Consider The Following Consumers Problem: U(x,y) = X1/4 Y3/4. Prices Are Px = $10, Py = $10 And Income

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Introduction

Understanding consumer behavior is fundamental in microeconomics, as it provides insights into how individuals allocate their limited resources among various goods and services to maximize utility. The problem at hand involves a consumer with a specific utility function, facing given prices for two goods, and an income constraint. Analyzing this scenario not only helps in understanding individual decision-making but also offers broader implications for market demand and pricing strategies.

In this article, we will explore a consumer's optimization problem characterized by the utility function:

$$
U(x,y) = x^{1/4} y^{3/4}
$$

with prices:

- $P_x = \$10$
- $P_y = \$10$

and a certain income level, which will be specified. The goal is to determine the consumer's optimal consumption bundle, analyze how changes in income and prices affect consumption, and discuss the economic intuition behind these results.

Understanding the Consumer's Utility Function

What Is a Utility Function?

A utility function represents a consumer's preferences over different bundles of goods. It assigns a real number to each possible combination of goods, indicating the level of satisfaction or happiness derived from that bundle. The higher the utility, the more preferred the bundle.

Characteristics of the Given Utility Function

The utility function:

$$
U(x,y) = x^{1/4} y^{3/4}
$$

is a form of a Cobb-Douglas utility function. Key features include:

- Diminishing Marginal Utility: As consumption of a good increases, the additional utility gained from consuming more diminishes.
- Constant Elasticity of Substitution: Cobb-Douglas functions have a unitary elasticity of substitution, meaning the consumer is willing to substitute between goods at a constant rate.
- Relative Weights: The exponents $1/4$ and $3/4$ indicate the relative importance or preference for each good. Here, the consumer values good y three times more than good x.

Budget Constraint and Its Role

Formulating the Budget Constraint

The consumer's budget constraint reflects the total income spent on goods x and y:

$$
P_x x + P_y y \leq I
$$

where:

- $P_x = \$10$
- $P_y = \$10$
- I is the consumer's total income.

Suppose the income is $I = \$200$ (as an example). The budget constraint becomes:

$$
10x + 10y \leq 200

\]

or simplified:

$$
x + y \leq 20
$$

The consumer can choose any combination of x and y on or below this line.

Goal: Maximize Utility Subject to Budget Constraint

The optimization problem:

$$
\max_{x,y} U(x,y) = x^{1/4} y^{3/4}
$$

subject to:

$$
10x + 10y = 200
$$

Solving the Consumer’s Optimization Problem

Step 1: Write the Budget Constraint

Expressed as:

$$
x + y = 20
$$

which allows us to express one good in terms of the other:

$$
y = 20 - x
$$

Step 2: Set Up the Utility Function in Terms of One Variable

Substitute $y = 20 - x$ into the utility function:

$$
U(x) = x^{1/4} (20 - x)^{3/4}
$$

Our goal is to find the value of x that maximizes $U(x)$.

Step 3: Use the Method of Lagrange Multipliers

Alternatively, directly maximizing the utility function with respect to the budget constraint can be done using the method of Lagrange multipliers.

Formulate the Lagrangian:

$$
\mathcal{L} = x^{1/4} y^{3/4} - \lambda (P_x x + P_y y - I)
$$

Substituting prices:

$$
\mathcal{L} = x^{1/4} y^{3/4} - \lambda (10x + 10y - 200)
$$

Set the partial derivatives to zero:

$$
\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{4} x^{-3/4} y^{3/4} - 10 \lambda = 0
$$

$$
\frac{\partial \mathcal{L}}{\partial y} = \frac{3}{4} x^{1/4} y^{-1/4} - 10 \lambda = 0
$$

$$
\frac{\partial \mathcal{L}}{\partial \lambda} = -(10x + 10y - 200) = 0
$$

From the first two conditions, equate the expressions for λ:

$$
\frac{1}{4} x^{-3/4} y^{3/4} = \frac{3}{4} x^{1/4} y^{-1/4}
$$

Simplify:

$$
x^{-3/4} y^{3/4} = 3 x^{1/4} y^{-1/4}
$$

Divide both sides by $x^{1/4} y^{-1/4}$:

$$
x^{-3/4} y^{3/4} / (x^{1/4} y^{-1/4}) = 3
$$

This simplifies to:

$$
x^{-3/4 - 1/4} y^{3/4 + 1/4} = 3
$$

$$
x^{-1} y^{1} = 3
$$

which implies:

$$
\frac{y}{x} = 3
$$

Step 4: Find the Optimal Consumption Bundle

From the above:

$$
y = 3x
$$

Using the budget constraint:

$$
10x + 10y = 200
$$

$$
10x + 10(3x) = 200
$$

$$
10x + 30x = 200
$$

$$
40x = 200
$$

$$
x = 5
$$

and

$$
y = 3 \times 5 = 15
$$

Thus, the optimal consumption bundle is:

$$
(x^, y^) = (5, 15)
$$

Interpreting the Results

Optimal Consumption and Utility Level

The consumer should purchase:

- 5 units of good x,
- 15 units of good y,

to maximize utility given the prices and income.

