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When analyzing discrete-time systems, understanding how an input signal interacts with the system's impulse response is fundamental. In this article, we delve into the problem of determining the output of a discrete-time system characterized by a given input $X[n]$ and a unit impulse response $H[n]$. Specifically, the input is defined as $X[n] = \left(\frac{1}{2}\right)^{n-2} U[n-2]$, where $U[n-2]$ is the unit step function shifted by 2 units. Our goal is to determine the system output, which involves convolution of the input with the impulse response, and analyze the resulting signal in detail.

This comprehensive exploration will include understanding the properties of the signals involved, the process of convolution in discrete-time systems, and the step-by-step calculation to derive the output. Additionally, we will discuss how to interpret the results and the importance of such analysis in signal processing applications.

Understanding the System Components

Before proceeding with the calculations, it is essential to understand the components involved:

The Input Signal $X[n]$

The input is given by:

$$
X[n] = \left(\frac{1}{2}\right)^{n-2} U[n-2]
$$

This can be interpreted as follows:

- The exponential term $\left(\frac{1}{2}\right)^{n-2}$ suggests a decaying exponential factor, starting from $n=2$.

- The unit step function $U[n-2]$ ensures the signal is zero for all $n < 2$; hence, the signal is active only for $n \geq 2$.

- The overall form indicates that $X[n]$ is a causal exponential sequence starting at $n = 2$.

The Impulse Response $H[n]$

The problem states that $H[n]$ is a unit impulse response, which in typical systems is denoted as $h[n]$. The unit impulse response defines the system's behavior, and the output $y[n]$ is given by the convolution:

$$
y[n] = (X H)[n] = \sum_{k=-\infty}^{\infty} X[k] H[n - k]
$$

Since the exact form of $H[n]$ isn't provided in the prompt, for the purpose of this discussion, we assume that $H[n]$ is a known sequence. Usually, in such problems, $H[n]$ could be:

- A finite-duration sequence, such as a simple exponential or finite impulse response.

- An infinite sequence, such as $H[n] = a^n U[n]$ for some $|a|<1$.

For completeness, we will analyze the general case and then illustrate specific examples.

Step-by-Step Solution Approach

To find the output $y[n]$, we perform the convolution of $X[n]$ and $H[n]$. The process involves:

1. Expressing $X[n]$ explicitly.

2. Understanding the support of $X[n]$.

3. Expressing $H[n]$, assuming a known form or a general form.

4. Calculating the convolution sum.

Let's break down these steps.

1. Expressing $X[n]$

Given:

$$
X[n] = \left(\frac{1}{2}\right)^{n-2} U[n-2]
$$

which implies:

$$
X[n] = \begin{cases}
\left(\frac{1}{2}\right)^{n-2}, & n \geq 2 \\
0, & n < 2
\end{cases}
$$

So, the sequence is zero for $n < 2$, and for $n \geq 2$, it decays exponentially.

2. Impulse Response $H[n]$

Assuming $H[n]$ is known; for example, a common choice might be:

$$
H[n] = a^n U[n]
$$

for some a, where $|a|<1$. Alternatively, if the original problem provides a specific $H[n]$, such as:

$$
H[n] = \delta[n] + \alpha \delta[n-1]
$$

then the convolution simplifies accordingly.

For this discussion, let's assume a general form:

\[

$$
H[n] = h[n]
$$

and later, we consider specific forms.

3. Convolution Sum

The output:

$$
y[n] = \sum_{k=-\infty}^{\infty} X[k] H[n - k]
$$

Given $X[k]$ is zero for $k < 2$, the sum reduces to:

$$
y[n] = \sum_{k=2}^{\infty} X[k] H[n - k]
$$

which simplifies the convolution integral to a sum over $k \ge 2$.

Calculating the Output for Specific Impulse Responses

Let's consider specific $H[n]$ for illustrative purposes, which helps to understand the process better.

Case 1: $H[n] = \delta[n]$

This corresponds to a system with an identity impulse response, meaning the output equals the input:

$$
y[n] = X[n]
$$

Thus,

\[

\boxed{
y[n] = \left(\frac{1}{2}\right)^{n-2} U[n-2]
}
\]

This is the simplest case, serving as a baseline.

Case 2: \(H[n] = a^n U[n] \), where \(|a| < 1 \)

Here, the convolution becomes:

\[
y[n] = \sum_{k=2}^{\infty} X[k] H[n - k]
\]

Given:

\[
X[k] = \left(\frac{1}{2}\right)^{k-2}
\]
and
\[
H[n - k] = a^{n - k} U[n - k]
\]

The sum is non-zero only when \(n - k \geq 0 \Rightarrow k \leq n \). Since \(k \geq 2 \), the limits of the sum are:

\[
k = 2, 3, \ldots, n
\]

for \(n \ge 2 \). For \(n < 2 \), the output is zero.

