

Consider The Following. $F(x) = 25x^2$ Find The Critical Numbers. (Enter Your Answers As A Comma-separated

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Introduction

Understanding the concept of critical numbers is a fundamental aspect of calculus, particularly when analyzing functions for their maximums, minimums, and points of inflection. Critical numbers often serve as potential candidates for local extrema and are essential in the process of optimization problems. In this article, we will explore how to find the critical numbers of the function $F(x) = 25x^2$, providing a detailed, step-by-step approach to ensure clarity and comprehension.

What Are Critical Numbers?

Definition

A critical number (or critical point) of a function is a value of x within the domain of the function where the derivative is either zero or undefined. These points are significant because they indicate possible locations where the function could have:

- A local maximum
- A local minimum
- An inflection point

Importance in Calculus

Identifying critical numbers is a crucial step in the first derivative test and second derivative test, which help determine the nature of the critical points. They are also instrumental in solving optimization problems, where the goal is to find the maximum or minimum value of a function within a certain interval.

Step-by-Step Guide to Find Critical Numbers of $F(x) = 25x^2$

Let's delve into how to find the critical numbers of the specific function:

$$[F(x) = 25x^2]$$

Step 1: Find the derivative of $F(x)$

The first step in locating critical numbers is to differentiate the function with respect to x .

Given:

$$F(x) = 25x^2$$

Using basic differentiation rules, namely the power rule:

$$\frac{d}{dx} [ax^n] = a n x^{n-1}$$

we get:

$$F'(x) = 25 \times 2 x^{2-1} = 50x$$

Step 2: Set the derivative equal to zero and solve for x

To find critical points, solve:

$$F'(x) = 0$$

Substituting the derivative:

$$50x = 0$$

Divide both sides by 50:

$$x = 0$$

Step 3: Check for points where the derivative is undefined

Since the derivative $F'(x) = 50x$ is a polynomial function, it is defined for all real numbers. Therefore, there are no points where the derivative is undefined.

Critical Numbers for $F(x) = 25x^2$

Based on the above steps, the critical number occurs where the derivative equals zero:

$$x = 0$$

And since the derivative is defined everywhere, no other critical points exist.

Answer:

$$\boxed{0}$$

\]

Understanding the Critical Number in Context

Graphical Interpretation

The function $f(x) = 25x^2$ is a parabola opening upwards, with its vertex at the origin (0, 0). The critical number at $x=0$ corresponds to this vertex, which is the global minimum of the function.

Significance

- At $x=0$, the slope of the tangent to the curve is zero.
- The function attains its minimum value at this point, which is:

\[

$$f(0) = 25 \times 0^2 = 0$$

\]

Additional Concepts Related to Critical Numbers

Critical Numbers and Extrema

While critical numbers are potential locations for local maxima or minima, not every critical point necessarily corresponds to an extremum. To determine whether a critical point is a maximum, minimum, or neither, additional tests are used:

- First Derivative Test: Analyze the sign changes of $f'(x)$ around the critical point.
- Second Derivative Test: Examine $f''(x)$ at the critical point.

Applying the Second Derivative Test to $f(x)=25x^2$

Let's compute the second derivative:

$$f''(x) = \frac{d}{dx} (f'(x)) = \frac{d}{dx} (50x) = 50$$

Since $f''(x) = 50 > 0$, the function is concave up everywhere, indicating that the critical point at $x=0$ is a local minimum.

Summary of Key Points

| Aspect | Details |

|-----|-----|

| Function | $f(x) = 25x^2$ |

Derivative	$F'(x) = 50x$
Critical Number	$x=0$
Derivative undefined?	No, derivative is defined everywhere
Nature of critical point	Global minimum at $x=0$
Function value at critical point	$F(0) = 0$

Practical Applications of Finding Critical Numbers

Understanding how to find critical points has numerous real-world applications, including:

- Optimization problems: Maximizing profits or minimizing costs.
- Physics: Finding points of equilibrium in motion or energy functions.
- Engineering: Structural analysis where maximum stress points are critical.

