Compute The Value Of The Discriminant And Give The Number Of Real Solutions Of The Quadratic Equation.

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Introduction to Quadratic Equations

Quadratic equations are polynomial equations of degree two, characterized by their standard form:

$$ ax^2 + bx + c = 0 $$

where a, b, and c are constants with $a \neq 0$, and x represents the variable. These equations are fundamental in algebra and appear frequently in various fields such as physics, engineering, economics, and mathematics.

Understanding the nature of their solutions is crucial. The solutions to quadratic equations can be real or complex, and identifying the number of real solutions helps in analyzing the behavior of the quadratic function.

The Discriminant: Definition and Significance

What is the Discriminant?

The discriminant of a quadratic equation is a specific part of the quadratic formula that determines the nature and number of solutions. It is given by:

$$ D = b^2 - 4ac $$

Here,

- D is the discriminant,
- a, b, and c are the coefficients from the quadratic equation.

The discriminant acts as a predictor of the solutions' nature without explicitly solving the equation.

Why is the Discriminant Important?

The value of the discriminant tells us:

- Whether the quadratic has two distinct real solutions,
- One real solution (a repeated root),
- Or no real solutions (complex roots).

This allows mathematicians and students to quickly analyze quadratic equations and understand their behavior graphically and algebraically.

Calculating the Discriminant

Step-by-Step Process

To compute the discriminant:

1. Identify the coefficients a, b, and c from the quadratic equation.
2. Plug these values into the formula:

$$ D = b^2 - 4ac $$

3. Calculate the value of D.

Example Calculation

Suppose the quadratic equation is:

$$ 2x^2 - 4x + 1 = 0 $$

- $a = 2$,
- $b = -4$,
- $c = 1$.

Calculating the discriminant:

$$ D = (-4)^2 - 4 \times 2 \times 1 = 16 - 8 = 8 $$

Thus, the discriminant is 8.

Determining the Number of Real Solutions

Once the discriminant is calculated, its value indicates the nature of the solutions:

Cases Based on the Discriminant Value

1. **D > 0**: Two distinct real solutions
2. **D = 0**: One real solution (a repeated root)
3. **D < 0**: No real solutions (complex conjugate roots)

Implications of Each Case

- When $D > 0$, the quadratic graph intersects the x-axis at two points, indicating two real roots.
- When $D = 0$, the graph just touches the x-axis at one point, indicating a perfect square trinomial.
- When $D < 0$, the graph does not intersect the x-axis, and solutions are complex conjugates.

Examples Illustrating Different Scenarios

Example 1: Two Real Solutions

Equation: $x^2 - 3x + 2 = 0$

- $a=1$, $b=-3$, $c=2$
- Discriminant:

$$ D = (-3)^2 - 4 \times 1 \times 2 = 9 - 8 = 1 $$

Since $D > 0$, there are two real solutions.

Example 2: One Real Solution

Equation: $x^2 - 4x + 4 = 0$

- $a=1$, $b=-4$, $c=4$
- Discriminant:

$$ D = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0 $$

Since $D=0$, there is one real solution (a repeated root).

Example 3: No Real Solutions

Equation: $x^2 + x + 1 = 0$

- $a=1$, $b=1$, $c=1$
- Discriminant:

$$D = (1)^2 - 4 \times 1 \times 1 = 1 - 4 = -3$$

Since $D < 0$, there are no real solutions; solutions are complex conjugates.

Additional Insights and Applications

Graphical Interpretation

The value of the discriminant correlates directly with the graph of the quadratic function:

- $D > 0$: The parabola cuts the x-axis at two points.
- $D = 0$: The parabola just touches the x-axis at its vertex.
- $D < 0$: The parabola lies entirely above or below the x-axis, with no x-intercepts.

Applications of Discriminant Analysis

Understanding the discriminant has practical implications:

- In physics, it can determine the number of times an object intersects a certain boundary.
- In engineering, it assesses system stability.
- In finance, it helps in analyzing quadratic models of profit or cost functions.
- In pure mathematics, it aids in solving quadratic equations efficiently.

