

Where $y = 0.06$.

Calculations:

- For years 1 through 9:

$$
PV_t = \frac{60}{(1.06)^t}
$$

- For year 10:

$$
PV_{10} = \frac{1,060}{(1.06)^{10}}
$$

---

3. Compute the Present Values

Let's compute each PV:

Year	PV of Coupon	PV of Total (Coupon + Redemption)
1	$\frac{60}{1.06} = 56.60$	N/A
2	$\frac{60}{1.1236} = 53.40$	N/A
3	$\frac{60}{1.191} = 50.38$	N/A
4	$\frac{60}{1.262} = 47.52$	N/A
5	$\frac{60}{1.338} = 44.78$	N/A
6	$\frac{60}{1.418} = 42.16$	N/A
7	$\frac{60}{1.501} = 39.75$	N/A
8	$\frac{60}{1.588} = 37.55$	N/A
9	$\frac{60}{1.689} = 35.45$	N/A
10	$\frac{1060}{1.790} = 592.24$	Total PV at Year 10

Note: The PV at year 10 includes both the coupon and the redemption amount.

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4. Calculate the Weighted Time of Each Cash Flow

For each cash flow, multiply the PV by the time period t:

$$
t \times PV_t
$$

Year	PV of Cash Flow	$t \times PV_t$
1	56.60	$1 \times 56.60 = 56.60$
2	53.40	$2 \times 53.40 = 106.80$
3	50.38	$3 \times 50.38 = 151.14$
4	47.52	$4 \times 47.52 = 190.08$
5	44.78	$5 \times 44.78 = 223.90$
6	42.16	$6 \times 42.16 = 252.96$
7	39.75	$7 \times 39.75 = 278.25$
8	37.55	$8 \times 37.55 = 300.40$
9	35.45	$9 \times 35.45 = 319.05$
10	592.24	$10 \times 592.24 = 5922.40$

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5. Sum of Present Values and Weighted Times

- Sum of PVs:

$$
PV_{\text{total}} = \sum_{t=1}^{10} PV_t = 56.60 + 53.40 + 50.38 + 47.52 + 44.78 + 42.16 + 39.75 + 37.55 + 35.45 + 592.24 = \text{approximately } 938.19
$$

- Sum of $t \times PV_t$:

$$
\sum t \times PV_t = 56.60 + 106.80 + 151.14 + 190.08 + 223.90 + 252.96 + 278.25 + 300.40 + 319.05 + 5922.40 = \text{approximately } 7434.08
$$

---

6. Calculate the Macaulay Duration

$$
\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} t \times PV_t}{\sum_{t=1}^{n} PV_t} =

\frac{7434.08}{938.19} \approx 7.93 \text{ years}
\]

Interpretation:

The ten-year Macaulay duration of this bond is approximately 7.93 years. This means that, on average, the present value-weighted cash flows are received around 7.93 years into the life of the bond.

Significance and Practical Implications

Understanding the duration of a bond like this has several practical applications:

- Interest Rate Risk Management: A duration of 7.93 years suggests that if interest rates rise by 1%, the bond

Frequently Asked Questions

How do you calculate the ten-year duration of a 6% Macaulay bond with a $1,000 face value, annual coupon, and redemption at maturity?

To calculate the ten-year duration, you need to determine the present value of each cash flow (coupons and redemption), multiply each by the time until receipt, sum these weighted cash flows, and then divide by the bond's current price. The process involves discounting all payments at the bond's yield to maturity and summing the weighted durations over the ten-year period.

What impact does the 6% coupon rate have on the duration of a $1,000 Macaulay bond with a ten-year maturity?

A 6% coupon rate provides regular cash flows, which generally reduce the bond's duration compared to a zero-coupon bond. Since coupons are paid annually, the duration is closer to the weighted average time of cash flows, typically less than the full ten years, making the bond less sensitive to interest rate changes.

How does the redemption at maturity influence the Macaulay duration of the bond?

Redemption at maturity ensures the principal is returned at the end of ten years, contributing to the overall duration calculation. It increases the weighted average time of cash flows, thus affecting the bond's duration by extending its sensitivity to interest rate fluctuations.

What is the significance of calculating the ten-year duration for a bond like this?

Calculating the ten-year duration helps investors assess the bond's interest rate risk over its lifespan. A higher duration indicates greater sensitivity to interest rate changes, aiding in risk management and portfolio strategy decisions.

Can the duration calculation be simplified for a 6% coupon bond with annual payments and ten-year maturity?

While the exact calculation involves discounting each cash flow, simplified formulas and financial calculators can approximate the duration, especially if the bond's yield to maturity is known. In practice, financial software or spreadsheets are commonly used to efficiently compute precise duration values.

Additional Resources

Calculate The Ten-Year Duration Of A 6% + $1,000 Macaulay Bond With An Annual Coupon And Redemption Of

Understanding bond duration is essential for investors and financial analysts seeking to manage interest rate risk, assess bond price sensitivity, and optimize portfolio strategies. In this detailed review, we explore the calculation of the ten-year duration for a specific bond characterized by a 6% coupon rate, a $1,000 face value, annual coupons, and redemption at maturity. We will dissect the bond's features, the theoretical foundations of duration, and step-by-step calculations, providing a comprehensive guide for practitioners and enthusiasts alike.

Introduction to Bond Duration

What Is Bond Duration?

Bond duration measures the weighted average time until a bond's cash flows are received. It is a crucial concept in fixed-income analysis because it indicates how sensitive a bond's price is to changes in interest rates. The higher the duration, the more the bond's price will fluctuate with interest rate movements.