For example, in designing a roller coaster, engineers need to find the critical points of the height function to ensure safety and thrill.

Additional Practice Problems

To reinforce your understanding, consider solving the following:

1. Find the critical numbers of $G(x) = -4x^3 + 12x$.
2. Determine the critical points of $H(x) = \frac{1}{3}x^3 - 3x + 5$.
3. For the function $J(x) = \sqrt{x}$, identify the critical points within its domain.

Conclusion

In this comprehensive guide, we've demonstrated how to find the critical numbers of the function $F(x) = 25x^2$. The process involves differentiating the function, setting the derivative equal to zero, and solving for x . For this particular quadratic function, the only critical number is at $x=0$, corresponding to the vertex of the parabola, which represents a global minimum. Recognizing and analyzing critical points is an essential skill in calculus, with broad applications across various fields.

Final Note

Always remember that critical numbers are just the starting point in analyzing a function's behavior. To fully understand the nature of these points, employ the first and second derivative tests, and consider the function's context and graph.

By mastering the process of finding critical numbers, you enhance your calculus problem-solving toolkit, enabling you to analyze and optimize functions with confidence.

Frequently Asked Questions

How do I find the critical numbers of the function $F(x) = 25x^2$?

To find the critical numbers, first compute the derivative $F'(x) = 50x$, then set it equal to zero: $50x = 0$, which gives $x = 0$. Thus, the only critical number is $x = 0$.

What are the critical numbers for the function $F(x) = 25x^2$?

The critical number is $x = 0$, since it is the point where the derivative equals zero. There are no points where the derivative is undefined.

Why are critical numbers important in analyzing the function $F(x) = 25x^2$?

Critical numbers help identify potential local maxima, minima, or points of inflection, which are essential in understanding the function's behavior.

If $F(x) = 25x^2$, how do I determine where the function has a minimum or maximum?

Since the only critical point is at $x = 0$ and the second derivative $F''(x) = 50 > 0$, the function has a local minimum at $x = 0$.

Can the critical number $x = 0$ for $F(x) = 25x^2$ be considered a maximum?

No, because the function is a parabola opening upwards with a minimum at $x = 0$; therefore, $x = 0$ is a critical point corresponding to a local minimum, not a maximum.

Enter the critical numbers for $F(x) = 25x^2$ as a comma-separated list.

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Additional Resources

Consider The Following. $F(x) = 25x^2$ Find The Critical Numbers. (Enter Your Answers As A Comma-separated

Introduction

Mathematics, particularly calculus, serves as a fundamental cornerstone in understanding the behavior of functions, optimizing processes, and analyzing real-world phenomena. Among the essential concepts in calculus are critical numbers, which enable mathematicians and scientists to identify potential points of maxima, minima, or inflection points on a graph. This article dives deeply into the problem: Given the function $F(x) = 25x^2$, find its critical numbers, dissecting each step to illuminate the process, the reasoning behind it, and its broader implications.

Understanding the Concept of Critical Numbers

What Are Critical Numbers?

In mathematics, critical numbers of a function are specific points in its domain where the function's derivative is either zero or undefined. These points are significant because they often mark locations where the function reaches local maximums, local minimums, or saddle points—collectively known as extrema and points of inflection.

To formalize, consider a function $f(x)$. The critical numbers are the values of x such that:

- $f'(x) = 0$, or
- $f'(x)$ does not exist.

Finding critical numbers is a crucial step in the process of optimization problems, where one seeks to find the best or worst values of a function within a domain.

Analyzing the Function $F(x) = 25x^2$

The Function's Structure

The function provided, $F(x) = 25x^2$, is a quadratic function with a leading coefficient of 25. It is a parabola opening upwards, symmetric about the y-axis, and its vertex at the origin (0,0). Such functions are well-understood in calculus and serve as classic examples of functions with straightforward derivatives.

