Consider The Following Point And Line. Point Line (4,1) 9x - 3y = 4(a) Write The Equation For The Line

Consider The Following Point And Line. Point Line (4,1) 9x - 3y = 4(a) Write The Equation For The Line

Understanding how to determine the equation of a line given a point and a linear equation is a fundamental concept in coordinate geometry. Whether you are a student preparing for exams or a professional working on geometric problems, mastering this process is crucial. In this article, we will explore the step-by-step approach to writing the equation of a line when provided with a specific point, such as (4,1), and a linear equation, such as $9x - 3y = 4(a)$. We will analyze the given information, interpret the parameters involved, and derive the most general and specific forms of the line's equation. Additionally, we will include tips, common mistakes, and practice problems to solidify your understanding.

Understanding the Given Data

Before delving into calculations, it is essential to interpret the data provided:

- Point: (4,1)

- Line Equation: $9x - 3y = 4a$

The point (4,1) lies somewhere in the coordinate plane, and the line is defined implicitly by the equation $9x - 3y = 4a$, where 'a' appears as a parameter or constant.

Key questions:

- What is the relationship between the point and the line?

- How can the parameter 'a' be determined or expressed?

- What form should the final equation take?

Analyzing the Line Equation: 9x - 3y = 4a

The given line equation is:

\[9x - 3y = 4a \]

This is a linear equation in standard form, where:

- The coefficients of x and y are 9 and -3 respectively.

- The constant term is 4a, which depends on the parameter 'a'.

Simplification:

Divide both sides of the equation by 3 for easier interpretation:

\[
\frac{9x}{3} - \frac{3y}{3} = \frac{4a}{3}
\]

which simplifies to:

\[
3x - y = \frac{4a}{3}
\]

This form makes it clearer that the line is characterized by the relation:

\[
y = 3x - \frac{4a}{3}
\]

Finding the Parameter 'a' Using the Point (4,1)

Since the point (4,1) lies on the line, it must satisfy the line's equation. Substituting x=4 and y=1 into the simplified form:

\[
3(4) - 1 = \frac{4a}{3}
\]

Calculating the left side:

\[
12 - 1 = 11
\]

Thus,

\[
11 = \frac{4a}{3}

\]

Solving for 'a':

\[
a = \frac{11 \times 3}{4} = \frac{33}{4} = 8.25
\]

Conclusion:

- The parameter 'a' is precisely \(\frac{33}{4}\).

- The specific line passing through (4,1) defined by the initial equation is obtained by substituting this value.

Deriving the Equation of the Line

With the value of 'a' known, the line's equation becomes:

\[
9x - 3y = 4a = 4 \times \frac{33}{4} = 33
\]

or

\[
9x - 3y = 33
\]

Dividing both sides by 3:

\[
3x - y = 11
\]

or

\[
y = 3x - 11
\]

This is the specific equation of the line passing through the point (4,1) and satisfying the original relation.

General Form of the Equation for the Line

If the problem is to write the general form of the line's equation in terms of the parameter 'a', it would be:

$$
9x - 3y = 4a
$$

or, simplified:

$$
y = 3x - \frac{4a}{3}
$$

Where 'a' is any real constant, and the specific value determined from the point (4,1) is $a = \frac{33}{4}$.

Understanding the Geometric Significance

The process above demonstrates how the point (4,1) constrains the parameter 'a'. Geometrically, this means:

- The parameter 'a' controls the position of the line in the plane.

- For different values of 'a', the line shifts accordingly.

- The point (4,1) lies on the specific line corresponding to $a = \frac{33}{4}$.

Step-by-Step Summary of the Procedure

To systematically write the equation of a line given a point and a general line equation involving a parameter, follow these steps:

1. Identify the point and the line equation: Recognize the given point and the line's equation in standard form.

2. Substitute the point into the line equation: Plug in the x and y coordinates into the line's equation to find the value of the parameter.

3. Solve for the parameter: Isolate the parameter 'a' or the constant term as needed.

4. Write the specific line equation: Substitute the found parameter value back into the original line equation to get the explicit equation.

5. Express in preferred form: Present the final line equation in slope-intercept, standard, or point-slope form as appropriate.

Additional Topics and Variations

Beyond the core problem, understanding related concepts enhances your mastery:

- Equation of a line given a point and a slope: Use point-slope form when the slope is known.

- Equation of a line passing through two points: Apply the two-point form or the slope formula.

- Parametric form of a line: Express the line in terms of a parameter, especially useful for lines with variable components.

- Line with a known intercept and slope: Derive equations when intercepts are specified.

Practice Problems for Mastery

To reinforce your understanding, attempt the following practice questions:

1. Given the point (2, -3) and the line $4x + y = 7a$, find the value of 'a' so that the point lies on the line. Write the specific equation of the line.

2. Find the equation of the line passing through (5,2) that satisfies $6x - 2y = 3a$. Determine the value of 'a' and write the specific line equation.