The maximum utility can be calculated as:

$$
U(5, 15) = 5^{1/4} \times 15^{3/4}
$$

Calculating:

$$
5^{1/4} \approx e^{(1/4) \ln 5} \approx e^{0.20} \approx 1.22
$$

$$
15^{3/4} = (15^{1/4})^3
$$

Calculate $15^{1/4}$:

$$
15^{1/4} \approx e^{(1/4) \ln 15} \approx e^{(1/4) \times 2.708} \approx e^{0.677} \approx 1.97
$$

Then,

$$
15^{3/4} \approx (1.97)^3 \approx 1.97 \times 1.97 \times 1.97 \approx 7.66
$$

Finally,

$$
U_{max} \approx 1.22 \times 7.66 \approx 9.36
$$

The consumer's maximum utility is approximately 9.36 units.

Economic Intuition

- The consumer allocates their income to buy more of the good with the higher preference weight (y), which aligns with the exponents in the utility function.
- Since prices are equal, the consumer chooses the ratio of goods that reflects their relative preferences.
- The marginal rate of substitution (MRS) between x and y at the optimum is equal to the ratio of their prices, ensuring optimality.

Impact of Income and Price Changes

Effect of Income Changes

If income increases, the budget constraint shifts outward, allowing the consumer to afford more of both goods. Given the Cobb-Douglas utility structure, consumption of both goods increases proportionally with income, maintaining the same ratio:

$$
\frac{y}{x} = 3
$$

For example, if income rises from \$200 to \$300:

\[
10x + 10y = 300 \Right

Frequently Asked Questions

What is the utility function in the given consumer problem?

The utility function is U(x, y) = X^{1/4} Y^{3/4}.

What are the prices of goods X and Y in this problem?

Both goods have prices Px = $10 and Py = $10.

How can we determine the consumer's optimal consumption bundle?

By setting up the budget constraint and maximizing the utility function subject to Px·x + Py·y = Income, then solving for x and y.

What is the marginal rate of substitution (MRS) for this utility function?

The MRS of X for Y is (3/4) (Y / X).

How does the consumer's income influence their optimal consumption bundle?

An increase in income shifts the budget line outward, allowing for higher consumption of both goods, possibly increasing utility depending on preferences.

Given the utility function and prices, how do we find the consumer's demand functions?

By solving the utility maximization problem using the method of Lagrange multipliers or by equating MRS to price ratio, then solving for x and y in terms of income and prices.

What is the effect of equal prices on the consumer's choice between goods X and Y?

When prices are equal, the consumer's optimal choice depends on the relative marginal utilities, often leading to proportional consumption based on the exponents in the utility function.

How does the Cobb-Douglas form of utility influence the expenditure shares on goods X and Y?

In a Cobb-Douglas utility function, the consumer spends a fixed proportion of income on each good, determined by the exponents (1/4 on X and 3/4 on Y in this case).

What happens if the consumer's income increases while prices stay the same?

The consumer can afford to buy more of both goods, increasing overall utility, with the demand for each good increasing proportionally to their expenditure shares.

Additional Resources

Consumers Problem: $U(x,y) = X^{(1/4)} Y^{(3/4)}$, Prices Px = $10, Py = $10, and Income

When analyzing consumer behavior and utility maximization, the problem involving the utility function $U(x, y) = x^{(1/4)} y^{(3/4)}$ with prices Px = $10 and Py = $10, and a given income, provides a compelling case study. This particular utility function captures the consumer's preferences and trade-offs between two goods, x and y, under a budget constraint. Understanding this problem involves delving into the concepts of utility maximization, budget constraints, and the resulting demand functions. This article explores the problem comprehensively, analyzing the consumer's optimal choice, the features of the utility function, and the implications of the given prices and income.

Understanding the Utility Function

The utility function $U(x, y) = x^{(1/4)} y^{(3/4)}$ belongs to the class of Cobb-Douglas preferences, which are widely used in economics due to their analytical tractability and realistic assumptions about consumer preferences.

Features of the Utility Function

- Cobb-Douglas Form: The utility function is multiplicative with powers summing to 1 (1/4 + 3/4 = 1), indicating constant expenditure shares over different income levels.
- Diminishing Marginal Utility: As consumption of either good increases, the additional utility gained from consuming an extra unit diminishes, which is typical in consumer theory.
- Relative Preference: The exponents (1/4 for x and 3/4 for y) reflect the consumer's relative preference or importance assigned to each good, with a stronger preference for y, given its higher exponent.

Pros:
- Simplicity in deriving demand functions.
- Reflects realistic consumer preferences with diminishing marginal utility.
- Allows for clear analysis of how preferences influence consumption choices.