Domain of the Function

Since the function is a quadratic polynomial, its domain is all real numbers:

$$\text{Domain} = (-\infty, \infty).$$

This unrestricted domain simplifies the analysis, as the critical points can be located anywhere along the real axis.

Step-by-Step Solution to Find Critical Numbers

The process of finding critical numbers involves several methodical steps:

Step 1: Compute the Derivative $(F'(x))$

Given $(F(x) = 25x^2)$, we differentiate with respect to (x) :

$$F'(x) = \frac{d}{dx} (25x^2) = 25 \times \frac{d}{dx} (x^2) = 25 \times 2x = 50x.$$

This derivative tells us the slope of the tangent line to the curve at any point (x) .

Step 2: Set the Derivative Equal to Zero

To find potential critical points, solve:

$$F'(x) = 0 \Rightarrow 50x = 0 \Rightarrow x = 0.$$

This is the only candidate where the slope of the tangent is zero.

Step 3: Check for Points Where the Derivative is Undefined

Since $(F'(x) = 50x)$ is a polynomial, it is defined for all real numbers. Therefore, there are no points where the derivative is undefined.

Step 4: Consolidate Critical Numbers

The only critical number here is $(\boxed{0})$.

Interpreting the Critical Number in Context

Is the Critical Number a Maximum, Minimum, or Inflection Point?

Given the nature of the function $(F(x) = 25x^2)$ —a parabola opening upward—the critical point at $(x=0)$ corresponds to its vertex, which is the absolute minimum point.

- At $(x=0)$:

$$F(0) = 25 \times 0^2 = 0,$$

which is the lowest point on the graph.

Visualization

Plotting the graph of $(F(x) = 25x^2)$ reveals a symmetric parabola with its vertex at the origin. The

critical number at $(x=0)$ marks this vertex, confirming it as the minimum point.

Broader Implications and Applications

Understanding and identifying critical numbers extend beyond pure mathematics into numerous applications:

- Optimization Problems: In fields like economics, engineering, and data science, critical points help determine optimal solutions—maximizing profit or minimizing cost, for example.
- Physics and Engineering: Analyzing the critical points of functions modeling physical systems allows for understanding equilibrium states or points of instability.
- Economics: Cost and revenue functions often involve finding critical points to determine the most profitable level of production.

In this specific context, analyzing $(F(x) = 25x^2)$ serves as an introductory example, illustrating how derivative calculations lead directly to critical points, which in turn reveal the function's extremal behavior.

Additional Considerations: Second Derivative Test and Concavity

While we've identified the critical number at $(x=0)$, further analysis can be performed to classify this point:

- Second Derivative:

$$\begin{aligned} & \left[\right. \\ F''(x) &= \frac{d}{dx} (50x) = 50. \\ & \left. \right] \end{aligned}$$

Since $(F''(x) = 50 > 0)$ for all (x) , the function is concave up everywhere, confirming that the critical point at $(x=0)$ is a local (and in this case, global) minimum.

Summary and Final Answer

In conclusion, the critical number of the function $(F(x) = 25x^2)$ is the point where the derivative equals zero, which occurs at $(x=0)$. Given the nature of the parabola, this critical point coincides with the absolute minimum of the function.

Final Answer:

$$\begin{aligned} & \left[\right. \\ & \boxed{0} \\ & \left. \right] \end{aligned}$$

Reflective Insights

This analysis underscores the elegance of calculus in providing clear, methodical pathways to understanding the behavior of functions. The simplicity of the quadratic function exemplifies core principles—derivative calculation, setting derivatives to zero, and analyzing the second derivative—to classify critical points efficiently. While the example is straightforward, the same principles extend to more complex functions, serving as foundational tools in scientific inquiry and problem-solving across disciplines.

Understanding the determination of critical numbers enhances not only mathematical literacy but also empowers professionals and students alike to approach real-world problems with analytical rigor and confidence.

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