Chang Knows One Side Of A Triangle Is 13 Cm. Which Set Of Two Sides Is Possible For The Lengths Of The

Chang Knows One Side Of A Triangle Is 13 Cm. Which Set Of Two Sides Is Possible For The Lengths Of The triangle? This question invites us to explore the fundamental properties of triangles, particularly focusing on the Triangle Inequality Theorem. Understanding the relationships between side lengths is crucial in geometry, whether for solving problems, constructing figures, or applying concepts in real-world scenarios. In this article, we will analyze the possible sets of side lengths when one side is known, and determine which combinations can form a valid triangle.

Understanding the Triangle Inequality Theorem

What Is the Triangle Inequality Theorem?

The Triangle Inequality Theorem states that for any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side. Mathematically, if the sides of a triangle are a, b, and c, then:

- $a + b > c$
- $a + c > b$
- $b + c > a$

This fundamental rule ensures that the three sides can connect to form a closed figure, i.e., a triangle.

Implications of the Theorem

When one side of a triangle is known, such as 13 cm, the theorem helps us determine the possible lengths of the other two sides. These lengths must satisfy the inequalities:

- $\text{Side}_1 + \text{Side}_2 > 13$ cm
- $\text{Side}_1 + 13 > \text{Side}_2$
- $\text{Side}_2 + 13 > \text{Side}_1$

Understanding these inequalities allows us to identify the range of possible lengths for the other two sides.

Determining Possible Sets of Two Sides When One Side Is 13 Cm

Suppose we are asked: If one side of a triangle is 13 cm, what are the possible pairs of lengths for the other two sides?

Let’s denote these sides as x and y. The goal is to find all pairs (x, y) that satisfy the triangle inequality conditions.

The Conditions for x and y

Applying the triangle inequalities, we get:

1. $x + y > 13$
2. $x + 13 > y$
3. $y + 13 > x$

These inequalities must hold simultaneously.

Analyzing the Conditions

Let’s analyze each inequality:

- $x + y > 13$: The sum of the other two sides must be greater than 13 cm.
- $x + 13 > y$: Side y must be less than $x + 13$.
- $y + 13 > x$: Side x must be less than $y + 13$.

From these, we see that:

- Both x and y must be positive numbers (lengths cannot be negative).
- The sum of x and y must be strictly greater than 13.
- Neither x nor y can be 13 or more, unless the other is sufficiently small to satisfy the inequalities.

Graphical Representation of Possible Side Lengths

Visualizing the possible values can clarify the constraints. Imagine plotting the possible pairs (x, y) on a coordinate plane.

Inequality Regions

The key inequality is:

- $x + y > 13$

This describes the region above the line $x + y = 13$.

Additionally, the inequalities:

- $y < x + 13$
- $x < y + 13$

define regions bounded by lines parallel to the axes.

Feasible Region for (x, y)

The intersection of these regions represents all the pairs (x, y) that satisfy the triangle inequalities. Specifically:

- Both x and y are positive.
- $x + y > 13$
- $y < x + 13$
- $x < y + 13$

The feasible region is an infinite set of pairs, but with clear bounds based on the inequalities.

Sample Sets of Possible Lengths for the Other Two Sides

Let's explore some specific examples to illustrate possible side lengths.

Example 1: $x = 10$ cm

To find y:

- $10 + y > 13 \Rightarrow y > 3$
- $y < 10 + 13 \Rightarrow y < 23$
- $y + 13 > 10 \Rightarrow y > -3$ (which is always true for positive lengths)

Since side lengths are positive, the key constraints are:

- $y > 3$
- $y < 23$

Thus, any y in the interval $(3, 23)$ cm will work when $x = 10$ cm.

Possible pairs: For $x = 10$ cm, y can be any value greater than 3 cm and less than 23 cm.

Example 2: $y = 15$ cm

Find x:

- $x + 15 > 13 \Rightarrow x > -2$ (always true)
- $15 < x + 13 \Rightarrow x > 2$
- $x + 13 > 15 \Rightarrow x > 2$

Since x must be greater than 2 cm, and positive:

Possible values: $x > 2$ cm, with no upper limit except that x must satisfy the inequalities.

Special Cases and Limitations

When Do Sides Fail to Form a Triangle?

If the sum of the two sides is less than or equal to 13 cm, then the triangle cannot exist. For example, if $x + y \leq 13$, the sides cannot form a triangle.

Example: $x = 6$ cm, $y = 7$ cm

- $6 + 7 = 13$

Since the sum equals 13, the sides do not form a proper triangle but rather a degenerate case (a straight line). Therefore, for a valid triangle, the sum must be strictly greater than 13 cm.

Implication for Side Length Selection

When choosing side lengths, ensure:

- $x + y > 13$
- $x, y > 0$

- Neither side is so large that it violates the inequalities when combined with the known side of 13 cm.

