Compute The Derlvative Of The Given Function In Two Different Ways. G(x)=4x 3(3x 3) A) Use The Product

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Understanding how to differentiate complex functions is a fundamental aspect of calculus. In this article, we will explore how to compute the derivative of the function G(x) = 4x 3(3x - 3) using two different methods: the product rule and simplifying the function before differentiating. Mastering these techniques not only enhances your calculus skills but also deepens your understanding of the underlying principles of derivatives.

Overview of the Function G(x)

Before diving into the differentiation methods, let's analyze the given function:

G(x) = 4x 3(3x - 3)

At first glance, the function appears as a product of two expressions:

- The first part: 4x
- The second part: 3(3x - 3)

Understanding the structure of G(x) is crucial for choosing the appropriate differentiation method. The function can be viewed as a product of two functions:

- u(x) = 4x
- v(x) = 3(3x - 3)

Alternatively, the entire expression can be rewritten for clarity.

Method 1: Using the Product Rule

The product rule is a fundamental differentiation technique used when differentiating the product of two functions. The rule states:

If G(x) = u(x) v(x), then

G'(x) = u'(x) v(x) + u(x) v'(x)

Let's apply this rule step-by-step.

Step 1: Identify u(x) and v(x)

- u(x) = 4x
- v(x) = 3(3x - 3)

Step 2: Find the derivatives u'(x) and v'(x)

- u'(x) = derivative of 4x = 4
- v(x) = 3(3x - 3)

Now, differentiate v(x):

v(x) = 3 (3x - 3)

Using the constant multiple rule:

v'(x) = 3 derivative of (3x - 3)

Derivative of (3x - 3):

d/dx (3x - 3) = 3

Therefore:

v'(x) = 3 3 = 9

Step 3: Apply the product rule formula

G'(x) = u'(x) v(x) + u(x) v'(x)

Substituting the derivatives and original functions:

G'(x) = 4 [3(3x - 3)] + 4x 9

Simplify each term:

First term:

4 [3(3x - 3)] = 4 3 (3x - 3) = 12 (3x - 3)

Second term:

4x 9 = 36x

Step 4: Simplify the derivative expression

Expand the first term:

12 (3x - 3) = 12 3x - 12 3 = 36x - 36

Now, combine all terms:

G'(x) = 36x - 36 + 36x

Combine like terms:

G'(x) = (36x + 36x) - 36 = 72x - 36

Final derivative using the product rule:

G'(x) = 72x - 36

Method 2: Simplifying the Function Before Differentiation

An alternative approach involves simplifying the original function first, then differentiating, which can often be more straightforward.

Step 1: Rewrite G(x) in a simplified form

Recall:

G(x) = 4x 3(3x - 3)

Distribute the 3 inside the parentheses:

G(x) = 4x (3 3x - 3 3) = 4x (9x - 9)

Now, distribute the 4x:

G(x) = 4x 9x - 4x 9 = 36x^2 - 36x

Step 2: Differentiate the simplified function

Now, differentiate G(x) = 36x^2 - 36x:

- Derivative of $36x^2$:

$\frac{d}{dx}(36x^2) = 72x$

- Derivative of $-36x$:

$\frac{d}{dx}(-36x) = -36$

Combine the derivatives:

$G'(x) = 72x - 36$

Step 3: Final result

The derivative obtained after simplification matches the one derived using the product rule:

$G'(x) = 72x - 36$

Comparison of Both Methods

Both approaches lead to the same derivative, but they differ in process and efficiency.

- Using the product rule is more systematic and is especially useful when the function is complex or not easily factored.
- Simplifying before differentiation can save time and reduce errors when the function can be easily rewritten, as in this case.

Understanding when to use each method is a valuable skill in calculus.

Additional Tips for Differentiation

- Always identify whether a function is a product, quotient, or composite to choose the appropriate differentiation rule.
- Simplify functions when possible to make derivatives easier to compute.
- Remember the basic rules: power rule, constant multiple rule, sum and difference rules, and chain rule for composite functions.
- Practice differentiating different types of functions to become more proficient.

Conclusion

In this comprehensive guide, we explored two methods to differentiate the function G(x) = 4x 3(3x - 3):

1. Applying the product rule, which involves differentiating each part separately and applying the rule's formula.
2. Simplifying the function before differentiation, making the process more straightforward.

Both methods yield the same derivative, G'(x) = 72x - 36, illustrating the importance of understanding multiple techniques for differentiation in calculus. Mastery of these methods enhances problem-solving skills and prepares you for more complex calculus problems.

Meta Description: Learn how to differentiate the function G(x)=4x 3(3x - 3) using two effective methods: the product rule and simplification. Step-by-step guide with detailed explanations.

Keywords: derivative, calculus, product rule, differentiation, simplify functions, G(x), derivative methods, calculus tips, advanced differentiation

Frequently Asked Questions

How do you compute the derivative of G(x) = 4x 3(3x - 3) using the product rule?

First, identify u = 4x and v = 3(3x - 3). Then, find u' = 4 and v' = 3 3 = 9. Using the product rule: G'(x) = u' v + u v' = 4 3(3x - 3) + 4x 9.

What is the step-by-step process to differentiate G(x) = 4x 3(3x - 3) using the product rule?

Step 1: Let u = 4x, so u' = 4. Step 2: Let v = 3(3x - 3), so v' = 3 3 = 9. Step 3: Apply the product rule: G'(x) = u'v + uv' = 4 3(3x - 3) + 4x 9.

Can you differentiate G(x) = 4x 3(3x - 3) by expanding first and then differentiating?