Summary and Conclusion

The discriminant is a vital component in the analysis of quadratic equations. Its calculation involves simple algebraic substitution into the formula $D = b^2 - 4ac$. The sign of the discriminant directly indicates the number of real solutions:

- Positive discriminant: two distinct real roots.
- Zero discriminant: one real, repeated root.
- Negative discriminant: no real roots; solutions are complex.

By mastering the computation and interpretation of the discriminant, students and professionals can quickly analyze quadratic equations' solutions without solving them explicitly. This understanding streamlines problem-solving in various scientific and mathematical contexts and deepens comprehension of quadratic functions' graphical behavior.

In essence, the discriminant serves as a powerful tool for algebraic analysis, enabling a clear understanding of the nature of solutions in quadratic equations across numerous applications.

Frequently Asked Questions

What is the formula for calculating the discriminant of a quadratic equation?

The discriminant D of a quadratic equation $ax^2 + bx + c = 0$ is calculated using the formula $D = b^2 - 4ac$.

How does the value of the discriminant determine the number of real solutions?

If $D > 0$, there are two real solutions; if $D = 0$, there is exactly one real solution; if $D < 0$, there are no real solutions.

Given the quadratic equation $2x^2 + 4x + 1 = 0$, what is its discriminant?

Discriminant $D = (4)^2 - 4(2)(1) = 16 - 8 = 8$.

For the quadratic equation $3x^2 - 6x + 2 = 0$, how many real solutions are there?

Calculate $D = (-6)^2 - 4(3)(2) = 36 - 24 = 12$. Since $D > 0$, there are two real solutions.

What does a zero discriminant indicate about the quadratic equation?

A zero discriminant indicates that the quadratic has exactly one real solution, also called a repeated root.

Can the discriminant be negative? What does that imply?

Yes, if the discriminant is negative, it implies that the quadratic equation has no real solutions; the solutions are complex conjugates.

How do you interpret the discriminant when solving a

quadratic graphically?

Graphically, a positive discriminant means the parabola intersects the x-axis at two points, zero means tangent at one point, and negative means it does not intersect the x-axis.

Why is calculating the discriminant important in solving quadratic equations?

Calculating the discriminant helps determine the nature and number of solutions without fully solving the equation, saving time and effort.

Additional Resources

Compute The Value Of The Discriminant And Give The Number Of Real Solutions Of The Quadratic Equation

Understanding the nature of solutions to quadratic equations is fundamental in algebra and various applied mathematics fields. The discriminant, a key component derived from the quadratic formula, provides invaluable insight into the number and nature of solutions without the need to explicitly solve the equations. This article delves into the concept of the discriminant, how to compute it, and how to interpret its value to determine the number of real solutions associated with any quadratic equation.

Introduction to Quadratic Equations

Quadratic equations are polynomial equations of degree two, generally expressed in the standard form:

$$ax^2 + bx + c = 0$$

where:

- $a \neq 0$,
- b and c are real coefficients,
- x is the variable.

Quadratic equations are ubiquitous in mathematics, physics, engineering, economics, and many other disciplines because they model a wide range of phenomena such as projectile motion, economic optimization problems, and equilibrium states.

The Quadratic Formula

The solutions (roots) of the quadratic equation can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula explicitly shows the role of the expression under the square root, known as the discriminant.

The Discriminant: Definition and Significance

What Is the Discriminant?

The discriminant D of a quadratic equation $ax^2 + bx + c = 0$ is the expression:

$$D = b^2 - 4ac$$

It is called the discriminant because it "discriminates" among the different types of solutions the quadratic equation can have.

Why Is the Discriminant Important?

The value of D determines the nature and number of solutions without explicitly solving the quadratic equation. This attribute is especially useful in graphing and analyzing the behavior of quadratic functions.

- If $D > 0$: The quadratic has two distinct real solutions.
- If $D = 0$: The quadratic has exactly one real solution (a repeated root).
- If $D < 0$: The quadratic has no real solutions; solutions are complex conjugates.

Understanding the discriminant allows students and professionals to quickly assess the solution landscape of quadratic equations, saving time and effort.

Computing the Discriminant

Step-by-Step Calculation

Calculating the discriminant involves straightforward arithmetic:

1. Identify the coefficients a, b, and c in your quadratic equation.
2. Substitute these values into $D = b^2 - 4ac$.
3. Simplify to find the value of D.