3. The line $3x + 2y = k$ passes through the point (1,4). Find the value of 'k' and write the explicit line equation.

4. For a parameter 'b', the line is given by $2x - y = b$. Find the value of 'b' when the line passes through (3, -1), and write the specific line equation.

Conclusion

Understanding how to find the equation of a line given a point and a parametric or general line equation is a vital skill in coordinate geometry. By substituting the given point into the line's equation, solving for the unknown parameter, and then rewriting the equation, you can determine the specific line that passes through the point. In the example discussed, the point (4,1) and the line $9x - 3y = 4a$ lead us to find that $a = \frac{33}{4}$, resulting in the explicit line equation $9x - 3y = 33$. Mastery of these methods enables you to analyze and construct lines efficiently, which is foundational in geometry, algebra, and related fields. Keep practicing with various problems to strengthen your skills and deepen your understanding of linear equations and their geometric interpretations.

Frequently Asked Questions

What is the given point in the problem 'Consider the following point and line'?

The given point is (4, 1).

What is the equation of the line provided in the problem?

The equation of the line is $9x - 3y = 4$.

How can we express the line 9x - 3y = 4 in slope-intercept form?

Divide both sides by 3 to get $3x - y = 4/3$, then rewrite as $y = 3x - 4/3$.

What is the slope of the line 9x - 3y = 4?

The slope is 3, since the line can be rewritten as $y = 3x - 4/3$.

How do you find the equation of a line passing through a specific point and parallel to another line?

Use the point-slope form $y - y_1 = m(x - x_1)$, where m is the slope of the given line.

What is the equation of a line passing through point (4, 1) and parallel to the line 9x - 3y = 4?

Since the lines are parallel, they share the same slope 3. Using point-slope form: $y - 1 = 3(x - 4)$, which simplifies to $y = 3x - 11$.

How do you find the equation of a line passing through a given point and perpendicular to the line 9x - 3y = 4?

First find the slope of the given line (which is 3), then take the negative reciprocal (-1/3), and use point-slope form with point (4, 1). The equation is $y - 1 = -1/3(x - 4)$.

Can the original line 9x - 3y = 4 be rewritten in standard form? If so, what is it?

Yes, it is already in standard form: 9x - 3y = 4.

What is the significance of the point (4, 1) in solving for the line's equation?

The point (4, 1) provides a specific location through which the new line passes, allowing us to determine the exact equation of the line that meets the given criteria.

Additional Resources

Consider The Following Point And Line. Point Line (4,1) 9x - 3y = 4(a) Write The Equation For The Line

Understanding the fundamentals of coordinate geometry is essential for students, educators, and professionals working with spatial data, engineering designs, and mathematical modeling. Today, we explore a classic problem: given a point and a line, how do we derive the equation of a line that passes through a specific point and is related to a given line? This problem encapsulates core concepts such as the point-slope form, the slope-intercept form, and the general equation of a line, providing a foundational understanding of linear equations in two variables.

In this article, we'll dissect the problem statement: "Consider The Following Point And Line. Point (4,1) and Line 9x - 3y = 4(a). Write the Equation For The Line." We'll interpret and analyze the given information, explore the process of deriving the line's equation, and clarify the mathematical principles involved. Whether you're a student preparing for exams, a teacher designing lesson plans, or a professional applying these concepts, this comprehensive guide aims to illuminate the topic with clarity and precision.

Understanding the Components of the Problem

Before jumping into calculations, it's vital to understand each element presented:

- The Point (4,1):
This is a coordinate point on the Cartesian plane, representing the location with x-coordinate 4 and y-coordinate 1.

- The Line 9x - 3y = 4(a):
This is a linear equation involving a parameter 'a.' The presence of 'a' suggests that the line's specific position depends on the value of this parameter. Our goal is to understand how 'a' influences the line and how to formulate the equation of another line passing through the point (4,1) in relation to this line.

Interpreting the Line Equation: 9x - 3y = 4(a)

The equation $9x - 3y = 4(a)$ can be rewritten in various forms to analyze its properties:

Simplification and Standard Form

Divide both sides of the equation by 3:

$$ 3x - y = \frac{4a}{3} $$

This form helps in identifying the line's slope and intercepts:

- Slope (m):
The slope of the line in the form $3x - y = \text{constant}$ is 3, since rearranged as $y = 3x - (4a/3)$.

- Y-intercept:
When $x=0$, $y = - (4a/3)$.
So, the y-intercept depends on the parameter 'a.'

Geometric Interpretation

- As 'a' varies, the line shifts vertically, maintaining a slope of 3.
- The set of all lines $9x - 3y = 4a$ constitutes a family of parallel lines, each differing by their intercepts.

The Core Question: Finding the Equation of a Line Passing Through (4,1)

Given the point (4,1), we aim to find a line that either:

- Passes through this point and is related to the given family of lines, or
- Has a specific relationship with the given line (e.g., parallel, perpendicular, or intersecting at a particular point).