Therefore,

\[
y[n] = \sum_{k=2}^{n} \left(\frac{1}{2}\right)^{k-2} a^{n - k}
\]

Rewrite:

\[
y[n] = a^{n} \sum_{k=2}^{n} \left(\frac{1}{2}\right)^{k-2} a^{-k}
\]

Expressed as:

\[
y[n] = a^{n} \sum_{k=2}^{n} \left(\frac{1}{2} \right)^{k-2} \left(a^{-1} \right)^{k}
\]

which simplifies to:

\[
y[n] = a^{n} \sum_{k=2}^{n} \left(\frac{1}{2} \times a^{-1} \right)^{k} \times 1
\]

This is a geometric series in \(k \):

\[
\sum_{k=2}^{n} r^{k}
\]

where

\[
r = \frac{1}{2} \times a^{-1}
\]

The sum becomes:

\[
\sum_{k=2}^{n} r^{k} = r^{2} \frac{1 - r^{n-1}}{1 - r}
\]

Therefore, the output simplifies to:

\[
\boxed{
y[n] = a^{n} \times r^{2} \frac{1 - r^{n-1}}{1 - r}
}
\]

Substituting back \(r \):

$$
r = \frac{1}{2a}
$$

and the complete expression can be analyzed accordingly.

General Approach for Arbitrary $H[n]$

If $H[n]$ is a known sequence, the convolution can be approached systematically:

1. Identify the support of $X[n]$ and $H[n]$.

2. Set the limits of the sum based on the support.

3. Express $y[n]$ as a finite or infinite sum, depending on the sequences.

4. Simplify the sum, often as a geometric series or other known series.

5. Interpret the result in the context of the system's response.

Practical Applications and Significance

Understanding the convolution of input signals with impulse responses is crucial in various fields such as:

- Signal Processing

Frequently Asked Questions

What is the definition of the input signal X[n] in the given problem?

The input signal $X[n]$ is defined as $(1/2)^{n-2}$ multiplied by the unit step function $U[n-2]$, which activates the signal starting from $n=2$.

How does the unit impulse response H[n] influence the output y[n]?

The output y[n] is obtained by convolving the input X[n] with the impulse response H[n], effectively shaping X[n] according to H[n]'s characteristics.

What is the significance of the shifting in the input signal, given by U[n-2]?

The shift by 2 in U[n-2] delays the activation of the input signal until n=2, affecting the timing of the system's response.

How can the convolution of X[n] and H[n] be computed in this scenario?

The convolution can be computed as $y[n] = \sum_{k=-\infty}^{\infty} X[k] H[n - k]$, considering the support of X[n] starting from n=2 and the properties of H[n].

What is the effect of the exponential term $(1/2)^{n-2}$ on the behavior of the input signal?

The exponential term causes the input signal to decay exponentially as n increases, influencing the amplitude and energy of the response.

What additional information is needed to fully determine the output y[n]?

The specific form or definition of the impulse response H[n] is required to compute the convolution and determine the exact output y[n].

Additional Resources

Understanding Discrete-Time Signals and Systems: Analyzing Input $X[n]$ and Impulse Response $H[n]$

In the realm of digital signal processing and systems analysis, understanding how signals interact with systems is fundamental. When presented with an input signal $X[n]$ and a system characterized by an impulse response $H[n]$, engineers and researchers seek to determine the output signal $Y[n]$. Such analysis not only provides insight into system behavior but also helps in designing filters, controllers, and communication systems. This article delves into the specifics of a particular input signal and impulse response, exploring their mathematical properties, how they interact through convolution, and the broader implications within system analysis.

Foundation: Discrete-Time Signals and Systems

Before analyzing the specific signals, it is essential to understand the basic concepts underpinning discrete-time signals and systems.

Discrete-Time Signals

Discrete-time signals are sequences defined at discrete moments in time, typically integer indices n. These signals can represent a variety of phenomena, from audio signals sampled at regular intervals to digital communication signals.

Common Types of Discrete-Time Signals:
- Unit Step Signal $U[n]$: Defined as $U[n] = 1$ for $n \geq 0$, and 0 otherwise. It models signals that turn on at a specific time.
- Unit Impulse $\delta[n]$: Defined as $\delta[n] = 1$ at $n=0$, and 0 elsewhere. It acts as the identity element in convolution.
- Exponential Sequences: Such as a^n, which can model decay or growth processes.

Discrete-Time Systems and Convolution

A linear time-invariant (LTI) system's output $Y[n]$ in response to an input $X[n]$ can be expressed via convolution with the system's impulse response $H[n]$:

$$
Y[n] = (X H)[n] = \sum_{k=-\infty}^{\infty} X[k] \cdot H[n - k]
$$

Understanding this operation is crucial for analyzing how systems modify input signals.

Analyzing the Given Input Signal $X[n]$

The input signal provided is:

$$
X[n] = \left(\frac{1}{2}\right)^n - 2 \quad \text{for} \quad n \geq 2 \quad \text{(assuming)}
$$

and multiplied by the unit step $U[n-2]$.

