Consider The Circle Centered At The Origin And Passing Through The Point (0, 2). (a) Give The Equation

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Understanding the fundamental properties of circles in coordinate geometry is essential for solving many mathematical problems. When a circle is centered at the origin, its equation simplifies significantly, making it easier to analyze and interpret. In this article, we will explore how to derive the equation of a circle centered at the origin that passes through a given point, specifically (0, 2). We will delve into the core concepts, formula derivations, and practical applications related to this problem.

Understanding the Standard Equation of a Circle

The General Equation

The standard form of a circle's equation in the coordinate plane is expressed as:

- $(x - h)^2 + (y - k)^2 = r^2$

where:
- (h, k) are the coordinates of the circle's center,
- r is the radius of the circle.

When the circle is centered at the origin, (h, k) simplifies to (0, 0), and the equation reduces to:

- $x^2 + y^2 = r^2$

This form is particularly straightforward to work with because it directly relates the x and y coordinates to the radius.

The Significance of the Radius

The radius (r) determines the size of the circle. Knowing the radius allows us to plot the circle accurately and understand its spatial relationship with other points and figures in the plane.

Determining the Equation of the Circle Passing Through a Given Point

Given Data

In our specific problem:
- The circle is centered at the origin: (0, 0).
- It passes through the point: (0, 2).

Using this information, our goal is to find the explicit equation of this circle.

Step-by-Step Solution

1. Identify the center and point:
   Center: (0, 0)
   Point on the circle: (0, 2)

2. Recall the standard form:
   Equation: $x^2 + y^2 = r^2$

3. Find the radius (r):
   Since the circle passes through (0, 2), and the center is at (0, 0), the radius is simply the distance between these two points:

   r = distance between (0, 0) and (0, 2):
   $$
   r = \sqrt{(0 - 0)^2 + (2 - 0)^2} = \sqrt{0 + 4} = 2
   $$

4. Write the equation:
   Plugging r = 2 into the standard form:
   $$
   x^2 + y^2 = 2^2 = 4
   $$

Final Equation of the Circle:

$$
x^2 + y^2 = 4
$$

This is the explicit equation of the circle centered at the origin passing through the point (0, 2).

Visualizing the Circle

Graphical Representation

Understanding the geometry of the circle helps in visualizing its position relative to axes and other points.

- The center is at (0, 0), the origin.
- The radius is 2 units.
- The point (0, 2) lies exactly 2 units above the origin on the y-axis, confirming our calculations.

Plotting Tips

- Draw the coordinate axes.
- Mark the origin.
- From the origin, measure 2 units along the y-axis to locate the point (0, 2).
- Draw a circle with radius 2 centered at the origin, passing through this point.

Additional Concepts and Applications

Other Points on the Circle

Since the circle's equation is $x^2 + y^2 = 4$, any point (x, y) satisfying this relation lies on the circle.

Examples:
- (2, 0) because $2^2 + 0^2 = 4$.
- (0, -2) because $0^2 + (-2)^2 = 4$.
- (√3, 1) because $(\sqrt{3})^2 + 1^2 = 3 + 1 = 4$.

Real-World Applications

Understanding circle equations is valuable in various fields:
- Engineering: Designing circular components and structures.
- Physics: Analyzing motion in circular paths.

- Computer Graphics: Rendering circles and curves.
- Navigation: Calculating distances and plotting routes.

Conclusion

Deriving the equation of a circle centered at the origin that passes through a specific point involves straightforward steps rooted in the distance formula and the standard form of a circle's equation. For the given point (0, 2), the radius is calculated as 2, leading to the simple, elegant equation:

\[
x^2 + y^2 = 4
\]

This equation encapsulates all points lying exactly 2 units from the origin, forming a perfect circle. Mastery of these fundamental concepts enables students and professionals to analyze circular shapes efficiently, interpret geometric relationships, and apply these principles across various disciplines.

Remember: The key to solving such problems is understanding the relationship between the circle's center, a point on the circle, and the radius, which together define the unique circle in the coordinate plane.

Frequently Asked Questions

What is the equation of the circle centered at the origin and passing through the point (0, 2)?

The equation of the circle is $x^2 + y^2 = 4$.

How do you determine the radius of a circle centered at the origin passing through a specific point?

The radius is the distance from the origin to the point, calculated using the distance formula: $r = \sqrt{(x^2 + y^2)}$.

Why is the equation of the circle centered at the origin written as $x^2 + y^2 = r^2$?

Because the circle's center is at (0, 0), the standard form simplifies to $x^2 + y^2 = r^2$, where r is the radius.

What is the radius of the circle passing through (0, 2) centered at (0, 0)?

The radius is 2, since the distance from (0, 0) to (0, 2) is $\sqrt{0^2 + 2^2} = 2$.

How can you graph the circle based on the given information?

Plot the center at the origin and draw a circle with radius 2, passing through points like (0, 2), (0, -2), (2, 0), and (-2, 0).

If the circle is centered at the origin and passes through (0, 2), what are its key features?

Key features include its radius of 2, center at (0, 0), and the equation $x^2 + y^2 = 4$.