Cons:
- Assumes constant expenditure shares, which may not hold in all real-world scenarios.
- Does not capture more complex or changing preferences over different income levels.

Budget Constraint and Income

The consumer faces a budget constraint determined by the prices and income:
$$ P_x \times x + P_y \times y = I $$
Given:
- $P_x = \$10$
- $P_y = \$10$
- I (income) is unspecified but can be treated as a parameter for analysis.

This constraint defines the feasible set of consumption bundles the consumer can afford:
$$ 10x + 10y = I $$
or, simplifying:
$$ x + y = \frac{I}{10} $$

The consumer's goal is to choose x and y within this budget to maximize utility.

Maximizing Utility Under Budget Constraints

The core of the consumer problem is:
$$ \max_{x,y} U(x,y) = x^{1/4} y^{3/4} $$
subject to:
$$ 10x + 10y = I $$

This can be approached via the method of Lagrange multipliers or by using the concept of marginal rate of substitution (MRS).

Using the Marginal Rate of Substitution

The MRS of x for y is given by:
$$ MRS_{x,y} = \frac{\partial U/\partial x}{\partial U/\partial y} $$

Calculating the partial derivatives:

$$ \frac{\partial U}{\partial x} = \frac{1}{4} x^{-3/4} y^{3/4} $$
$$ \frac{\partial U}{\partial y} = \frac{3}{4} x^{1/4} y^{-1/4} $$

Therefore,
$$ MRS_{x,y} = \frac{\frac{1}{4} x^{-3/4} y^{3/4}}{\frac{3}{4} x^{1/4} y^{-1/4}} = \frac{1/4}{3/4} \times \frac{y^{3/4}}{x^{1/4}} \times \frac{y^{1/4}}{x^{-3/4}} $$

Simplifying:
$$ MRS_{x,y} = \frac{1}{3} \times \frac{y}{x} $$

The consumer maximizes utility where the MRS equals the ratio of prices:
$$ MRS_{x,y} = \frac{P_x}{P_y} $$

Given prices are equal (\$10 each),
$$ \frac{1}{3} \times \frac{y}{x} = 1 $$
$$ \frac{y}{x} = 3 $$

This indicates the consumer prefers to consume y three times as much as x at the optimum.

Deriving the Demand Functions

From the ratio $y = 3x$, we can substitute into the budget constraint:
$$ 10x + 10(3x) = I $$
$$ 10x + 30x = I $$
$$ 40x = I $$
$$ x^{\wedge} = \frac{I}{40} $$

And,
$$ y^{\wedge} = 3x^{\wedge} = 3 \times \frac{I}{40} = \frac{3I}{40} $$

Demand functions:
$$ x^{\wedge}(I) = \frac{I}{40} $$
$$ y^{\wedge}(I) = \frac{3I}{40} $$

These demand functions reveal how consumption of each good depends on income, given prices and preferences.

Implications and Analysis

The derived demand functions and optimal choices provide several insights into consumer behavior under the specified utility function and prices.

Income Elasticity

- Both \(x^\) and \(y^\) increase proportionally with income, indicating unit elasticity relative to income.
- The consumer allocates a fixed share of income to each good, consistent with the Cobb-Douglas assumption.

Relative Preference and Consumption Ratios

- The consumer prefers to consume y three times as much as x, regardless of income level, due to the exponents in the utility function.
- This fixed ratio is a characteristic feature of Cobb-Douglas preferences, where expenditure shares are constant.

Price Changes and Demand Response

- Since prices are equal, any change in the prices would alter the MRS and thus the optimal ratio of consumption.
- For instance, if P_x decreases relative to P_y, the consumer would allocate more income to x, shifting demand accordingly.

Pros and Cons of the Model

Pros:
- Provides clear, analytical solutions for demand functions.
- Demonstrates the impact of preferences, prices, and income on consumer choices.
- Suitable for teaching basic consumer theory concepts.

Cons:
- Assumes constant expenditure shares, which may not reflect real-world variability.
- Utility function is simplified and may not capture complex preferences or changing tastes.
- Prices are assumed to be fixed and known, ignoring market dynamics.

Conclusion

The consumer problem involving the utility function $U(x, y) = x^{1/4} y^{3/4}$ with equal prices of \$10 for both goods and a given income exemplifies the elegant analytical framework of consumer theory. The derivation of demand functions shows that consumers allocate income proportionally to their preference exponents, maintaining a fixed consumption ratio. The model underscores the importance of preferences in shaping demand and highlights the straightforward nature of Cobb-Douglas utility maximization. While the model's simplicity offers clarity and ease of analysis, its assumptions may limit its application to more complex or realistic scenarios. Nonetheless, this problem serves as a fundamental illustration of how consumers make optimal choices in the face of constraints, providing valuable insights for economic analysis, policy-making, and understanding market behavior.

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