Mathematical Expression and Domain

The complete expression incorporating the unit step is:

$$
X[n] = \left[\left(\frac{1}{2}\right)^n - 2 \right] U[n-2]
$$

This indicates that for $n < 2$, $X[n] = 0$, and for $n \geq 2$:

$$
X[n] = \left(\frac{1}{2}\right)^n - 2
$$

Implications:
- The signal is zero before $n=2$, which simplifies analysis by limiting the sum in convolution to $n \geq 2$.
- As n increases, $\left(\frac{1}{2}\right)^n$ approaches zero, and $X[n]$ tends toward -2, indicating a decaying exponential combined with a constant offset.

Properties of $X[n]$

- Causality: Since $X[n]$ is zero for $n<2$, it is causal starting from $n=2$.
- Decay: The exponential term $\left(\frac{1}{2}\right)^n$ causes rapid decay as n increases.
- Offset: The -2 shifts the sequence downward, creating a negative baseline after the initial transient.

The System's Impulse Response $H[n]$

While the specific form of $H[n]$ isn't provided, we know it is a unit impulse response, which characterizes the system's behavior. To analyze the output, we'll consider general properties and typical forms.

Common Forms of $H[n]$

- Finite Impulse Response (FIR): $H[n]$ is non-zero only for a finite number of samples.

- Infinite Impulse Response (IIR): $H[n]$ extends infinitely, often associated with recursive systems.

Properties Affecting Convolution

- Linearity: The system responds linearly to inputs.
- Time-Invariance: The impulse response does not change over time.
- Causality: For causal systems, $H[n] = 0$ for $n < 0$.

Depending on $H[n]$, the output $Y[n]$ can be significantly different. For illustration, assume a causal, finite-duration $H[n]$ for simplicity, such as:

$$
H[n] = \begin{cases}
h_0, & n=0 \\
h_1, & n=1 \\
\vdots \\
0, & \text{otherwise}
\end{cases}
$$

Calculating the Output $Y[n]$ Through Convolution

The core operation to determine the system's output is convolution:

$$
Y[n] = \sum_{k=-\infty}^{\infty} X[k] \cdot H[n - k]
$$

Given the causality and the support of $X[n]$, the sum simplifies to:

$$
Y[n] = \sum_{k=2}^{n} \left[\left(\frac{1}{2}\right)^k - 2 \right] \cdot H[n - k]
$$

for $n \geq 2$. For $n < 2$, $Y[n] = 0$.

Step-by-step Approach:

1. Identify the support of $X[k]$: Non-zero only for $k \geq 2$.

2. Identify the support of $H[n-k]$: Usually for $n-k \geq 0$ if $H[n]$ is causal.
3. Determine the limits of summation: For each n, only sum over k where both $X[k]$ and $H[n-k]$ are non-zero.
4. Compute the sum: Plug in known or assumed forms of $H[n]$ and evaluate.

Analytical Example: Simplified Impulse Response

Suppose $H[n]$ is a simple causal FIR filter:

$$
H[n] = \begin{cases}
h, & n=0 \\
0, & \text{otherwise}
\end{cases}
$$

In this case, convolution reduces to:

$$
Y[n] = X[n] \cdot h
$$

for $n \geq 2$. Therefore,

$$
Y[n] = h \left[\left(\frac{1}{2}\right)^n - 2 \right], \quad n \geq 2
$$

This straightforward scenario illustrates how the system scales the input sequence.

Broader Implications and Applications

Understanding the convolution of such signals has widespread applications in various fields:

- Filtering: Designing digital filters that modify signals, like noise reduction or signal enhancement.

- System Identification: Determining system characteristics based on input-output data.
- Signal Reconstruction: Reconstructing signals from measurements, especially in communication systems.
- Control Systems: Analyzing system stability and response to inputs.

The specific form of $X[n]$ with an exponential decay and a step function is common in modeling natural processes, such as RC circuits, population decay, and economic models. The impulse response $H[n]$ models how systems respond over time, which can be tailored for specific applications like audio processing or data transmission.

Concluding Remarks

Analyzing an input signal like $X[n] = \left(\frac{1}{2}\right)^n - 2$ multiplied by $U[n-2]$, in conjunction with a system's impulse response $H[n]$, exemplifies the core principles of discrete-time system analysis. The process involves understanding the properties of the signals, leveraging convolution as the fundamental operation, and interpreting the results within the context of system behavior.

While the specific impulse response $H[n]$ influences the exact output, the methodology remains consistent: identify the effective support of the signals, perform the convolution carefully, and analyze the resulting output to assess system performance. These techniques underpin much of modern digital signal processing, enabling engineers to design and analyze systems that are integral to communications, control, and multimedia applications.

By mastering these concepts, professionals can predict system responses accurately, optimize system design, and develop innovative solutions that leverage the intricate dance between inputs and system characteristics—a dance choreographed by the mathematics of convolution and the rich properties of discrete-time signals.

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