Conclusion: Summary of Possible Sets of Two Sides

In summary, if one side of a triangle is 13 cm, then the other two sides can be any positive lengths x and y satisfying:

- $x + y > 13$
- $|x - y| < 13$

The second inequality arises from the triangle inequalities $|x - y| < 13$, ensuring the difference between the two sides is less than 13 cm.

Key takeaways:

- The other two sides must have a combined length greater than 13 cm.
- The absolute difference between the two sides must be less than 13 cm.
- Both sides must be positive numbers.

Practical examples include:

- $x = 10$ cm, $y \in (3, 23)$ cm
- $x = 20$ cm, $y \in (7, 33)$ cm
- $x = 5$ cm, $y \in (8, 18)$ cm

By understanding and applying the Triangle Inequality Theorem, we can confidently determine which sets of side lengths are feasible for forming a triangle when one side is known, such as 13 cm. This knowledge is essential for solving geometric problems, designing constructions, and understanding the properties of triangles in mathematics.

Frequently Asked Questions

If one side of a triangle is 13 cm, what are the possible lengths for the other two sides?

The other two sides must satisfy the triangle inequality theorem: each side must be less than the sum of the other two sides and greater than their difference. For example, if one side is 13 cm, the other sides can be any lengths where each is greater than 0 and less than 13 + the other side, and greater than the absolute difference of 13 and the other side.

Can the set of sides 13 cm, 7 cm, and 6 cm form a triangle?

Yes, because the sum of any two sides exceeds the third: $13 + 7 = 20 > 6$, $13 + 6 = 19 > 7$, and $7 + 6 = 13 = 13$, which satisfies the triangle inequality (equal sums are acceptable for degenerate triangles).

Is the set of sides 13 cm, 10 cm, and 5 cm possible for a triangle?

No, because $10 + 5 = 15$, which is greater than 13, so it satisfies the triangle inequality. But check the other sums: $13 + 5 = 18 > 10$, and $13 + 10 = 23 > 5$. All conditions are satisfied, so yes, these sides can form a triangle.

What set of two sides, with one side being 13 cm, cannot form a triangle?

Any set where the sum of the other two sides is less than or equal to 13 cm cannot form a triangle. For example, 13 cm, 4 cm, and 8 cm: since $4 + 8 = 12 \leq 13$, they cannot form a triangle.

If one side of a triangle is 13 cm, what is the maximum length the other side can have?

The other side must be less than the sum of the two sides, so it must be less than 13 + the third side. If the third side is also 13 cm, then the other side must be less than 26 cm. More specifically, for a valid triangle, the other side must be less than 26 cm.

What is the minimum length the other side can be if one side is 13 cm and the sides form a triangle?

The other side must be greater than the difference of the two sides. So, if the third side is also 13 cm, the other side must be greater than 0 cm, i.e., any positive length less than 26 cm, but practically, greater than 0 cm.

Can sides of lengths 13 cm, 20 cm, and 5 cm form a triangle?

No, because $5 + 20 = 25$, which is greater than 13, so that part is okay. But check the other sums: $13 + 5 = 18$, which is less than 20, so the sides violate the triangle inequality and cannot form a triangle.

How does the triangle inequality affect the possible side lengths when one side is fixed at 13 cm?

It restricts the other side lengths to be greater than the difference (which is 0 if considering positive lengths) and less than the sum (which is 13 plus the other side). Specifically, the other sides must satisfy: $|13 - x| < y < 13 + x$, where x and y are the other sides.

Additional Resources

Chang Knows One Side Of A Triangle Is 13 Cm: Exploring the Possible Lengths of the Other Sides

Understanding the properties of triangles is fundamental in geometry, with applications spanning architecture, engineering, and various sciences. When a specific side length is known, such as Chang's knowledge that one side measures 13 centimeters, the question naturally arises: what are the feasible lengths for the other sides? This inquiry is not merely academic; it encapsulates core principles of triangle inequality, geometric constraints, and the conditions necessary for a valid triangle formation. This article delves into the mathematical underpinnings of this problem, exploring the possible combinations of the remaining two sides, and providing an in-depth analysis suitable for students, educators, and enthusiasts alike.

Understanding the Triangle Inequality Theorem

The foundation of any exploration into the possible side lengths of a triangle is the triangle inequality theorem. This fundamental principle states that, for any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.

Mathematically, if the sides of a triangle are labeled a, b, and c, then:

- $a + b > c$
- $a + c > b$
- $b + c > a$

This set of inequalities ensures the three sides can indeed form a closed, non-degenerate triangle. If any one of these inequalities is violated, the sides cannot constitute a triangle.

Given Side Length: 13 cm

Suppose Chang knows that one side of the triangle is exactly 13 cm. Without loss of generality, let's designate this known side as $c = 13\,\text{cm}$. The task then becomes to determine which pairs of lengths (a, b) are possible for the other two sides, considering the constraints imposed by the triangle inequality.

Key considerations:

- The other sides a and b must be positive real numbers.
- The inequalities must be satisfied to form a valid triangle.

Possible Lengths for the Remaining Sides

To analyze the problem systematically, let's consider the possible ranges for a and b.