Yes. Expand G(x): G(x) = 4x 3(3x - 3) = 4x (9x - 9) = 4x 9x - 4x 9 = 36x^2 - 36x. Then, differentiate term-by-term: G'(x) = 72x - 36.

What are the advantages of using the product rule versus expanding the function before differentiating?

Using the product rule is often more efficient and less error-prone, especially for complex functions, as it avoids expanding large expressions. Expanding first can simplify the differentiation but may be cumbersome for complicated functions.

How do you verify the derivative obtained via the product rule matches the one obtained after expansion?

Calculate the derivative using both methods—product rule and expansion—and simplify both results. If both expressions are identical after simplification, it confirms the correctness of the derivative.

What is the derivative of G(x) = 4x 3(3x - 3) using the product rule, and what is its simplified form?

Using the product rule: G'(x) = 4 3(3x - 3) + 4x 9 = 12(3x - 3) + 36x = 36x - 36 + 36x = 72x - 36.

Additional Resources

Compute The Derivative Of The Given Function In Two Different Ways. G(x)=4x^3(3x^3) A) Use The Product

Deriving the derivative of a function is a fundamental skill in calculus, providing insights into the function's rate of change, slopes of tangent lines, and behavior over intervals. In this article, we focus on the function $G(x) = 4x^3(3x^3)$ and explore two systematic methods to compute its derivative. The first approach employs the product rule, which is particularly suited for functions expressed as the product of two differentiable functions. The second method involves simplifying the function first and then differentiating, showcasing the benefits of algebraic manipulation before applying differentiation rules.

Understanding the Function $G(x) = 4x^3(3x^3)$

Before diving into the differentiation techniques, it's crucial to comprehend the structure of the given function. It is a product of two functions:

$$
G(x) = 4x^3 \times 3x^3
$$

This structure hints that the product rule could be an effective method. Additionally, recognizing that the function can be simplified algebraically might streamline the differentiation process.

Method 1: Using the Product Rule

Overview of the Product Rule

The product rule states that if a function $G(x)$ is the product of two differentiable functions $u(x)$ and $v(x)$, then its derivative is given by:

$$
G'(x) = u'(x)v(x) + u(x)v'(x)
$$

This rule is especially useful when the function naturally appears as a product, and directly applying it can sometimes be more straightforward than expanding the entire function.

Applying the Product Rule to $G(x)$

Let's identify:

$$
u(x) = 4x^3 \quad \text{and} \quad v(x) = 3x^3
$$

Calculating their derivatives:

$$
u'(x) = 12x^2
$$

$$
v'(x) = 9x^2
$$

Now, applying the product rule:

$$
G'(x) = u'(x)v(x) + u(x)v'(x)
$$

Substituting the expressions:

$$
G'(x) = 12x^2 \times 3x^3 + 4x^3 \times 9x^2
$$

Simplify each term:

$$
G'(x) = 36x^5 + 36x^5
$$

Combine like terms:

$$
G'(x) = 72x^5
$$

Result from Method 1:

$$
\boxed{
G'(x) = 72x^5
}
$$

Pros of Using the Product Rule:

- Directly applicable without prior simplification.
- Clear and systematic, especially for complex products.
- Preserves the structure of the original function, useful in cases where the product's components are more intricate.

Cons:

- Can be lengthier if the functions are simple and easily simplified.
- Slightly more computational steps compared to algebraic simplification.

Method 2: Simplifying First then Differentiating

Algebraic Simplification of $G(x)$

Before differentiating, it's often beneficial to simplify the function algebraically. Given:

$$
G(x) = 4x^3 \times 3x^3
$$

Multiply the coefficients and combine like powers:

$$
G(x) = (4 \times 3) \times x^{3 + 3} = 12x^6
$$

Now, the function is simplified to:

$$
G(x) = 12x^6
$$

Differentiating the Simplified Function

Using the power rule, which states:

$$
\frac{d}{dx} [ax^n] = a n x^{n-1}
$$

Apply to $G(x) = 12x^6$:

$$
G'(x) = 12 \times 6 x^{6-1} = 72x^{5}
$$

Result from Method 2:

$$
\boxed{
G'(x) = 72x^5
}
$$

Pros of Simplification First:

- Fewer steps, often faster, especially for monomials.
- Reduced chance of algebraic mistakes when the function is straightforward.
- Provides a clearer understanding of the function's structure.

Cons:

- Not always straightforward; some functions are complex and easier to differentiate in their factored form.
- May obscure the original structure if the context demands it.

Comparison and Final Remarks

Both methods yield the same derivative:

$$
G'(x) = 72x^5
$$

This confirms the consistency of calculus rules and the importance of choosing the most efficient method based on the function's form.

When to Use Which Method?

- Use the product rule when the function is naturally expressed as a product, especially if the factors are complicated or involve different variables or functions.
- Simplify first when the algebraic form is manageable, and the function is a straightforward monomial or polynomial, to save time and reduce complexity.

Additional Tips for Derivative Computations

- Always analyze the structure of the function before choosing a method.
- Simplify as early as possible to streamline calculations.
- Double-check derivative steps, especially when applying higher-order rules or combining multiple rules.

Conclusion

Calculating derivatives is a vital skill in calculus, with multiple approaches available depending on the function's form. In the case of $G(x) = 4x^3(3x^3)$, applying the product rule and simplifying first both lead to the same derivative, $72x^5$. Mastery of these techniques enhances problem-solving efficiency and deepens understanding of the underlying mathematical principles. Whether choosing the product rule or algebraic simplification, understanding their features and optimal contexts ensures accurate and efficient differentiation in various scenarios.

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