Example:

Given the quadratic equation:

$$ 2x^2 - 4x + 1 = 0 $$

- $a = 2$,
- $b = -4$,
- $c = 1$.

Calculate:

$$ D = (-4)^2 - 4 \times 2 \times 1 = 16 - 8 = 8 $$

Since $D = 8 > 0$, the equation has two real solutions.

Computational Tips

- Always double-check signs and coefficients.
- Use a calculator for large numbers to avoid arithmetic errors.
- When coefficients are fractional or decimal, ensure consistent handling of the numbers.

Interpreting the Discriminant: Number and Nature of Solutions

Case 1: $D > 0$ — Two Distinct Real Solutions

When the discriminant is positive, the quadratic formula yields two different real roots:

$$ x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} $$

Graphically, this corresponds to the parabola intersecting the x-axis at two points.

Implications:

- The parabola opens upward if $a > 0$, downward if $a < 0$.
- The roots are real and different, which can be used for real-world applications where solutions must be real numbers, such as maximum profit or projectile ranges.

Case 2: $D = 0$ — One Repeated Real Solution

When the discriminant equals zero, the quadratic has a single, repeated root:

$$ x = \frac{-b}{2a} $$

This indicates the parabola just touches the x-axis at its vertex, known as the point of tangency.

Implications:

- The quadratic has a double root.
- The solution is real but not distinct.
- This scenario often corresponds to a critical point, such as an extremum in optimization problems.

Case 3: $D < 0$ — No Real Solutions

A negative discriminant indicates that the solutions are complex conjugates:

$$ x_{1,2} = \frac{-b \pm i \sqrt{|D|}}{2a} $$

where i is the imaginary unit.

Implications:

- The parabola does not intersect the x-axis.
- Solutions cannot be represented on the real number line.
- Such equations are common in oscillatory systems, quantum mechanics, and other fields involving complex analysis.

Applications and Examples

Practical Scenarios

Understanding the discriminant's role extends beyond pure mathematics:

- Physics: Determining the nature of projectile trajectories.
- Engineering: Analyzing system stability.
- Economics: Solving optimization problems involving quadratic profit functions.
- Computer Graphics: Calculating intersection points of curves and surfaces.

Example 1: Real Solutions Assessment

Suppose a quadratic equation:

$$ 5x^2 + 2x - 3 = 0 $$

Compute the discriminant:

$$ D = (2)^2 - 4 \times 5 \times (-3) = 4 + 60 = 64 $$

Since $D > 0$, the equation has two real solutions:

$$ x_{1,2} = \frac{-2 \pm \sqrt{64}}{2 \times 5} = \frac{-2 \pm 8}{10} $$

Calculating:

- $x_1 = \frac{-2 + 8}{10} = \frac{6}{10} = 0.6$
- $x_2 = \frac{-2 - 8}{10} = \frac{-10}{10} = -1$

Example 2: Complex Solutions

Given:

$$ x^2 + 4x + 5 = 0 $$

Calculate discriminant:

$$ D = 4^2 - 4 \times 1 \times 5 = 16 - 20 = -4 $$

Since $D < 0$, solutions are complex:

$$ x = \frac{-4 \pm i \sqrt{4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i $$

These solutions are not real, implying the parabola does not intersect the x-axis.

Conclusion and Summary

The discriminant serves as a powerful, quick diagnostic tool in algebra for understanding the solution set of quadratic equations. By computing $D = b^2 - 4ac$, mathematicians and practitioners can determine whether the roots are real or complex, and whether they are distinct or repeated, without resorting to full solutions.

Key takeaways:

- The value of the discriminant directly indicates the number and nature of solutions.
- A positive discriminant signifies two real roots.
- A zero discriminant indicates a single real, repeated root.
- A negative discriminant indicates complex conjugate roots.
- Calculating the discriminant involves simple substitution and arithmetic, but its interpretation requires understanding the quadratic formula's structure.

In conclusion, mastery of discriminant calculation and interpretation is essential in both academic settings and real-world applications, enabling efficient analysis of quadratic models across disciplines.

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