Additional Resources

Circle Equation: An In-Depth Exploration of Centered at the Origin and Passing Through (0, 2)

Introduction

Mathematics is often described as the language of the universe, offering tools and concepts to describe the shapes, patterns, and relationships that define our world. Among its foundational elements are geometric figures like circles, which appear ubiquitously—from the wheels of vehicles to the orbits of planets, and even in the design of everyday objects. Understanding the equations that define these figures is essential for students, educators, and enthusiasts alike.

Today, we delve into a specific yet fundamental problem in coordinate geometry: Given a circle centered at the origin and passing through the point (0, 2), determine its equation. This seemingly straightforward question opens the door to exploring circle properties, coordinate systems, and algebraic techniques that underpin much of analytic geometry.

The Significance of the Circle Centered at the Origin

Before we derive the equation, it’s crucial to understand why the circle's position and attributes matter.

Why the Center at the Origin?

- Simplicity in Equation Formation: When a circle is centered at the origin (0,0), its equation assumes a straightforward form, making calculations and visualizations easier.

- Fundamental Geometric Properties: The origin is a natural reference point in the coordinate plane, serving as a baseline for many geometric problems.
- Educational Value: Many introductory lessons on conic sections use circles centered at the origin as initial examples for clarity.

The Passing Point (0, 2)

This point lies on the circle, providing a vital piece of information to determine the circle's radius once the center is known. Since (0, 2) is directly above the origin on the y-axis, this aligns with the symmetry of the circle about the origin, simplifying calculations.

Mathematical Foundations

The General Equation of a Circle

The standard form of a circle's equation in the Cartesian coordinate plane is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:

- (h, k) is the center of the circle,
- r is the radius.

When the circle is centered at the origin, $(h, k) = (0, 0)$, and the equation simplifies to:

$$
x^2 + y^2 = r^2
$$

Using the Given Information to Find the Radius

Since the circle passes through the point $(0, 2)$, we can substitute this point into the simplified equation to determine r^2:

$$
(0)^2 + (2)^2 = r^2
$$
$$
0 + 4 = r^2
$$
$$
r^2 = 4
$$

Therefore, the radius r is:

$$
r = \pm 2
$$

But since radius is a distance (a scalar quantity), it must be positive:

$$
r = 2
$$

Deriving the Equation of the Circle

Having established the radius, the circle's equation is now:

$$
x^2 + y^2 = 4
$$

This is the concise, explicit equation representing the circle centered at the origin and passing through the point $(0, 2)$.

Geometric Interpretation and Properties

Visualizing the Circle

- Center: At the origin $(0, 0)$.
- Radius: 2 units.
- Points on the Circle: All points (x, y) satisfying $x^2 + y^2 = 4$.

Symmetry

- The circle exhibits perfect symmetry across both axes.
- The point $(0, 2)$ confirms the radius extends 2 units along the positive y-axis.
- Similarly, points $(0, -2)$, $(2, 0)$, and $(-2, 0)$ also lie on the circle, exemplifying its symmetry.

Applications

Understanding this specific circle can be useful in various contexts:

- Physics: Modeling circular motion with a fixed radius.
- Engineering: Designing components with symmetric circular features.
- Computer Graphics: Rendering circles centered at the origin in coordinate systems.

Extending the Concept: Variations and Related Problems

While the problem focuses on a circle centered at the origin passing through a specific point, similar questions can be extended or modified:

1. Circle with a Different Center

- Question: What is the equation of a circle passing through $(0, 2)$ with center at (h, k)?
- Approach:
- Use the general form: $(x - h)^2 + (y - k)^2 = r^2$.
- Substitute the point $(0, 2)$ into the equation to find r^2:

$$
(0 - h)^2 + (2 - k)^2 = r^2
$$

- The values of h, k, and r can then be determined based on additional constraints.

2. Circle Passing Through Multiple Points

- Question: How to find the equation of a circle passing through three given points?
- Method:
- Use the general form of the circle's equation.
- Set up a system of equations based on the points.
- Solve for the center and radius.

3. General Equation from the Center and Radius

- Once the center (h, k) and radius r are known, the equation is straightforward.
- Conversely, given an equation, one can extract the center and radius by rewriting it in standard form.

Practical Tips for Students and Educators

- Always verify the point lies on the circle by substituting into the equation.
- Remember the geometric meaning of the radius and center to interpret your equations meaningfully.
- Use symmetry properties to verify your results, especially for circles centered at the origin.
- Practice with different points to strengthen understanding of how passing points influence the circle's size and position.

Conclusion

The task of determining the equation of a circle centered at the origin passing through a specific point exemplifies the elegance and utility of coordinate geometry. Starting from fundamental principles, we derived that the circle passing through $(0, 2)$ with the origin as its center has the simple equation:

\[

\boxed{
x^2 + y^2 = 4
}
\]

This result not only reinforces the core concepts of the standard form of a circle but also illustrates how algebra and geometry intertwine to describe the shapes that define our world. Whether you're a student embarking on your journey in mathematics or an educator designing engaging lessons, understanding such problems provides a solid foundation for exploring more complex conic sections and geometric relationships.

In essence, mastering the derivation and interpretation of circle equations serves as a stepping stone to a deeper appreciation of geometry's role in science, engineering, and art—truly a testament to the enduring power of mathematical insight.

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