2.1. Constraints Derived from Triangle Inequality

Given $c = 13$, the inequalities become:

- $a + b > 13$ (1)
- $a + 13 > b$ (2)
- $b + 13 > a$ (3)

From inequalities (2) and (3), we see that:

- $b < a + 13$
- $a < b + 13$

Since a and b are positive, they must satisfy:

- $a > 0$, $b > 0$

From inequality (1):

$$
a + b > 13
$$

which can be rewritten as:

$$
b > 13 - a
$$

for a in $(0, \infty)$.

2.2. Visualizing the Constraints

Plotting these inequalities on the (a, b) coordinate plane helps visualize the feasible region:

- The region where $a > 0$ and $b > 0$.
- The inequality $a + b > 13$ indicates that the points lie above the line $a + b = 13$.
- The inequalities $a < b + 13$ and $b < a + 13$ define bounds that are always satisfied for positive a, b, except when sides become degenerate.

2.3. Range of Possible Side Lengths

From the inequalities, the possible lengths of a and b are:

- Both a and b are strictly positive.
- The sum $a + b$ must be greater than 13 for the triangle to exist.

Thus, the set of feasible side lengths (a, b) is:

$$
a > 0,\quad b > 0,\quad a + b > 13
$$

Specific Scenarios and Examples

To better understand the possible side lengths, consider specific cases:

3.1. Equal Lengths for the Other Two Sides

Suppose $a = b$. Then, the inequality $a + b > 13$ becomes:

$$
2a > 13 \implies a > 6.5\,\text{cm}
$$

Implication:

- When $a = b$, both sides must be greater than 6.5 cm.
- The sides are then $(a, a, 13)$ with $a > 6.5$.

Examples:

- $a = b = 7\,\text{cm}$: Sides are (7, 7, 13). Check inequalities:
- $7 + 7 = 14 > 13$ ✓
- $7 + 13 = 20 > 7$ ✓
- $7 + 13 = 20 > 7$ ✓

- $a = b = 6.6\,\text{cm}$: Sides are (6.6, 6.6, 13). Check:
- $6.6 + 6.6 = 13.2 > 13$ ✓
- $6.6 + 13 = 19.6 > 6.6$ ✓
- Same for the other side.

This illustrates that the minimal equal side length for the other two sides is just over 6.5 cm.

3.2. One Side Slightly Less Than 13 cm

Suppose a is just under 13 cm, say $a = 12\,\text{cm}$. Then, for b:

$$
b > 13 - a = 1\,\text{cm}
$$

- b must be greater than 1 cm.
- Also, $b < a + 13 = 25\,\text{cm}$ (from inequality $b < a + 13$).

Example:

- $a = 12\,\text{cm}$, $b = 10\,\text{cm}$:

- $a + b = 22 > 13$ ✓
- $a + 13 = 25 > 10$ ✓
- $b + 13 = 23 > 12$ ✓

All inequalities satisfied, so (12, 10, 13) forms a valid triangle.

Implications for Triangle Formation and Design

The analysis reveals several key points relevant to both theoretical and practical applications:

4.1. Flexibility in Side Lengths

Given a fixed side of 13 cm, the other two sides can vary widely, provided they satisfy the triangle inequalities. Specifically, they:

- Must be positive.
- Must satisfy $a + b > 13$.

This means the other two sides can be any lengths summing to more than 13, with no strict upper bounds, as long as the inequalities hold.

4.2. Range of Possible Triangle Types

Depending on the side lengths chosen, the triangle can be:

- Acute: All angles less than 90°, typically when the sides are relatively balanced.
- Right-angled: When the sides satisfy the Pythagorean theorem, e.g., if $a^2 + b^2 = 13^2 = 169$.
- Obtuse: When one angle exceeds 90°, usually when one side is significantly longer than the other two.

Example of right triangle:

Suppose a and b satisfy:

$$
a^2 + b^2 = 169
$$

Possible solutions include:

- $a = 12\,\text{cm}$, $b = 5\,\text{cm}$:

- Check $a + b = 17 > 13$, valid.
- Sides: (12, 5, 13), which form a right triangle (since $12^2 + 5^2 = 144 + 25 = 169$).

Note: The side lengths (12, 5, 13) satisfy the triangle inequality, as:

- $12 + 5 = 17 > 13$,
- $12 + 13 = 25 > 5$,
- $5 + 13 = 18 > 12$.

4.3. Degenerate Cases and Limitations

Theoretically, as the sum $a + b$ approaches 13 from above, the triangle becomes degenerate—flattened with the three points almost colinear. The sides satisfy $a + b = 13$, but the triangle inequality becomes equality, and no proper triangle exists.

Practical Applications and Considerations

Understanding the feasible side lengths when one side is fixed at 13 cm has real-world implications across various fields.

5.1. Engineering and Structural Design

Engineers often work with specific side lengths to optimize materials and stability. Knowing that two other sides can vary freely (within the

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