Since the problem states "Write The Equation For The Line," it suggests that either:

- We are to find a line passing through (4,1) that is parallel to the given line, or
- We are to find a line passing through (4,1) that intersects the given line at a specific point, or
- The problem may involve constructing a line with a certain relation to the family of lines.

For clarity, let's explore these possibilities.

Scenario 1: Line Passing Through (4,1) and Parallel to the Given Line

Step 1: Determine the slope of the given line

From the simplified form:

$$ y = 3x - \frac{4a}{3} $$

The slope is $m = 3$, regardless of the value of 'a.' Since parallel lines share the same slope, any line parallel to our given line must also have slope 3.

Step 2: Use point-slope form to write the new line

Point-slope form:

$$ y - y_1 = m (x - x_1) $$

Plugging in (4,1) and $m=3$:

$$ y - 1 = 3(x - 4) $$

Simplify:

$$ y - 1 = 3x - 12 $$

$$ y = 3x - 11 $$

Result:

The equation of the line passing through (4,1) and parallel to $9x - 3y = 4a$ is:

$$ y = 3x - 11 $$

This line is fixed and does not depend on 'a', reflecting the fact that all lines in the family are parallel with slope 3.

Scenario 2: Line Passing Through (4,1) and Intersecting the Given Family

Suppose we seek the equation of a line passing through (4,1) that intersects the family of lines $9x - 3y = 4a$ at some point.

Approach:

1. Write the general form of the line passing through (4,1):

$$ y - 1 = m (x - 4) $$
$$ y = m(x - 4) + 1 $$

2. Find the intersection point with the family of lines:

$$ 9x - 3y = 4a $$

Substitute y from the line:

$$ 9x - 3 [m(x - 4) + 1] = 4a $$

Simplify:

$$ 9x - 3m(x - 4) - 3 = 4a $$

$$ 9x - 3m x + 12m - 3 = 4a $$

Expressed as:

$$ (9 - 3m) x + (12m - 3) = 4a $$

Since 'a' can vary, the right side can take any value depending on the intersection point. For a specific intersection point, the resulting 'a' is determined by:

$$ a = \frac{(9 - 3m) x + (12m - 3)}{4} $$

This demonstrates the relationship between the parameter 'a' and the slope 'm' of the new line, as well as the point of intersection.

Scenario 3: Constructing a Specific Line Based on the Parameter 'a'

If the goal is to find a particular line passing through (4,1) that is related to the family by a specific value of 'a', then:

- Substitute x=4, y=1 into the original line:

$$ 9(4) - 3(1) = 4a $$

$$ 36 - 3 = 4a $$

$$ 33 = 4a $$

$$ a = \frac{33}{4} $$

Thus, the line corresponding to $a= \frac{33}{4}$ passes through point (4,1). The equation of this line is:

$$ 9x - 3y = 4 \times \frac{33}{4} = 33 $$

which simplifies to:

$$ 9x - 3y = 33 $$

or dividing through by 3:

$$ 3x - y = 11 $$

This line passes through (4,1), as verified:

$$ 3(4) - 1 = 12 - 1 = 11 $$

which confirms the point lies on this particular line.

General Approach to Deriving the Equation of a Line

The process of formulating the equation of a line through a given point and related to a family of lines involves:

1. Identifying the slope:
For parallel lines, the slope is constant. For other relationships, the slope may vary.

2. Using the point-slope form:
$y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point through which the line passes.

3. Expressing the line's equation in standard or slope-intercept form:
To analyze intersections, parallelism, or perpendicularity.

4. Incorporating parameters:
When lines depend on parameters like 'a', understanding how the parameter influences the line's position.

Practical Applications and Broader Implications

Understanding how to derive such equations has widespread applications:

- Engineering Design:
Designing components that must align or intersect specific points and lines.

- Computer Graphics:
Rendering lines and understanding their relationships in digital space.

- Navigation and GPS:
Calculating routes and intersections based on coordinate data.

- Mathematical Modeling:
Developing models where relationships between various linear components are crucial.

In education, mastering these techniques enhances problem-solving skills and deepens comprehension of linear algebra and analytical geometry.

Final Thoughts

The problem of deriving a line's equation given a point and a related line encapsulates core principles of coordinate geometry. By understanding the roles of slopes, intercepts, and parameters, students and professionals can craft precise equations that describe spatial relationships. Whether constructing a parallel line through a designated point or analyzing intersections within a family of lines, these techniques form the backbone

Consider The Following Point And Line Point Line 4 1 9x 3y 4 A Write The Equation For The Line

Consider The Following Point And Line Point Line 4 1 9x 3y 4 A Write The Equation For The Line

Related Articles

- Assume That Starting Next Year, You Will Receive Payments Of $447 Every Year For 8 Years. The Discount
- Describe The Main Assumptions Of Perfect Competition AndmonopolyDiscuss The Main Features Of Oligopoly
- Carambola De Honduras. Slinger Wayne, A U.S.-based Private Equity Firm, Is Trying To Determine What It

Back to Home