Types of Duration

- Macaulay Duration: The weighted average time to receive the bond's cash flows, expressed in years.
- Modified Duration: Derived from Macaulay Duration, it estimates the percentage change in price for a 1% change in yield.
- Effective Duration: Used for bonds with embedded options, accounting for potential changes in cash

flows.

In this analysis, our focus will be on Macaulay Duration, which provides a foundational understanding of the timing of cash flows.

Bond Characteristics and Assumptions

To accurately calculate the duration, we need precise details about the bond's features:

- Coupon Rate: 6% annually
- Face Value (Par Value): $1,000
- Coupon Payment: 6% of $1,000 = $60 annually
- Maturity: 10 years
- Redemption Value: $1,000 (at maturity)
- Payment Frequency: Annual
- Yield to Maturity (YTM): For the purpose of illustration, assume a YTM of 6% (matching the coupon rate). This simplifies calculations but can be adjusted for different yields.
- Interest Payments: Fixed, annual coupons
- Redemption: Lump sum of $1,000 at the end of 10 years

It's important to note that the actual market yield may differ, and variations directly influence the duration calculation.

Understanding the Calculation Framework

The Present Value of Cash Flows

The core of duration calculation involves discounting each cash flow (coupons and redemption) to its present value, based on the current yield (YTM). The formula for present value (PV) of a future cash flow is:

PV = Cash Flow / (1 + YTM)^t

Where:
- Cash Flow: Coupon payment or redemption amount
- t: Year in which the cash flow occurs
- YTM: Yield to maturity (as a decimal)

Weighted Average Time

Macaulay Duration is calculated as:

Duration = (Sum of (t PV of cash flow at time t)) / (Price of the bond)

In essence, it's the weighted average of the time until each cash flow is received, with weights proportional to the present value of each cash flow.

Step-by-Step Calculation of the Ten-Year Duration

Step 1: Calculate the Present Value of Each Cash Flow

Since the YTM equals the coupon rate (6%), the bond is priced at par, i.e., $1,000. This simplifies the calculations:

- Each coupon: $60
- Redemption value at year 10: $1,000 + $60 = $1,060 (but since redemption is at par, it's just the face value plus final coupon)

The PV of each coupon payment:

PV_coupon = $60 / (1 + 0.06)^t

The PV of the redemption value:

PV_redemption = $1,000 / (1 + 0.06)^10

Calculations for each year:

Year (t)	Cash Flow	PV of Cash Flow
1	$60	$60 / (1.06)^1 = $56.60
2	$60	$60 / (1.06)^2 = $53.40
3	$60	$60 / (1.06)^3 = $50.38
4	$60	$60 / (1.06)^4 = $47.55
5	$60	$60 / (1.06)^5 = $44.83
6	$60	$60 / (1.06)^6 = $42.33
7	$60	$60 / (1.06)^7 = $39.94
8	$60	$60 / (1.06)^8 = $37.66
9	$60	$60 / (1.06)^9 = $35.58
10	$1,060	$1,060 / (1.06)^10 = $629.89

(Note: PV calculations are approximate and rounded to two decimal places)

Step 2: Calculate the Total Present Value (Bond Price)

Since the sum of the PVs of all cash flows equals the bond’s price, and we're assuming it’s priced at par:

Total PV ≈ $1,000

This confirms that YTM equals the coupon rate, simplifying the calculations.

Step 3: Calculate the Weighted Time for Each Cash Flow

For each cash flow, multiply the PV by the time period:

Year (t)	PV of Cash Flow	PV t
1	$56.60	1 $56.60 = $56.60
2	$53.40	2 $53.40 = $106.80
3	$50.38	3 $50.38 = $151.14
4	$47.55	4 $47.55 = $190.20
5	$44.83	5 $44.83 = $224.15
6	$42.33	6 $42.33 = $253.98
7	$39.94	7 $39.94 = $279.58
8	$37.66	8 $37.66 = $301.28
9	$35.58	9 $35.58 = $320.22
10	$629.89	10 $629.89 = $6,298.90

Note: The PV of the final cash flow combines coupon and redemption:

PV of year 10 cash flow = $1,060 / (1.06)^10 ≈ $629.89

Step 4: Sum the Weighted PVs and Divide by the Bond Price

Sum of weighted PVs:

Total weighted PV = $56.60 + $106.80 + $151.14 + $190.20 + $224.15 + $253.98 + $279.58 + $301.28 + $320.22 + $6,298.90 ≈ $8,186.35

Since the total PV (bond price) is approximately $1,000, the Macaulay Duration is:

Duration = Total weighted PV / Total PV = $8,186.35 / $1,000 ≈ 8.19 years

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Interpreting the Results

The calculated Macaulay Duration of approximately 8.19 years indicates that, on average, the bond's cash flows are received after about 8.19 years. This is somewhat shorter than the maturity of 10 years because earlier coupon payments contribute to the weighted average, reducing the overall duration.

This duration suggests the bond's price sensitivity to interest rate movements. For example, with a duration of 8.19 years, a 1% increase in interest rates would approximately cause an 8.19% decrease in the bond's price, all else equal.

Impact of Yield Changes and Other Considerations

Varying the Yield to Maturity

- When YTM exceeds the coupon rate, the bond trades at a discount, and the duration typically increases.
- Conversely, if YTM is lower than the coupon rate, the bond trades at a premium, and duration decreases.

Adjusting the YTM in calculations can significantly influence the duration, especially for bonds with longer maturities or embedded options.

Embedded Options and Callable Bonds

If the bond had embedded options like call features, the calculation of effective duration would require more sophisticated models accounting for

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